A MODELING STUDY OF THE FOUR-ROD RFQ

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 Summary

The four-rod RFQ structure, under investigation at Frankfurt, is here studied as a candidate for a large-bore high-power cw proton accelerator. The basic single module configuration was considered theoretically using a combined distributed-lumped equivalent circuit model to predict voltage distributions, circuit $Q$, shunt impedance and power required for operating conditions. Results have been calculated for a 108 MHz unit with 35 mm bore diameter and an outer rf tank diameter of 1 metre. A water cooled copper structure of this configuration with 90% of theoretical $Q$ should develop a vane-to-vane voltage of 260 kV at an rf power of 220 kW/m. This is competitive with the four-vane structure in a high beam loading application, and suggests that mechanical characteristics such as long term stability and ease of construction and maintenance could govern the choice of structure. A "half-scale" two module cold model was constructed to test the general predictions and to study details of tuning, module coupling and field tilts.

 Introduction

The launching of a high current cw proton beam is being studied at CRNL as part of a long term program to develop accelerator breeder technology. Eventual commercial viability requires average proton currents of 300 mA or more, and the radiofrequency quadrupole (RFQ) is the preferred method of confining and accelerating the required high quality, high current beams at low energy.

Most studies have considered only the four-vane RFQ for this application, but other configurations should be reviewed that would better satisfy the demand for reliability and ease of maintenance of a production breeder accelerator system. In this paper the four-rod RFQ configuration (Fig. 1), the "Q-mode RFQ structure" under investigation at Frankfurt, is considered as a high current proton accelerator. The electric field distribution has very good azimuthal symmetry from the inherent vane "strapping" by the rod support structure. The surface currents are largely confined to the "U" shaped support structures, and are theoretically zero on the rods at the symmetry point between support structures. Thus the rf joint between the structure and the vacuum shell carries lower currents than the vane-base joint on the four-vane RFQ, making design of a demountable configuration easier. The magnetic fields at the outer shell are also lower than on the four-vane structure, giving lower surface current densities and simplifying shell cooling. The zero current point on the rods would be an ideal location for breaking a long structure into shorter modules. Alignment of the rods in the individual modules could be done before insertion into the shell, as opposed to the requirement for vane alignment after mounting with the four-vane system.

The disadvantages of the four-rod structure are the possible lack of rod rigidity and stability, the very high currents on the "U" shaped support structures which necessitate very good cooling, and the possibly lower efficiency. In addition, there is a general lack of understanding and appreciation of the configuration. Some of these points will be considered here by first presenting a simple theoretical analysis of a single module, then comparing the efficiency with a four-rod RFQ high current injector design, and finally by studying a half-scale cold model made up of two coupled modules.

Theoretical Analysis of a Single Four-Rod RFQ Module

The four-rod RFQ unit module (e.g., half of Fig. 1(b), length = $L$) can be regarded as a TEM mode transmission line with open circuit terminations loaded at the centre with a lumped shunt inductance. When viewed from the centre, each half of the transmission line has impedance

$$Z_{k/2} = -j Z_0 \cot \left( \frac{\beta L}{2} \right)$$

(1)

where $\beta = 2\pi/\lambda = w/c$, and $Z_0$ is the characteristic impedance of the line, given by

$$Z_0 = \sqrt{\frac{C_T}{L_T}} = \frac{1}{c} \sqrt{\frac{L}{C_T}}$$

(2)

For the lowest TEM mode, $L_T/C_T = c^{-2}$ where $C_T$ is the equivalent capacitance per unit length (i.e., total capacitance), $L_T$ is the equivalent inductance per unit length and $c$ is the velocity of light. If the line halves are shorter than a quarter-wavelength, then they present a capacitive reactance, which, for the two halves in parallel is $Z_c = 0.5 \times Z_{k/2}$.
The rod-to-rod voltage distribution on the four-rod line is a maximum, \( V_0 \), at the open ends and the variation with distance, \( \Delta x \), from the ends is given by

\[
V(\Delta x) = V_0 \cos \left(2\pi \frac{\Delta x}{\lambda}\right)
\]

(3)

Thus a 10% longitudinal voltage variation is obtained with

\[
\Delta x = 0.072 \lambda
\]

(4)

The "U" shaped vane support can be viewed either as a lumped inductance or as a short length of two rod transmission line terminated with a short circuit. In either case, the support can be viewed as an effective inductance, \( L_{\text{eff}} \), with a reactance

\[
X_{\text{eff}} = j \omega L_{\text{eff}}
\]

Resonance occurs when \( L_{\text{eff}} = -1/C \), that is,

\[
\omega L_{\text{eff}} = \frac{1}{\omega C} \cot \left(\frac{\omega L_{\text{eff}}}{C}\right)
\]

(5)

Expanding for \( (\omega L_{\text{eff}}/C) \ll 1 \), the resonance condition is

\[
L_{\text{eff}} = \frac{1}{\omega^2 (4C/T) (1 + \frac{1}{3} \frac{(\omega L_{\text{eff}})^2}{C} + \ldots)}
\]

(6)

Fig. 2 Schematic view of the inductor and vane attachment, showing cutouts for voltage standoff.

As the effective inductance of the two parallel inductors is approximately half that of the single one alone, an appropriate adjustment in the dimensions of the single lumped inductor is required to maintain the original frequency. For the symmetric "double" inductor, \( L_{\text{eff}} = 0.5 * L_s \).

