ANALYSIS OF THE TRANSPORT OF MUON POLARIZATION FOR THE FERMILAB G-2 MUON EXPERIMENT*
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Abstract
The Muon g-2 Experiment at Fermilab aims to measure the anomalous magnetic moment of the muon to a precision of 140 ppb — a fourfold improvement over the 540 ppb precision obtained in BNL experiment E821. Obtaining this precision requires controlling total systematic errors at the 100 ppb level. One form of systematic error in the measurement of the anomalous magnetic moment occurs when the muon beam injected and stored in the ring has a correlation between the muon's spin direction and its momentum. In this paper, we first analyze the creation and transport of muon polarization from the production target to the g-2 storage ring. Then, we detail the spin-momentum correlations and their evolution at various beamline positions. Finally, we outline mitigation strategies that could potentially circumvent this problem.

INTRODUCTION
The Muon g-2 Experiment, at Fermilab [1], will measure the muon anomalous magnetic moment, \( g-2 \), to unprecedented precision: 0.14 parts per million. This four-fold improvement in experimental precision compared to Brookhaven’s experiment [2], could establish beyond a doubt a signal for new physics if the central value of the measurement remains unchanged. To perform the experiment, a polarized beam of positive muons is injected into a storage ring with a uniform magnetic field in the vertical direction. Since the positron direction from the weak muon decay is correlated with the spin of the muon, the precession frequency is measured by counting the rate of positrons above an energy threshold versus time. The g-2 value is then proportional to the precession frequency divided by the magnetic field of the storage ring.

Most of the new experiment improvements will be based on increased statistics. Therefore, achieving the targeted precision requires optimum transmission of polarized muons within the g-2 storage ring acceptance. The goal of this paper is to develop a detailed simulation model for the Fermilab Muon g-2 experiment. In particular, in this study, we analyze numerically and theoretically the creation and transport of muon polarization from the production target up to the entrance of the g-2 storage ring.

MUON CAMPUS OVERVIEW
Protons with 8 GeV kinetic energy are transported via the M1 beamline to an Inconel target at AP0. Within a 1.33 s cycle length, 16 pulses with \( 10^{12} \) protons and 120 ns full length, are arriving at the target. Secondary beam from the target will be collected using a lithium lens, and positively-charged particles with a momentum of 3.1 GeV/c (± 10%) will be selected using a bending magnet. Secondary beam leaving the Target Station will travel through the M2 and M3 lines which are designed to capture as many muons with momentum 3.094 GeV/c from pion decay as possible. The beam will then be injected into the Delivery Ring (DR). After several revolutions around the DR, essentially all of the pions will have decayed into muons, and the muons will have separated in time from the heavier protons. A kicker will then be used to abort the protons, and the muon beam will be extracted into the new M4 line, and finally into the new M5 beamline which leads to the (g-2) storage ring. Note that the M3 line, DR, and M4 line are also designed to be used for 8 GeV proton transport by the Mu2e experiment.

TRANSPORT OF POLARIZATION
The primary purpose of the DR [4] is to allow enough time for all pions to convert into muons. The DR is a rounded 505 m long triangle and is divided into 6 sectors numbered 10-60. Each sector contains 19 quadrupoles and 11 dipoles. Other magnetic devices include correction dipoles and sextupoles. There are three straight sections – 10, 30, and 50, which are located directly beneath service buildings AP10, 30, and 50 respectively. The straight sec-

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* Operated by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the United States Department of Energy.
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tions are regions of low dispersion while the arcs are dispersive regions. A typical cell in the arcs is comprised of an F-quadrupole with similarly oriented sextupoles on either side followed by a dipole or drift region, then a D-quadrupole also surrounded by sextupoles of the same convention and another dipole or drift region.

A detailed simulation model using the tracking code G4Beamline [5] has been developed in order to evaluate the performance and track the beam polarization. Figure 2 illustrates the number of pions, muons and protons after each turn in the DR. Note that after the second turn the beam consists of muons and protons, with the proton intensity being greater by a factor of 100. Current studies [1,3] suggest that at least four turns are required so that protons can be safely removed, via an extraction kicker, without any muon losses.

Figure 2: Overall performance versus turn number as the beam loops the DR. Protons are two orders of magnitude more than muons.

Figure 3 shows the muon polarization when the beam enters the DR (Fig. 3(a)) and after the second turn (Fig. 3(b)). When the beam enters the ring the beam is 96% polarized in the longitudinal direction. This is a direct result of the lattice acceptance, which accepts only forward decayed pions. While in the DR, the spin precesses in the horizontal plane due to the vertical magnetic field. As a result, the polarization will be split in the horizontal and longitudinal direction. When the beam exits the DR (Fig. 4(b)), the net polarization remains 96% and is almost equally split between x and z directions (i.e. Px= -0.71 and Pz= -0.65).

Systematic effects on the measurement of \(a_\mu\) occur when the beam has a correlation between the muon spin direction and its momentum [1,3]. Due to this anomaly, the muon spin precesses relative to muon momentum, in the DR, by an angle:
\[ \varphi_a = 2\pi N\gamma \alpha_\mu, \]  
(1)

where \(N\) is the turn number and \(\gamma\) is the relativistic factor. The slope of the spin-momentum correlation is then:

\[ \frac{d\varphi_a}{dp} = \frac{2\pi N\alpha_\mu}{m_\mu \beta c}, \]

(2)

where \(m_\mu\) is the muon rest mass, \(c\) is the speed of light and \(\beta\) is the speed in units of \(c\). The correlation between spin precession angle and muon momentum versus the turn number is illustrated in Fig. 5 and Fig. 6. The red curve is a linear fit of the data. Clearly, as the turn number increases the slope becomes steeper, a result that would have been anticipated directly from Eq. (2).

Figure 5: Spin-momentum correlations after the second turn (a) and third turn (b). Red curve is a fit to the simulation results.

Quantitatively, we estimate the degree of correlation from the slope of the linear fit and compare it to values predicted by the theory in Eq. (2). Our results are displayed in Table 1. Simulated and theoretical values are in a reasonable agreement.

Table 1: Comparison between Theoretical and Numerical Results for Different Turns (T) along the DR

<table>
<thead>
<tr>
<th>Parameter</th>
<th>T2</th>
<th>T3</th>
<th>T4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi_a) [rad]</td>
<td>0.43</td>
<td>0.64</td>
<td>0.86</td>
</tr>
<tr>
<td>(d\varphi_a/dp) [mrad/MeV/c]</td>
<td>0.14</td>
<td>0.21</td>
<td>0.28</td>
</tr>
<tr>
<td>(\varphi_a) (sim)</td>
<td>0.41</td>
<td>0.62</td>
<td>0.83</td>
</tr>
<tr>
<td>(d\varphi_a/dp) (sim)</td>
<td>0.18</td>
<td>0.20</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Figure 6: Spin-momentum correlation after the fourth turn (a) and fifth turn (b). Red line is a linear fit to the simulation data. Note that in the current g-2 commissioning scenario the beam exits the DR after four turns.

FUTURE WORK

A fast kicker system (rise time <180 ns) could allow proton removal at fewer turns and thus substantially improve the systematics. The fast system will also play a key role in improving the transport of the Mu2e Experiment since it will reduce machine activation risks associated with magnet or steering errors as it will allow early detection and beam removal.

ACKNOWLEDGMENT

Thanks to B. Drendel, M. Korostelev, and V. Tishchenko for many fruitful discussions.

REFERENCES