SENSITIVITY ANALYSIS OF FEEDFORWARD BEAM CURRENT COMPENSATION FOR IMPROVED BEAM LOADING ROBUSTNESS

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Abstract
The planned SIS100 heavy ion synchrotron at the Facility for Antiproton and Ion Research (FAIR) in Darmstadt, Germany will possess twenty ferrite accelerating cavities in its final stage of extension. During the intended acceleration cycles, the cavities will encounter significant beam loading effects, which have to be handled by the control systems. As both the generator- and beam-current act on the same system input, a feedforward disturbance compensation can be a promising approach to improve beam qualities and suppress instabilities induced by the beam current. Particle tracking simulations, incorporating twenty ferrite cavities and their attached LLRF control systems, are performed to analyse the sensitivity of the beam quality with respect to errors in the feedforward beam current compensation. The main focus lies on the time after injection from a pre-accelerator, where most cavities in the SIS100 do not provide any gap voltage and thus are particularly sensitive to induced voltages by beam currents if the cavities are not or only partly short-circuited.

INTRODUCTION
Significant beam current can seriously deteriorate the beam quality by inducing a parasitic gap voltage which has a destabilizing effect on the beam dynamics and thus leads to emittance growth. In order to guarantee satisfactory beam characteristics, appropriate counter measures have to be taken. Particularly critical are the phases of injection and slow extraction where the required gap voltages are typically low and the majority of the ferrite cavities operate in idle mode. For the precursor accelerator SIS18, detuning of the cavities is a convenient procedure in order to assure robust operation related to high beam currents. However, significant side bands arise due to the existence of empty buckets in the SIS100 accelerator, which makes beam current handling via detuning particularly challenging, especially as the quality factors of the ferrite cavities are relatively low, c.f. [1, 2].

As an alternative approach we consider a feedforward disturbance compensation and analyze the sensitivity of the beam dynamics with respect to parametric errors of the beam current cancellation. The primary focus lies on the injection phase, where bunches from the pre-accelerator are successively injected pairwise until eight out of ten buckets are filled.

In the following we will give a brief overview about the system dynamics and the assumptions underlying the simulation model. Thereupon the results of extensive numerical simulations are presented in order to analyze the disturbance compensation.

LONGITUDINAL DYNAMICS MODEL
Within the scope of this work solely the longitudinal dynamics are considered. This simplification is justifiable as transient and longitudinal dynamics are sufficiently time scale separated. The planned accelerator design for SIS100 will consist of twenty tunable accelerating ferrite cavities, each being able to provide a maximum gap voltage up to 20 kV. Their individual system dynamics can be modeled sufficiently accurate around a given operating point by a lumped parallel circuit with the transfer function

\[
U_{\text{gap}}(s) = \frac{1}{s^2 + C_p L_p s + \frac{1}{L_p C_p}} \left[ I_G(s) - I_B(s) \right],
\]

with \(U_{\text{gap}}\) being the gap voltage, \(I_G\) the generator current, \(I_B\) the beam current and \(R_p\), \(C_p\), \(L_p\) the parallel resistance, capacitance and inductance, respectively [3]. The parameter \(R_p \in [2k\Omega, 3k\Omega]\) is set point dependent and therefore modeled as a nonlinear characteristic. In contrast, the
capacitance $C_p = 740$ pF is largely constant over the operating range, [4]. In combination with the target resonance frequency range, these values yield a comparably low quality factor whose minimal value can reach $Q_{\text{min}} = 14.5$. The inductance $L_p$ can be modified via a magnetization bias current $I_A$ in order to tune the cavity to some given resonance frequency. Each cavity is equipped with a low-level RF control system (LLRF) which is depicted in Fig. 1 and consists of PI controllers for the amplitude and resonance frequency and an controller of integral type for the phase. Amplitude and phase discriminators are simplified and modeled based on the algorithm presented in [5]. The generator produces the single harmonic cavity driving current $I_G(t)$ based on the commanded phase and amplitude values of the corresponding controllers.

Finally the beam dynamics are modeled and simulated via a macro particle tracking approach. Herein the state of each particle is discontinuously mapped each turn by the sum of the individual gap voltages.

### SENSITIVITY ANALYSIS

Clearly, the disturbance $I_D$ lies in the image space of the system input $I_G$, see Eq. (1). Given an appropriate estimation or measurement of the disturbance, it is particularly tempting to cancel out its influence via a feed-forward compensation, before the system dynamics are significantly perturbed. However, in the forefront it is unclear how accurate a feed-forward compensation has to be. Thus, in the following we will analyze the sensitivity of the beam dynamics w.r.t. compensation errors by numerical simulations of the longitudinal dynamics of a reference cycle. The results may be particularly useful for defining requirements of both estimations or beam current sensors, when a closed feedback structure is considered.

As no analytic methods do exist to establish any performance criteria or even stability proof, we have to resort to numerical simulations which cover the relevant dynamics described before, to analyze the influence of the compensation. As test case we consider the planned $^{238}$U$^{28+}$ extremum cycle whose essential parameters are summarized in Table 1. This scenario does possess significant beam loading and is therefore particularly interesting for control system validation. At injection the peak beam current is about 1.6 A and rises to 9 A during acceleration. Despite the lower beam current, the injection phase is more sensitive as only two cavities are active providing a summed up gap voltage of 30.55 kV, while the others are running idle. Without counter measures to handle the beam current in the accelerating cavities, this cycle would lead to unstable beam dynamics and complete particle loss.

