K-MODULATION IN FUTURE HIGH ENERGY COLLIDERS∗

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Abstract

K-Modulation of the quadrupoles closest to the interaction point (IP) has been an indispensable tool to accurately measure the beta-function in the interaction point (β∗) in the Large Hadron Collider (LHC) at CERN. K-Modulation may become even more important to control the lower β∗ and reach the design luminosities in the High-Luminosity LHC (HL-LHC) and the Future Circular Collider (FCC). K-Modulation results also provide important input for the luminosity calibration and help in the identification and correction of errors in the machines. This paper presents a method for determining β∗ using K-Modulation adapted to the characteristic layout of both colliders. Using the latest models for the HL-LHC and the FCC-hh, estimated uncertainties on the measurements are presented. The results are compared to the accuracy of an alternative modulation scheme using a different powering scheme.

INTRODUCTION

Future circular colliders like the HL-LHC [1] and the FCC-hh [2] are designed to increase the peak luminosity by lowering the betatron-functions in the interaction point (IP) [2, 3] compared to the LHC design [4]. To achieve these higher luminosities, ensuring machine protection and avoiding luminosity imbalances between the experiments accurate measurements and good control of β∗ are required. The currently preferred method to determine the β∗ in the LHC is K-Modulation of the quadrupole closest to the IP. This method relies on the modulation of the gradient of the quadrupoles closest to the IP. The induced tune shifts allow to determine the average β-function in the quadrupoles. The waist shift w and β∗ can then be calculated via propagation. The accuracy of the average β-function and of the β∗ relies on the measurements uncertainty of tune, gradient and good knowledge of machine parameter such as the L∗. The evaluation of the impact of these systematic errors allows a better understanding of the influence of those and possible options to improve the accuracy. In the following, an adaptation to better represent the HL-LHC and FCC-hh interaction regions is presented. Using this, additional sources of error can be taken into account and an error analysis for these cases is presented.

ADAPTATION TO SPLIT INNER QUADRUPOLES

Contrary to the LHC, the triplet quadrupoles closest to the IP in the HL-LHC and FCC-hh will have to be split for manufacturing reasons, both pieces having the same length and, in case of the HL-LHC, both being powered by the same power supply with an additional 35 A trim supply connected to the innermost quadrupole facing the IP [5]. From here on, these magnets will be referred to as Q1A and Q1B. A schematic layout of the HL-LHC and FCC-hh interaction region is illustrated in Fig. 1. In the following it is assumed that the power supply used for both Q1A and Q1B is used for the modulation and an adaptation to [6] is presented to take into account new arising error sources such as an error in the interquadrupole distance. In case of only trimming Q1A, the formulas shown in [6] are used. Using the measured tune shifts from the modulation, the average β-function in the split quadrupoles is determined by [7]

$$
\bar{\beta}_{x,y}(\Delta Q_{x,y}) = \pm \left( \cot(2\pi Q_{x,y}) [1 - \cos(2\pi \Delta Q_{x,y})] \right) + \sin(2\pi \Delta Q_{x,y}) \frac{2}{2\Delta KL} = \pm \frac{4\pi \Delta Q_{x,y}}{2\Delta KL}
$$

where ΔK is the change in the quadrupole gradient, L the length of each quadrupole, Qx,y the tunes and ΔQx,y the tune shift induced by the modulation. Note that here a factor 2 has already been introduced to account for both parts of the split Q1 and it is assumed that both Q1A and Q1B have the same length and modulation amplitude. Following the derivations in [6], to determine β∗ an analytic expression for the average β-function in the quadrupole depending only on the design parameters, the waist-shift w, and the β-function at the waist βw is needed. The average β-function $\bar{\beta}$ over both magnets can be calculated via

$$
\bar{\beta} = \int_{Q1A}^{Q1B} \beta_{Q1A}(s) ds + \int_{Q1A}^{Q1B} \beta_{Q1B}(s) ds
$$

where $\beta_{Q1A}$ and $\beta_{Q1B}$ are the β-functions in the first and second quadrupole, and $L_{Q1A}$ and $L_{Q1B}$ the length of each quadrupole, respectively. The Twiss parameters at the beginning of the focusing quadrupole closer to the IP are given

