HYBRID METHODS FOR SIMULATION OF MUON IONIZATION COOLING CHANNELS∗

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Abstract

COSY Infinity is an arbitrary-order beam dynamics simulation and analysis code. It uses high-order transfer maps of combinations of particle optical elements of arbitrary field configurations. New features have been developed and implemented in COSY to follow charged particles through matter. To study in detail the properties of muons passing through a material, the transfer map approach alone is not sufficient. The interplay of beam optics and atomic processes must be studied by a hybrid transfer map–Monte Carlo approach in which transfer map methods describe the average behavior of the particles including energy loss, and Monte Carlo methods are used to provide small corrections to the predictions of the transfer map, accounting for the stochastic nature of scattering and straggling of particles. This way the vast majority of the dynamics is represented by fast application of the high-order transfer map of an entire element and accumulated stochastic effects. The gains in speed simplify the optimization of muon cooling channels which are usually very computationally demanding. Progress on the development of the required algorithms is reported.

INTRODUCTION

A prime example of why matter-dominated lattices are relevant comes from the prospect of a neutrino factory or a muon collider [1]. As muon branching fractions are 100% $\mu^- \rightarrow e^-\bar{\nu}_e\nu_\mu$ and $\mu^+ \rightarrow e^+\nu_e\bar{\nu}_\mu$, there are obvious advantages of a muon-sourced neutrino beam. Also, because muons are roughly 200 times heavier than electrons, synchrotron radiation is not an issue. As a result, a high-energy muon collider ($\sqrt{s} \approx 6$ TeV) could be built on a compact site. Such energy levels are experimentally unprecedented in the leptonic sector, since a circular electron accelerator is restricted by synchrotron radiation. At lower energy, a muon collider could serve as a Higgs Factory ($\sqrt{s} \approx 126$ GeV), with possible new physics via the observation of Higgs to lepton coupling. This is advantageous since the Higgs coupling to leptons scales as mass squared.

However, muon-based facilities are not without their challenges. Synthetic muon creation comes from the collision of protons with a fixed target. The resultant spray of particles is evaluated using differential algebra. Along with tracking of particles through a lattice, COSY includes spectrographs, beam lines, electron microscopes, and other such devices, with its use in accelerator lattice design being of particular interest here. COSY uses the transfer map approach, in which the overall effect of the optics on a beam of particles is evaluated using differential algebra. Along with tracking of particles through a lattice, COSY includes many analysis and optimization tools.

COSY INFINITY

COSY Infinity (COSY) [3] is a simulation tool used in the design, analysis, and optimization of particle accelerators, spectographs, beam lines, electron microscopes, and other such devices, with its use in accelerator lattice design being of particular interest here. COSY uses the transfer map approach, in which the overall effect of the optics on a beam of particles is evaluated using differential algebra. Along with tracking of particles through a lattice, COSY includes many analysis and optimization tools.

Currently supported elements in COSY include various magnetic and electric multipoles (with fringe effects), homogeneous and inhomogeneous bending elements, Wien filters, wigglers and undulators, cavities, cylindrical electromagnetic lenses, general particle optical elements, and deterministic absorbers of intricate shapes described by polynomials of arbitrary order, with the last element being of particular interest for this study. This element only takes into
account deterministic effects (producing the same final result every time for a given initial condition), not stochastic effects (intrinsically random effects such as multiple scattering and energy straggling).

To take into account stochastic effects, the transfer map paradigm needs to be augmented by implementing the corrections from stochastic effects directly into the fabric of COSY.

**STOCHASTIC PROCESSES**

The stochastic processes of interest are straggling (fluctuation about a mean energy loss) and multiple angular scattering. The general outline to simulate these two beam properties is discussed more thoroughly in [4]. Straggling follows Landau theory and has the form [5]

\[
 f(\lambda) = \frac{1}{\xi} \cdot \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \exp(x \ln x + \lambda x)dx, \tag{1}
\]

where \( \xi = ZpL/\beta^2 A \), and \( \lambda \sim dE/\xi - \beta^2 - \ln \xi \). Here \( Z, A, \) and \( \rho \) are the atomic charge, atomic mass, and density of the material; \( L \) is the amount of material that the particle traverses; \( \beta = v/c; \) and \( dE \) is the fluctuation about the mean energy. The algorithm based on Eq. (1) has been implemented in COSY.

The derivation of the scattering function \( g(u) \) (where \( u = \cos \theta \)) is done separately for small angles and large angles. For small angles, the shape is very nearly Gaussian in \( \theta \) [6]. For large angles, the distribution follows the Mott scattering cross section and is Rutherford-like [7]. The resulting peak and tail are continuous and smooth at some critical \( u_0 \), which yields the final form of \( g(u) \):

\[
 g(u) = \begin{cases} 
 \exp\left(-\frac{1}{2} \frac{1-u}{1-u_{tr}}\right) & \text{if } u_0 < u \\
 \xi \cdot \frac{1}{(1-u+b)^2} & \text{if } u \leq u_0 
\end{cases} \tag{2}
\]

Here the parameters \( \xi \) and \( b \) are chosen to ensure continuity and smoothness, \( \gamma = 1/\sqrt{1 - \beta^2} \), \( u_0 \) is a fitted parameter chosen as \( u_0 = 9u_{tr} - 8 \), \( u_{tr} \) is the \( \sigma \)-like term for a Gaussian in \( \theta \). It is another fitted parameter based off [8] and taking the form

\[
 u_{tr} = \cos \left( \frac{13.6 \text{ MeV}}{\beta pc} \cdot \frac{L}{L_0} \left( 1 + 0.103 \ln \frac{L}{L_0} \right) + 0.0038 \left( \ln \frac{L}{L_0} \right)^{3/2} \right). 
\]

In [4], the hybrid method presented here was benchmarked against two other beamline simulation codes, ICOOL [9] and G4Beamline [10], and (in the case of angular scattering) against experimental data obtained by MuScat [11].

