Intrinsic Landau Damping of Bunched Beams at Coupling Resonance

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Introduction

- Incoherent tune spread caused by space charge contributes to Landau damping.
- The damping mechanism caused by space charge is not completely understood.
- Numerical investigations are necessary.
- Developing tools for modal analysis in particle tracking simulations is necessary.
  - Single value decomposition is not so good (appropriate for stationary problems).
  - Dynamic mode decomposition works (provides damping/growing rates).

Investigating the Landau damping of space charge modes at coupling resonance we found a novel damping mechanism.
Synergia simulations

Synergia simulation package

- **Single-particle physics**
  - direct symplectic tracking
  - arbitrary-order polynomial maps
- **Apertures** (different shapes)
- **Collective effects** (single and multiple bunches)
  - space charge (different solvers)
  - wake fields (arbitrary wakes)

https://cdcvs.fnal.gov/redmine/projects/synergia2

Space charge modes simulation:

- 10 x OFORODO lattice
- $10^8$ macroparticles
- 3D space charge Poisson solver with open boundary conditions (Hockney)
- Gaussian beams with equal transverse emittance
- Beam initially excited with an approximate (guessed) mode shape
Dynamic Mode Decomposition

C.W. Rowley et al., Journal of Fluid Mechanics, 641, 2009
A. Macridin et al., PRSTAB, 074402, 2015

- Store $X(z, \delta, t_n)$ at every turn
  
  $X(z, \delta, t_0), X(z, \delta, t_1), ..., X(z, \delta, t_N)$

  $$X(z, \delta, t) = \int dx \; dy \; x \rho(x, y, z, \delta, t)$$

  dipole density

- DMD assumption: A linear operator $A$ exists such as:
  
  $$X(z, \delta, t_{k+1}) = A \cdot X(z, \delta, t_k) \quad \text{for all} \quad k = 1, N-1$$

- The eigenvalues and the eigenvectors of $A$ describe the dynamics
  
  $$A \phi_j(z, \delta) = \mu_j \phi_j(z, \delta)$$

  $$X(t_k) = A^k X(t_0) = \sum_j \mu_j^k \alpha_j \phi_j = \sum_j e^{-\gamma_j k} e^{-i \omega_j k} \psi_j$$

  $$X(t_0) = \sum_j \alpha_j \phi_j$$

- DMD finds the best approximation for the eigenvalues and eigenvectors of $A$ for a given set of data
Strong space charge

Analytical results, transverse modes

Good agreement between the simulation and the analytical predictions

A. Burov, PRSTAB 12, 109901, 2009.
V. Balbekov. PRSTAB12, 124402, 2009.

Simulation

Comparison
In the strong interaction regime the damping rate is proportional to $k^4/q^3$, $k$ is the mode number, $q$ is the space charge parameter (agreement with A. Burov, *PRSTAB* 12, 109901, 2009)
Mode 1 at coupling resonance

Coupling resonance, $Q_{x0} = Q_{y0}$

- Nonlinear coupling resulting from the term proportional to $x^2y^2$ in space charge potential
- Montague's resonance, $2Q_x - 2Q_y = 0$

2 times larger damping at CR in strong space charge regime.

Why?
Landau damping mechanism

\[ \ddot{x}_i + \omega_0^2 \left( Q_{0x} - \delta Q_i \right)^2 x_i = -2 \omega_0^2 Q_{0x} \delta Q_i \bar{x} \]

- The mode energy is transferred to the resonant particles.
- The resonant particles tune equal the coherent tune \( Q_c \).
Landau damping off-resonance

\[ \ddot{x} + \omega_0^2 (Q_{0x} - \delta Q)^2 x = -2 \omega_0^2 Q_{0x} \delta Q \bar{x}, \quad \delta Q(z, J_x, J_y), \quad \bar{x} \propto e^{i \omega_0 (\nu - Q_s) t} \]

- Conventional LD mechanism.
- The tune of the LD responsible particles is at the coherent tune.
- LD responsible particles increase their energy with time
Landau damping at coupling resonance, $Q_{x0} = Q_{y0}$

- The tune of most of the LD responsible particles is not in the vicinity of coherent tune. **Why do these particles absorb the mode's energy?**

- The picture does not fit the conventional LD paradigm.
Particles dynamics at coupling resonance

- Montague’s resonance, simple model for round beams
  \[ H_{sc} = \alpha (x^2 + y^2)^2 \]

- Stable point: \( \cos 2(\Phi_x - \Phi_y) = -1 \), \( J^* \mu Q_x - Q_y \)

- Trapped particles properties:
  - \( J_s = J_x + J_y \), constant of motion
  - \( J_d = J_x - J_y \), oscillates around the stabled point
  - the trapping frequency is particle dependent
    \[ Q_t \propto \alpha \sqrt{J_s - J^*_d} \]

Synergia simulations

Poincaré plots, \( J_d \text{ vs } \Phi_x - \Phi_y \)

- Most of the LD responsible particles are trapped in resonance islands.
- Their actions oscillates with particle dependent frequency.
Parametric Landau damping

\[ \ddot{x}_i + \omega_0^2 (Q_0 - \delta Q_i)^2 x_i = -2 \omega_0^2 Q_0 \delta Q_i (z_i, J_{si}, J_{di}) \bar{x} \]

mode-particle coupling

\[ \ddot{x}_i + \omega_0^2 Q(z, J_{si}, J_{di})^2 x_i = -A(z, J_{si}) \bar{x} - B(z, J_{si}) J_{di} \bar{x}, \quad \bar{x} \propto e^{i \omega_0 (\nu - Q_s) t}, \quad J_{di} \propto e^{i \omega_0 Q_{ti} t} \]

resonance condition:

- **Ax coupling** \( Q_i = \nu - Q_s \)
  - conventional LD
- **BJ_dx coupling** \( Q_i + Q_{ti} = \nu - Q_s \)
  - parametric LD
  - mode-particle coupling modulated by \( Q_t \)
  - the tune of the LD resonant particles is not at the coherent tune

\[ \rho(Q) = \sum_i \delta(Q_i - Q) \quad h(Q) = \sum_i \delta(Q_i + Q_{ti} - Q) \]
Conclusions

- We employed Synergia and DMD techniques to analyze the transverse space charge modes in bunched beams.

- The simulations reveal a novel Landau damping mechanism driven by the modulation of mode-particle interaction.

- The amplitude oscillations of the trapped particles at the coupling resonance enhance the Landau damping rate in bunched beams.