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by
\[
\begin{align*}
\begin{pmatrix}
\beta_{\text{entry}}^{Q1A} \\
\alpha_{\text{entry}}^{Q1A} \\
\gamma_{\text{entry}}^{Q1A}
\end{pmatrix} &= \begin{pmatrix}
\beta_w + \frac{(L^* - w)^2}{\beta_w} \\
\frac{(L^* - w)^2}{\beta_w} \\
\frac{(L^* - w)^2}{\beta_w}
\end{pmatrix},
\end{align*}
\]
(3)

where \(L^*\) is the distance from the interaction point to the beginning of the first quadrupole and \(\beta_{\text{entry}}, \alpha_{\text{entry}},\) and \(\gamma_{\text{entry}}\) the Twiss parameter at the entry of the quadrupole. The \(\beta\)-function in the quadrupole is determined by propagating the Twiss parameter \(\beta_{\text{entry}}, \alpha_{\text{entry}},\) and \(\gamma_{\text{entry}}\) from the entry of the quadrupole using
\[
\beta(s) = C^2 \beta_{\text{entry}} - 2CS\alpha_{\text{entry}} + S^2\gamma_{\text{entry}}
\]
(4)

where \(C = \cos(\sqrt{K_s})\) and \(S = \frac{1}{\sqrt{K}} \sin(\sqrt{K_s})\) in case of a focusing quadrupole. The Twiss parameter at the beginning of the second quadrupole in dependence of \(\beta_w\) and \(w\) are obtained similarly via propagation
\[
\begin{align*}
\begin{pmatrix}
\beta_{\text{entry}}^{Q1B} \\
\alpha_{\text{entry}}^{Q1B} \\
\gamma_{\text{entry}}^{Q1B}
\end{pmatrix} &= D(L_d) \cdot Q(K^{Q1A}, L^{Q1A}) \cdot \begin{pmatrix}
\beta_w + \frac{(L^* - w)^2}{\beta_w} \\
\frac{(L^* - w)^2}{\beta_w} \\
\frac{(L^* - w)^2}{\beta_w}
\end{pmatrix},
\end{align*}
\]
(5)

where \(Q(K^{Q1A}, L^{Q1A})\) represent the propagation matrix of the first quadrupole and \(D(L_d)\) the one for a drift space with \(L_d\) being the length of the interconnect between Q1A and Q1B. With the presented formulas, Eq. (2) can then evaluated and the average \(\beta\)-function is
\[
\bar{\beta} = \left( \beta_w + \frac{(L^* - w)^2}{\beta_w} \right) F_0 + \left( \frac{(L^* - w)}{\beta_w} \right) F_1 + \frac{1}{\beta_w} F_2,
\]
(6)

where \(F_0, F_1,\) and \(F_2\) are obtained by integration of the three terms in Eq. (4) over the length of the quadrupole. Using the assumption of squeezed optics
\[
\frac{(L^* - w)^2}{\beta_w^2} \gg 1
\]
(7)

and following the derivations in [6] \(\beta_w\) is given by
\[
\beta_w = \frac{1}{\bar{\beta}} \left[ (L^* - w)^2 F_0 + (L^* - w) F_1 + F_2 \right]
\]
(8)

The derivation for the case of a defocusing quadrupole follows analogously by using \(C = \cosh(\sqrt{K_s})\) and \(S = \frac{1}{\sqrt{K}} \sinh(\sqrt{K_s})\) and changing the sign of the waist shift \(w\) in Eq. (3). We note that a similar derivation was used to analyze K-Modulation measurements in SuperKEKB in order to account for quadrupole fields from one ring leaking into the other [8].

**Uncertainty of \(\beta^*\)**

The HL-LHC aims to achieve an even lower \(\beta^*\) than the LHC, utilizing wider aperture triplet quadrupoles and by use of the Achromatic Telescopic Squeezing (ATS) scheme [9]. During the telescopic squeeze of said scheme, MC1: Circular and Linear Colliders

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![Figure 2: Calculated uncertainties for a modulation of both parts of the quadrupole with and without scaling of the tune uncertainty.](image)