Fig. 3 Schematic of the twin-inductor 4-rod RFO basic module developed at CRNL.

Experimentally, most of the inductor current (i.e., large magnetic fields near surfaces) is found on the inside faces of the inductor plates, and \( y \) is defined as the ratio of the "exterior" to "inner" surface currents. Simple approximations were used to derive approximate expressions for \( Z_T \), the transverse shunt impedance per unit length, and the quality factor, \( Q \), of modules of length "s".

\[
Q = \frac{1}{(\omega L_{\text{eff}}) C} \left(\frac{2W(1 + y)}{R_s(2H + h)}\right)
\]

(8)

\[
Z_T = \frac{V^2_{\text{Peak}}}{P_{\text{Loss/Unit Length}}} = \frac{20}{\omega L_{\text{eff}}}
\]

(9)

where \( R_s = 2.68 * 10^{-3} \) ohms for pure copper at 108 MHz.

Efficiency Comparison of 108 MHz Four-Vane and Four-Rod RFO Structure

Calculations for the ZEBRA accelerator\(^3\) showed that a vane-to-vane voltage of 260 kV is required (1.75 times the Kilpatrick limit with modulated vanes) for an RFO with \( r_0 = 17.4 \) mm (mean bore radius) and \( p = 12.7 \) mm (mean vane tip radius). The theoretical transverse shunt impedance of such a copper structure at 108 MHz, without manifold, is \( Z_T = 0.56 \) M\( \Omega \)/m.

A four-rod test capacitor with unmodulated rods of 12.7 mm radius and 17.4 mm clear bore had a measured \( C_T = 74 \pm 4 \) pF/m. A balanced inductor four-rod module with these rod dimensions and with \( H = 340 \) mm, \( h = 50.9 \) mm, \( W = 178 \) mm and \( L = 457 \) mm had a measured frequency of 108 MHz. Using formulas (9) and (7), and

\[
L_s = \frac{\mu_0 A}{W} = \mu_0 \frac{B H}{W}
\]

(7)
f = 104 MHz, the following values were derived with y = 0.2 (an estimate based on surface field measurements):

\[ Q = 9550 \text{(theoretical)} \]
\[ Z_T = 0.38 \Omega \text{m} \]

Because of its lower shunt impedance, the four-rod RFQ structure requires approximately 1.5 times more structure power (if the loss in a manifold is included for the four-vane structure, the factor is 1.2). For an accelerator with high beam loading and in which the RFQ is a small part of the total system, this penalty in increased power may be offset by the expected easier maintenance for the four-rod system.

A Study of a Coupled Two Module Four-Rod Model

A 104 MHz, two module cold model was constructed (Fig. 4) to test the effects of module coupling, to measure field distributions both near the rods and near the outer walls, to determine the location and size of the coupling loops, and to check the mode spectrum for possible interfering resonances.

The single modules have the dimensions \( r_0 = 8.64 \text{ mm}, \rho = 6.35 \text{ mm}, h = 220 \text{ mm}, \) \( W = 102 \text{ mm}, \lambda = 470 \text{ mm}, \) and outer cylinder I.D. = 508 mm.

A single module operating outside the cylinder had a frequency of 102.56 MHz. The coupled two module system frequency under the same conditions was 103.13 MHz. This indicates that the mutual inductance between the support inductors is 1.1% of the individual inductor value.

The calculated properties of the coupled two module system in the cylindrical shell \((f = 104.37 \text{ MHz}, C_T = 74 \text{ pf/m}, A_T = 0.33 \text{ \Omega m})\) are \( Q = 9790, \) and \( Z_T = 0.33 \Omega \text{m} \). A power dissipation of 50 kW \((=20 \text{ watts/cm}^2)\) on the inductor surfaces should produce a vane-to-vane voltage of 130 kV and surface fields equivalent to 1.7 times the Kilpatrick limit. The mode spectrum showed no resonance below 104 MHz, and the nearest higher one, presumably the \( r \)-mode, is at 178 MHz. Thus, interfering modes are no problem as expected.

The small change in frequency upon insertion into the cylinder suggests only 1.2% of the stored magnetic field energy is intercepted by the cylinder walls. Indeed, direct magnetic field measurements showed the peak magnetic field on the inner surface of the inductor to be \( 15 \) times larger than the peak field on the cylinder wall. Thus the cylinder wall currents and surface power densities are much lower than those on the inductors, reducing the cooling requirements on the shell and lowering the current carrying requirements for the rf joint at the cylinder wall. A large coupling loop of 0.02 m\(^2\) area was required near the base of an inductor to produce critical coupling.

**Discussion**

The basic rf properties and field distributions of the four-rod structure can be easily understood and parameterized. The azimuthal electric field distributions are very flat and stable; the longitudinal voltage distribution has small predictable variations, within acceptable limits for beam dynamics.

A comparison of the four-vane and four-rod RFQ configuration at 100 MHz shows that the efficiency of the four-rod structure is between 65 and 80% that of the four-vane device. In spite of this penalty, the four-rod structure has some advantages where an easily demountable and serviceable structure is required for a high beam loading, high average power application.

![Fig. 4 Photograph of a 104 MHz cold model composed of two twin-inductor basic modules.](image-url)