During stationary operation, i.e. without acceleration, the beam current of a single matched bunch can be well approximated by a Gaussian pulse given by

$$I_B^{SB}(t) = \bar{I}_B \exp \left( -\frac{t^2}{2\sigma^2} \right),$$

where $\bar{I}_B$ denotes the peak value of the beam current and $\sigma$ characterizes its width. Given appropriate measurements or estimations $\bar{I}_B$ and $\sigma$, a compensation signal can be synthesized to cancel out the beam current in each cavity. As time synchronization of the compensation can be very cumbersome from an implementation viewpoint, we consider to synchronize the synthetic compensation signal to the zero-crossing of the also synthetic reference gap voltage signal. It is particularly interesting if adequate results can be achieved by this setup as implementation barriers are significantly reduced. On the other hand a pure synthetic compensation signal has the advantage that no sensor noise is directly fed back into the cavity. For each revolution of the bunchtrain we consider the compensation signal

$$I_C = \sum_{i=1}^{10} \epsilon_i \bar{I}_B \exp \left( \frac{(\tau_i - \tau_i^\text{ref})^2}{2\sigma^2} \right),$$

where $\epsilon_i \in \{0, 1\}$ indicates if the corresponding bunch is filled. The quality of each single bunch can be assessed by the RMS-emittance given by $\pi \epsilon_i \sigma_i^\text{ref} \sqrt{\sigma_i^2 + \sigma_i^\text{ref}^2}$, where $\tau$ and $W$ are the canonical conjugate phase space coordinates, namely the time and energy deviation of each particle, with respect to the reference particle. In order to be able to evaluate the beam quality of the whole bunchtrain, let us define the quality criterion $J(t) = 100 \cdot \|\Delta e(t)/e_0\|_\infty$, with $e_0 := e_i(t = 0)$, where $\|\cdot\|_\infty$ denotes the supremum norm. This criterion corresponds to the worst centred emittance growth over the bunchtrain. Figure 2 shows the sensitivity of the emittance over time w.r.t. errors of $\bar{I}_B$. It can be observed that around the ideal value of $\bar{I}_B = 1.6$ A, a large plateau of about 0.4 A exists where the emittance growth stays well below 5%. However, even at $\bar{I}_B = 1$ A, corresponding to an error in amplitude of 37.5%, the induced emittance gain is less than 15%. The same non-problematic behaviour can be observed for errors in the compensation width as shown in Fig. 3. Around the ideal value of 31 deg, it is noticeable, that the beam dynamics are more sensitive if the compensation signal is too narrow compared to the real beam current.

However the impacts of the parameter errors are not independent from one another, i.e. the beam quality does not steadily degrade as the true values are left. Figure 4 shows

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>ion mass</td>
<td>238.05078</td>
<td>amu</td>
</tr>
<tr>
<td>number of ions</td>
<td>$5 \times 10^{11}$</td>
<td></td>
</tr>
<tr>
<td>injection energy</td>
<td>195.7</td>
<td>MeV/u</td>
</tr>
<tr>
<td>extraction energy</td>
<td>2700</td>
<td>MeV/u</td>
</tr>
<tr>
<td>RF frequency</td>
<td>1.56 - 2.67</td>
<td>MHz</td>
</tr>
<tr>
<td>gap voltage (max)</td>
<td>372.53</td>
<td>kV</td>
</tr>
<tr>
<td>synchronous phase (max)</td>
<td>59.28</td>
<td>deg</td>
</tr>
<tr>
<td>ramping rate (max)</td>
<td>4</td>
<td>T/s</td>
</tr>
<tr>
<td>momentum compaction</td>
<td>0.005</td>
<td></td>
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</tbody>
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an extensive parameter scan for errors in both height and width of the compensation. The true values are marked by the red cross. A conspicuous feature is the diagonal band of low emittance growth even in the area of considerable errors in the compensation signal. For example the points in the upper right still guarantee an adequate quality with an emittance growth of less than 5%. This phenomenon can be illustrated by regarding the frequency components of the disturbance signal \( I_D = I_B - I_C \). By using the Fourier transform of the idealized beam current (c.f. [2]) and processing it by the cavity dynamics (1), the average power of the induced stationary disturbance gap voltage signal follows by Parseval’s theorem for power signals. Figure 5 shows the normalized analytic disturbance signal power in dependence of the normalized amplitude and width errors. The good qualitative accordance with the numerical results fortify the validity of the approach.

### CONCLUSION AND OUTLOOK

The results of the cycle simulations indicate that feedforward compensation is a valid approach for beam quality improvement. Even though the compensation is synchronized with the reference signal and not the measured beam signal, satisfactory and robust beam dynamics can be achieved. For implementation this result is important as time lag compensation for measured signals can be very cumbersome to achieve. The results can be further improved when a feedback scheme is considered either by measuring and estimating the beam parameters or by an iterative learning-based control. Finally it has to be emphasized that a beam current compensation does not inflict other counter measures but can be deployed in parallel in order to improve the beam quality. For instance it can drastically reduce the settling time of the LLRF control system when a new bunch is injected into the synchrotron.

### REFERENCES