The \(\beta\)-functions in some of the arc dipoles adjacent to the experimental insertions increase significantly, which also increases the sensitivity of the tune to power supply ripple in these arcs. Without any upgrades of these power supplies, the tune stability is expected to be \(4.2 \cdot 10^{-5}\) in case of the \(\beta^* = 15\) cm optics, whereas for the same optics and with the upgrade of the power supplies the tune stability is reduced to \(2.9 \cdot 10^{-5}\) [10]. Using the same approach as in [6] and the previously presented adaptation to the HL-LHC triplet, the error for different \(\beta^*\) have been calculated and are presented in Fig. 2. Here two cases are presented, one making the pessimistic assumption that the tune uncertainty is independent from the chosen \(\beta^*\) and the second case assuming that the tune uncertainty scales with \(\alpha \frac{1}{\beta^*}\). For the misalignment and uncertainty in the magnetic length a uniform distribution has been assumed with a maximum deviation of \(\pm 2\) mm [11] and \(\pm 5\) mm [12], respectively. For the uncertainty of the quadrupole gradient, it was assumed that both parts of the Q1 have the same uncertainty of 10 units. In addition, each magnet has an uncertainty of 2 units. Note that here the nominal LHC injection tunes of 0.28/0.31 were used as these allow a higher tune shift before approaching the coupling stop band. For this case the modulation amplitude was chosen such that the tune shift does not exceed 0.01. Using the LHC collision tunes of 0.31/0.32 with the subsequently reduced tune shift of 0.003 the error for the \(\beta^* = 15\) cm optics would increase to 103% and 51% for the \(\delta Q = 4.2 \cdot 10^{-5}\) and \(\delta Q = 2.9 \cdot 10^{-5}\), respectively. To keep the luminosity imbalance between the two high luminosity experiments below 5% a maximum error on the \(\beta^*\) of 2.5% is specified which is currently exceeded in all scenarios for a \(\beta^* < 20\) cm. As a mitigation, an additional trim power supply was introduced, allowing to modulate only the innermost magnet, which has been called Q1A modulation. Using the same error specifications as in the previous case and the formulas derived in [6] for the single quadrupole case, the resulting accuracy in the \(\beta^*\) calculation is presented in Fig. 3. In both cases, a reduction in the uncertainty of close to a factor 2 is observed. Still, with the current specifications, the target accuracy is not reached for the \(\beta^* = 15\) cm optics. To investigate the impact of the error contributions further studies were conducted and the results are presented in Fig. 4. The biggest contribution for both the Q1 and Q1A modulation is the error from the tune uncertainty. As expected, both the error from mispowering and from misalignment have a smaller effect in case of the Q1A modulation and in both cases make...
up less than half of the error bar. From this it also becomes apparent that in order to meet the goal of 2.5% the tune jitter would need to decrease as even without any magnet errors or misalignment the error on the $\beta^*$ is above the target. In Fig. 5, a scan of tune uncertainty is presented for both modulation types. To meet the target accuracy at $\beta^* = 15$ cm, indicated with a black dashed line, a tune uncertainty below $2 \cdot 10^{-5}$ is required in a case of Q1A modulation.

**Status in the LHC**

To benchmark simulations calculating the tune jitter from the power supply ripple and see the impact of additional tune jitter coming from other contributions, measurements in the LHC were carried out. The result can be found in [13, 14]. Here it is shown that for example in case of $\beta^* = 30$ cm optics the tune jitter from simulations is a factor 2 below the measured jitter. Using a $\delta Q$ of $6 \cdot 10^{-5}$ instead of $3 \cdot 10^{-5}$ increases the uncertainty on the $\beta^*$ from 1.4% to 5.1% when taking no other errors into account. Similarly, in the case of the $\beta^* = 25$ cm the increase is even more drastic, going from 2% to 7.2%. This is in line with various K-Modulation measurements conducted during 2018, where unreliable results were obtained, partly attributed to tune uncertainty [15]. This significantly-increased tune jitter compared to simulations suggests that further investigations into sources and possible mitigation techniques should be conducted, both in view of LHC Run 3 and the HL-LHC, in order to minimise any possible luminosity imbalance between experiments. One example is presented in [16] where it is shown how after application of normal octupole correction the noise in the online tune measurement is significantly reduced.

**Outlook for the FCC-hh**

For the case of the FCC-hh, currently no information on the expected tune jitter is available. Due to the higher $\beta$-function in the arc cells compared to the LHC, an increased sensitivity is expected. Assuming the same power supply parameters as in the HL-LHC, this would lead to an even higher tune jitter compared to the HL-LHC. However, even when using the optimistic HL-LHC error parameters and Q1A modulation, the uncertainty on the $\beta^* = 1.1$ m is 4.5%. In the case of the ultimate optics with a $\beta^* = 0.3$ m the error goes to 18.9% indicating, similarly to the HL-LHC, that control of the lowest $\beta^*$ will be challenging. Due to the considerable length of the Q1A in this case, the average $\beta$-functions in the different planes differ significantly. This in turn leads to the tune shift in one plane being smaller, thus increasing the impact of the tune jitter. To overcome this effect, a shorter innermost quadrupole could be introduced, for example via an asymmetrical split. Provided that the difference of the average $\beta$-functions in this quadrupole is sufficiently small, the tune shift in both planes is similar and thus the impact of the tune jitter in the plane of the previously smaller tune shift is partly mitigated.

**CONCLUSIONS**

A systematic error analysis for the K-Modulation method has been presented for both the HL-LHC and FCC-hh. Using an analytic expression, adapted to the specific design of these colliders, the error on $\beta^*$ measurements in the HL-LHC was presented. The results are compared to an alternative modulation scheme, made possible by the powering scheme of the HL-LHC triplet. In all the presented cases the measurement error for the lowest targeted $\beta^*$ exceeds the currently acceptable levels. In order to decrease the impact of the tune jitter, the introduction of a short innermost quadrupole could be considered in the design of future colliders, which would allow to reach the maximum allowed tune shift in both planes. The high sensitivity of the K-Modulation method to tune jitter also motivates studying alternative techniques to measure $\beta^*$ and any waist displacement in order to establish good control over $\beta^*$ in these future high energy colliders. As shown for the LHC case, further investigations into additional sources of tune jitter should be conducted and mitigations, where possible, could be put in place in order to limit any potential luminosity imbalance between experiments.
REFERENCES


