OVERMODED HIGH-POWER RF MAGNETIC SWITCHES AND CIRCULATORS*

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Abstract
We present design methodology for active rf magnetic components which are suitable for pulse compression systems of future X-band linear colliders. These components comprise an array of active elements arranged together so that the total electromagnetic field is reduced and the power handling capabilities are increased. The active element of choice is a magnetic material (garnet), which can be switched by changing a biasing magnetic field. A novel design allows these components to operate in the low loss circular waveguide mode TE_01. We describe the design methodology, the switching elements and circuits.

1 INTRODUCTION
Because of the requirements of the Next Linear Collider high-power rf systems, passive microwave components have developed considerably during the last few years [1,2]. The power handling capabilities of these components have increased considerably [3]. This has been achieved by increasing the size of these components with respect to the operating wavelength, i.e., by overmoding these components. Similar attempts have been made to increase the power handling capabilities of active semiconductor components. A switch made from a PIN diode array has been developed to operate in an overmoded waveguide carrying the TE_{01} mode [4]. Optically controlled semiconductor switches operating in overmoded waveguides have also been reported [5].

In this spirit we suggest geometries that allow nonreciprocal devices to operate in an overmoded waveguide structures. We present the theory for a nonreciprocal device that operates in the coaxial circular waveguide mode TE_{01}. This device has the potential of handling tens of megawatts at X-band. It could be used as a circulator or as a switch.

2 DEVICE GEOMETRY
The implementation of a circulator or a switch can be achieved using a two-port nonreciprocal network plus a 3dB hybrid and a splitter, see Fig. 1. The simplest implementation of the nonreciprocal element in an overmoded waveguide using the TE_{01} mode is shown in Fig. 2. This implementation depends on the so-called wrap-around mode converter [3]. In this system a mode converter launches the TE_{01} mode, which has both axial and radial magnetic fields. At the end of the mode converter the coaxial guide becomes narrow and only the coaxial TEM mode can propagate, see Fig. 2. A pulsed current signal could be launched from that narrow port (port-A in Fig. 2) This pulse would have only azimuthal magnetice field. This field is used to bias a piece of garnet wrapped around the center conductor of the coaxial structure. This structure has several advantages for handling high power rf signals:

1- Since we are operating with an overmoded waveguide that has a larger cross sectional area for a given wavelength, the device should be able to handle more power.
2- All rf electric field lines are parallel to the interface between the garnet material and vacuum; see the theory section below.
3- The center conductor could be used to cool the garnet material. It could be made as a tube with water flowing in the middle.

Several variations on this geometry are possible. To implement the system shown in Fig. 1 elegantly we...
suggest the geometry shown in Fig. 3. In this geometry
the splitter is realized by dividing the power between the
TE$_{01}$ and the TE$_{02}$ modes. These two modes interact
with the garnet coated section in different manners. To
implement the circulator, one could design the system to
make the phase difference between these two modes in
the forward direction differ from that of the backward
direction by $\pi$. To implement a switch, one can control
that phase shift by varying the current in the center
conductor.

![Diagram of waveguide structure](image)

Figure 3. Implementation of the overmoded
circulator/switch.

In the reminder of this paper we will analyze and
present a theory for propagating modes with no azimuthal
variation in the coaxial structure coated with a layer of
garnet.

### 3 THEORY

Consider the waveguide structure shown in Fig. 4. We
assume a DC current in the center conductor. Using
cylindrical coordinates, the magnetic permeability tensor
of the garnet material is given by:

$$
\mu_g = \mu_0 \begin{pmatrix}
\mu & 0 & -jk \\
0 & 1 & 0 \\
jk & 0 & \mu
\end{pmatrix}.
$$

