Simulated Measurements of Beam Cooling in Muon Ionization Cooling Experiment
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Abstract

The international Muon Ionization Cooling Experiment (MICE) aims to demonstrate ionization beam cooling:
- Muon beam is passed through an absorbing material to reduce its phase-space volume (emittance).
- Why cooled muon beams:
  - Neutrino Factory: for intense and pure neutrino beams.
  - Muon Colliders: for compact lepton colliders with energies of up to several TeV.
- The figure of merit for cooling: root-mean-square (RMS) emittance reduction.

Introduction

- How MICE demonstrates beam cooling:
  - Ensure muon beam purity using PID detectors (time-of-flight, Cherenkov, electron muon range).
  - Reconstruct muon transverse coordinates \(X_i = (x_i, p_{x_i}, y_i, p_{y_i})\) using the trackers.
  - Compute RMS emittance from transverse coordinates.
- BUT a different measure of cooling is needed because of the sensitivity of the RMS emittance to non-linear effects.

Kernel Density Estimation in MICE

- Kernel Density Estimation (KDE) technique: Well known in image processing.
- No assumptions are made about the distribution.
- How MICE demonstrates beam cooling using KDE:
  - Center a four dimensional Gaussian kernel function (weighting function shaped as multi-dimensional ellipse of variance \(h = h_f \sum\)) at each muon.
  - Estimate the density at an arbitrary point \(x = (x, p_x, y, p_y)\) by summing the contributions from all muons.
  \[ f(x) = \frac{1}{n} \sum_{i=1}^{N} \exp \left( -\frac{1}{2h^2} (x - X_i)^T \Sigma^{-1} (x - X_i) \right) \]
- \(h\) and \(h_f\) are the bandwidth factor and parameter. \(\Sigma\) is the covariance matrix of the muon coordinates.
- \(h\) has a strong effect on the estimated density. Scott's rule of thumb was used here, \(h = \frac{3M^{-1/5}}{n^{2/9}}\).

Bandwidth Factor Effect

- Estimated density vs. \(x\) position plot for 500 muons.
- Scott's rule of thumb bandwidth parameter multiplied by a large factor oversmoothes the density.
- A smaller factor leads to a noisier density.

Simulation Results

- Preliminary density, volume and emittance evolution plots in the MICE Step IV channel:
  - The yellow curves represent a channel with no absorber.
  - The blue curves represent a channel with a 65 mm LiH absorber.
  - The evolution curve remains constant for an empty channel except at \(z=1.5\) m due to the turned off downstream Match 1 and Match 2 coils.

Conclusion

- Studied a MICE Step IV lattice with Match 2 and the in-operative Match 1 coil fields set to zero in the downstream Spectrometer Solenoids.
- Demonstrated cooling through phase-space density increase and phase-space volume decrease using KDE.

MICE

- The current configuration: MICE Step IV.

Before and after MICE photos: the cooling channel (left, 2015) enclosed by the partial return yoke (PRY) (right, 2016).

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