COMPARISON OF NONLINEAR DYNAMICS OPTIMIZATION METHODS FOR APS-U

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Abstract

Many different objectives and genetic algorithms have been proposed for storage ring nonlinear dynamics performance optimization. These optimization objectives include nonlinear chromaticities and driving/detuning terms, on-momentum and off-momentum dynamic acceptance, chromatic detuning, local momentum acceptance, variation of transverse invariant, Touschek lifetime, etc. In this paper, the effectiveness of several different optimization methods and objectives are compared for the nonlinear beam dynamics optimization of the Advanced Photon Source upgrade (APS-U) lattice. The optimized solutions from these different methods are preliminarily compared in terms of the dynamic acceptance, local momentum acceptance, chromatic detuning, and other performance measures.

OVERVIEW

Next-generation storage ring light sources, such as the Advanced Photon Source (APS) Multi-Bend Achromat (MBA) upgrade, will improve the x-ray brightness by roughly two orders of magnitude. For APS upgrade (APS-U), the equilibrium emittance is pushed from the current 3 nm to below 100 pm. To achieve this low emittance, the original double-bend achronomat lattice is replaced by a hybrid seven-bend-achromat (H7BA) lattice [1], where seven bending magnets with either transverse or longitudinal gradients, plus strong quadrupole focusing, are employed in each of the 40 arc cells. The strong nonlinearities in the chromatic sextupoles make it hard to achieve large dynamic acceptance (DA) and long Touschek lifetime, even for the on-axis swap-out injection scheme [2].

A direct-tracking-based multi-objective genetic algorithm (MOGA) [3–6] is employed to vary the linear optics and optimize the nonlinear elements (typically sextupoles, but also octupole magnets) for better beam dynamics performance. The optimization objectives include: DA; Touschek lifetime computed from local momentum acceptance (LMA); and the desired positive chromaticity for high bunch charge mode. The algorithm can include realistic errors and find robust solutions. The disadvantage is that the LMA simulation takes significant computing time.

In this paper, several alternate optimization objectives are explored which may be faster and yet provide good nonlinear optics solutions. These optimization objectives include: analytically calculated nonlinear chromaticities and driving/detuning terms; on-momentum and off-momentum dynamic acceptance [7, 8]; and minimization of variation of the Courant-Snyder invariant [9] [10, 11]. Consideration of the chromatic detuning is also included in most of these cases. In the following sections, details are presented on the applications of these nonlinear optics optimization methods.

OPTIMIZATION KNOBS

For all the nonlinear optics optimizations, a same APS-U H7BA lattice is employed which emittance is 41 pm with reverse bending fields [12]. The TWISS parameters are shown in Figure 1 for one sector. The nominal betatron tunes are (95.1, 36.1) and the linear chromaticities are all corrected to be (5, 5) for the high bunch charge mode. For the hybrid achronomat lattice scheme [1], there are 3 pairs of sextupole magnets in each sector, with betatron phase advance of $\Delta \nu_x = 3\pi$ and $\Delta \nu_y = \pi$ between each pair (designed to cancel geometric aberrations [1]). Usually a two-sector translational symmetry is adopted, giving a maximum of 12 families of sextupoles. Octupole fields may be integrated in the 8-pole fast corrector magnets. The algorithms are allowed to vary up to 10 families of sextupole magnets, with two families reserved for the linear chromaticity, plus up to 4 families of octupole magnets. Simulation is performed with ELEGANT [13].

OBJECTIVE: DA AND LMA

The nominal optimization method is a direct-tracking-based MOGA [5, 6]. It is employed to directly optimize the Touschek lifetime (through local momentum acceptance) and the injection efficiency (through dynamic acceptance with physical apertures). Recently chromatic detuning from direct tracking is also included as another optimization objective. The optimized solutions are robust in ensemble evaluations after commissioning simulation [2]. The disadvantage is that the local momentum acceptance takes a long time to compute. A MOGA optimization process is

Figure 1: Twiss parameters in one sector of a 41-pm, H7BA lattice design for APS-U. Blue blocks represent quadrupoles, red blocks represent dipoles, and green blocks represent sextupoles.
Figure 2: MOGA [5, 6] progress showing DA area (top), chromatic detuning (middle) and Touschek lifetime (bottom). Red dots denote the initial condition from a previous optimization. Green dots denote the best seeds.

shown in Figure 2, where the DA area, chromatic detuning and Touschek lifetime are all improved.