Here, $\mu_0$ is the free space permeability and $\mu$ and $k$
are complex parameters that depend on the biasing current.
For a detailed derivations of this tensor see, for example,
[6]. Consider a solution for the time harmonic Maxwell’s

$$
Z_g \equiv \frac{E_\phi}{H_z} = \frac{j \omega \mu_0}{(\beta^2 - k_g^2 \mu)} \left( \delta - \frac{2(CM'(\delta/2,1,2r\gamma) + U'(\delta/2,1,2r\gamma))}{CM(\delta/2,1,2r\gamma) + U(\delta/2,1,2r\gamma)} \right);
$$

where $U(\ldots)$ and $M(\ldots)$ are the confluent
hypergeometric functions, $'$ means a derivative with
respect to the third argument, $\delta \equiv 1 + q\beta / \gamma$, and $C$
is a constant. Eq. (6) becomes valid also for the vacuum

$$
Z_0 = j \frac{\omega \mu_0}{\gamma_0} \frac{K_i(\gamma_0 r) - CI_0(\gamma_0 r)}{K_0(\gamma_0 r) + CI_0(\gamma_0 r)};
$$

region if one lets $\mu \to 1$ and $k \to 0$. The resultant

These two conditions immediately imply that $H_\varphi = 0$,
and even, when applying Eq. (1) it follows that $E_z = 0$.
Maxwell’s equations inside the garnet region reduce to the
following set of equations:

$$
\frac{\partial H_z}{\partial r} = -j (\omega E_\varphi + \beta H_\varphi), \quad (2)
$$

$$
\frac{\beta}{\omega \mu_0} E_\varphi = j k H_z - \mu H_r, \quad (3)
$$

$$
\frac{1}{\omega \mu_0} \left( \frac{E_\varphi}{r} + \frac{\partial E_\varphi}{\partial r} \right) = k H_r - j \mu H_z; \quad (4)
$$

where $\omega$ and $\beta$ are the angular frequency and
propagation constant respectively, and $\epsilon$ is the
permittivity of the garnet material. Solving Eq. (2) and (3)
for $H_r$ and $E_\varphi$ and substituting in (4) one get the
following second order deferential equation for $H_z$,

$$
r \frac{\partial^2 H_z}{\partial r^2} + \frac{\partial H_z}{\partial r} - (q\beta + r\gamma^2)H_z = 0; \quad (5)
$$

where $\gamma \equiv \sqrt{\beta^2 + k_g^2(k^2 - \mu^2)/\mu}$, $k_g \equiv \omega^2 \mu_0 \epsilon$, and $q \equiv k / \mu$. Here we make the simplifying
assumption that the permeability tensor is independent of
radius. Solving Eq. (5) while using Eqs. (2) and (3) yield
an expression for the radial impedance,

$$
Z_0 = j \frac{\omega \mu_0}{\gamma_0} \frac{K_i(\gamma_0 r) - CI_0(\gamma_0 r)}{K_0(\gamma_0 r) + CI_0(\gamma_0 r)}; \quad (7)
$$
where $K$ and $I$ are the modified Bessel functions, $\gamma_0 \equiv \sqrt{B^2 - k_0^2}$, $k_0 \equiv \omega^2 \mu_0 \varepsilon_0$, $\varepsilon_0$ is the free space permittivity, and $C$ is a constant.

The boundary conditions of the problem can now be applied by setting both $Z_g$ and $Z_0$ equal to zero at the inner and outer radii respectively. This would yield the two Constants $C$ and $\bar{C}$. Now, equating $Z_g$ and $Z_0$ at the interface between the two media, at $r = r_g$, yield the required equation for the propagation constant $\beta$.

4 RESULTS

For a proof of principle numerical experiment we considered the properties of calcium vanadium doped garnet [7]. We choose this material because of its narrow line width. The calculated values for $\mu$ and $k$ are shown in Fig. 5 and 6. The operating frequency used to generate these curves is 11.424 GHz. The dielectric constant of this material is about 14.

![Figure 5](image1.png)

Figure 5. Variations of the diagonal elements of the relative magnetic permeability tensor, $\mu$, with magnetic field.

![Figure 6](image2.png)

Figure 6. Variations of the off diagonal elements of the relative magnetic permeability tensor, $k$, with magnetic field.

The biasing magnetic field was chosen so that the material would operate below the resonance frequency.

Figure 7. Shows the required garnet thickness and length for a $\pi$ phase difference between forward and backward TE$_{01}$ mode waves. For the optimum thickness, the rf losses are less than one percent. This is a manageable losses level, which be cooled through the center conductor.

![Figure 7](image3.png)

5 CONCLUSION

We presented a novel design for circulators and switches in an overmoded waveguide. These designs increase the power handling capabilities of these devices. We also presented a theory for the design and estimated the dimensions of the nonreciprocal material and rf losses for these devices.

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7 REFERENCES