**THE MUON IONIZATION COOLING EXPERIMENT**

The Muon Ionization Cooling Experiment (MICE [12]) is an experiment currently being developed at the Rutherford Appleton Laboratory in Oxfordshire, U.K. Its goal is to show a proof-of-principle demonstration of muon ionization cooling. MICE Step IV configuration is explored in this work. The Step IV cell includes 12 magnetic coils positioned symmetrically around a flat absorber. Figure 1 shows a schematic of this lattice with 350 mm of liquid hydrogen as the absorber.

![Figure 1: MICE Step IV cell. Magnetic coils are shown in yellow and the absorber is shown in blue. The green and blue axes are the \( y \) and \( z \) axes, here drawn to scale as 500 mm each. The aperture (invisible for display purposes) is 300 mm. Image rendered via G4Beamline.](image)

10⁶ muons were simulated through the cell shown in Figure 1. The coil parameters may be found in Table 1. The absorber was a 350 mm cylindrical block of liquid hydrogen centered at \( z = 0 \). The aperture was set to 300 mm. Note that other materials such as safety windows were not accounted for in this simulation. The decay process was disabled in all simulation codes. The beam started at \(-2.45 \text{ m} \) and ended at \(2.45 \text{ m} \).

<table>
<thead>
<tr>
<th>Name</th>
<th>z position</th>
<th>Length</th>
<th>Inner radius</th>
<th>Outer radius</th>
<th>Current density A/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>End2</td>
<td>±3200</td>
<td>111</td>
<td>258</td>
<td>326</td>
<td>±126</td>
</tr>
<tr>
<td>Center</td>
<td>±2450</td>
<td>1314</td>
<td>258</td>
<td>280</td>
<td>±148</td>
</tr>
<tr>
<td>End1</td>
<td>±1700</td>
<td>111</td>
<td>258</td>
<td>319</td>
<td>±133</td>
</tr>
<tr>
<td>Match2</td>
<td>±1300</td>
<td>199</td>
<td>258</td>
<td>289</td>
<td>±132</td>
</tr>
<tr>
<td>Match1</td>
<td>±861</td>
<td>201</td>
<td>258</td>
<td>304</td>
<td>±133</td>
</tr>
<tr>
<td>Focus</td>
<td>±202</td>
<td>213</td>
<td>268</td>
<td>362</td>
<td>±104</td>
</tr>
</tbody>
</table>

Precalculated field maps were used by both G4Beamline (which used the `f1681dmap` command) and ICOOL (which used the GRID command operating in G43D mode).

The runtimes of ICOOL, G4Beamline, and COSY are listed in Table 2. Note that the initialization time for
Table 2: Run Times (in seconds) for MICE Step IV Simulation

<table>
<thead>
<tr>
<th>Number of particles:</th>
<th>10^6</th>
<th>10^5</th>
<th>10^4</th>
<th>10^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>COSY:</td>
<td>367</td>
<td>31</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>G4BL (coils):</td>
<td>3973</td>
<td>392</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>G4BL (field map):</td>
<td>662</td>
<td>75</td>
<td>15</td>
<td>9</td>
</tr>
<tr>
<td>ICOOL (field map):</td>
<td>1091</td>
<td>117</td>
<td>19</td>
<td>9</td>
</tr>
</tbody>
</table>

Note that the G4Beamline initialization time was not added to the run time values. G4BL (coils) represents the simulation in G4Beamline when the coil parameter was used. G4BL (field map) represents the simulation when G4Beamline (like ICOOL) read the field map from a file.

G4Beamline to create the field maps was 33 seconds. The time it took to create a text file for ICOOL input was 11 seconds. Since G4Beamline only has to create the field map once, the initialization time is added to neither ICOOL nor G4Beamline the run times in Table 2. COSY did not have any initialization time.

As a second test, MICE configuration in Figure 1 was simulated using 65 mm of lithium hydride. Lithium hydride is an attractive material because, unlike liquid hydrogen, it does not require cryogenic conditions, but still maintains a low Z value. It can be seen from Figure 3 that 65 mm of lithium hydride has a similar effect on the beam as 350 mm of liquid hydrogen.

**SUMMARY**

The addition of stochastic processes in COSY Infinity for the use of muon ionization cooling has been successful. Using MICE as a case study, Figures 2 and 3 show agreement within 1%. The hybrid method implemented into COSY works well for both liquid hydrogen and lithium hydride. Furthermore, Table 2 shows that COSY uses roughly half the computational time as G4Beamline and a third of the computational time of ICOOL.

**REFERENCES**