**OBJECTIVE: CHROMATICITY AND DRIVING/DETUNING TERMS**

The nonlinear elements (sextupole and octupole magnets) drive transverse amplitude detuning terms, betatron resonance terms, and nonlinear chromaticities. A direct minimization of these analytically calculated terms [14] (nonlinear chromaticities and driving/detuning terms) may improve the nonlinear dynamics performance. Here the second- and third-order chromaticities, plus the transverse amplitude detuning terms are employed as the optimization objectives. The calculation is done with ELEGANT [13], and an external optimizer is used to pick the best seed and feed next generation. The optimization results after 20 iterations (50 seeds for each iteration) are listed in Table 1. This method is fairly fast. The disadvantage is that the analytically calculated terms may not be accurate, and the higher order terms, such as \( d^2v_x/dJ_x^2 \), may be hard to include.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Target</th>
<th>Initial</th>
<th>Final</th>
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</thead>
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<tr>
<td>( d^2v_x/dp^2 )</td>
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<td>1043</td>
<td>261</td>
</tr>
<tr>
<td>( d^2v_y/dp^2 )</td>
<td>250</td>
<td>526</td>
<td>255</td>
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<td>( 0.99\times 10^5 )</td>
</tr>
<tr>
<td>( d^2v_y/dJ_y )</td>
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<td>( -2.66\times 10^4 )</td>
<td>( 5\times 10^4 )</td>
</tr>
<tr>
<td>( d^2v_y/dJ_y )</td>
<td>( 1\times 10^5 )</td>
<td>( -2.52\times 10^4 )</td>
<td>( 0.96\times 10^5 )</td>
</tr>
</tbody>
</table>

**OBJECTIVE: CS INVARIANT [10, 11] [9]**

For a system without nonlinear fields (with only dipole and quadrupole fields), the Courant-Snyder invariant, \( A^2 = 2\alpha xx' + \beta (x')^2 + \gamma x^2 \), is a constant for a given transverse amplitude. In this case, when tracking for many turns in a storage ring, the CS invariant does not change. However, in the presence of the nonlinear elements (sextupole and octupole magnets), the CS invariant changes [10, 11]. Y. Li and L. Yu et al. [9] proposed to use minimization of the variation of CS invariant for a set of particles initially on an ellipse in phase space (same CS invariant). For this method [9], only one super-period or at most one turn is needed for tracking, so it is fast. Here the original proposed method [9] was modified to include 100-200 particles that cover the whole x-y space of interest. In addition, chromatic detuning from direct tracking is included as an optimization objective.

**OBJECTIVE: ON- AND OFF-M DA**

On-momentum and off-momentum DA (or dynamic aperture, without physical apertures) [7, 8] are also used as optimization objectives in genetic algorithms. Off-momentum DA and chromatic detuning may be used to indirectly optimize the local momentum acceptance and thus, one hopes, the Touschek lifetime [7, 8]. A MOGA optimization process is shown in Figure 3, where the on- and off-momentum DA, and the chromatic detuning are all improved.

**COMPARISON OF PERFORMANCE**

The optimized solutions from these 4 different methods are preliminarily compared in terms of the dynamic accep-
tance, local momentum acceptance, chromatic detuning, and other performance measures. The following notation is used: **LMA**: objective of dynamic acceptance, chromatic detuning and local momentum acceptance; **ANA**: objective of nonlinear chromaticity and driving/detuning terms; **CSI**: objective of CS invariant and chromatic detuning; **DA**: objective of on- and off-momentum dynamic acceptance, and chromatic detuning.

**Figure 5**: Comparison of dynamic acceptance without errors (all observed at ID center). Real physical apertures with narrow IDs are included.

**Figure 6**: Comparison of LMA in two sectors without errors. Real physical apertures with narrow IDs are included. RF bucket height is ±4%.

The linear chromaticities are corrected to be (5, 5) in all cases and the RF bucket height is ±4%. Tracking simulations to validate the optimized results are performed for around two synchrotron periods. Real physical apertures are also included for these methods, in particular, the nominal physical apertures (half aperture of 10 mm by 3 mm at all IDs), and narrow ID apertures of 4 mm by 3 mm (half) at some IDs.

As shown in Figure 4, the chromatic tune shifts are all well optimized, especially in the vertical plane. The dynamic acceptance and local momentum acceptances without errors are shown in Figs. 5 and 6, where the direct tracking based methods seem to achieve better results than the analytical approach (case ANA). The frequency map in x-y plane, as shown in Figure 7, illustrates similar resonance properties between the cases of LMA and DA. Case ANA seems to have smallest diffusion rates.

**Figure 7**: Comparison of frequency map in x-y plane without errors. Real physical apertures with narrow IDs are included. Left top: LMA; right top: ANA; left bottom: CSI; right bottom: DA.

**CONCLUSIONS**

The effectiveness of 4 different optimization methods and objectives are compared for the nonlinear beam dynamics optimization of the Advanced Photon Source upgrade (APS-U) lattice. Preliminary comparisons of the optimized solutions from these 4 different methods show similar nonlinear beam dynamics performance. These optimized solutions will be evaluated with realistic errors/corrections and ensemble evaluations, which may reveal strengths or weaknesses of the various methods.

**ACKNOWLEDGMENT**

The authors would like to thank ESRF-II design team for their original work on the HMBA lattice scheme; APS upgrade design group for helpful discussions; M. Ehrlichman for triggering our interest in the DA method; and Y. Li for introducing us to the CSI method.

**REFERENCES**

