

# EFFECT OF MODEL ERRORS ON THE CLOSED ORBIT CORRECTION AT THE SIS18 SYNCHROTRON OF GSI\*

S. H. Mirza<sup>†1</sup>, P. Forck, H. Klingbeil<sup>1</sup>, R. Singh  
 GSI, Darmstadt, Germany

<sup>1</sup>also at TU Darmstadt, Darmstadt, Germany

## Abstract

The influence of model errors on the closed orbit correction for the SIS18 synchrotron at GSI has been simulated. The systematic model drift over the ramp due to the transition of triplet to doublet quadrupole configuration and the non-systematic tune shifts due to image charge and beta beating are considered. The study is aimed to draw hints for the robust stability requirements of the closed orbit feedback controller against model mismatch.

## INTRODUCTION

A closed orbit feedback (COFB) system is under development for the whole acceleration cycle in GSI SIS18 synchrotron in order to preserve the beam quality before injection into the upcoming SIS100 synchrotron. The orbit response matrix (ORM) which represents the spatial response of the closed orbit to the kicks of the dipolar corrector magnets [1] is given as

$$\mathbf{R}_{mn} = \frac{\sqrt{\beta_m \beta_n}}{2 \sin(\pi Q)} \cos(Q\pi - |\mu_m - \mu_n|) \quad (1)$$

where  $\beta$  and  $\mu$  denote the beta function and phase advance at BPMs and corrector locations marked as  $m$  and  $n$ , respectively.  $Q$  is the coherent tune of the machine. The matrix inversion (or pseudo-inversion  $R^+$  for non-quadratic matrices) is required for the calculation of the corrector settings according to

$$\theta = \mathbf{R}^{-1} \Delta y. \quad (2)$$

where  $\theta$  is the corrector settings vector and  $\Delta y$  is the vertical ( $\Delta x$  for horizontal plane) perturbed orbit measured at BPM locations. Singular value decomposition (SVD) is commonly used for the inversion (or pseudo-inversion) of the ORM. In case of circulant symmetry of the synchrotron, DFT based decomposition and inversion has also been proposed [2].

SIS18 accelerates a wide range of ions to desired energies limited by the maximum rigidity of 18 Tm. One of the challenges for SIS18 COFB is to accommodate the variation of the actual machine model relative to the assumed model for such a flexible machine, detailed account of challenges is given in [2]. The only known study of feedback system robust to model errors was reported from Diamond light source [3], however it mainly focused on the robustness against potential tune deviations. Here we extend it to the comparison of the sources and localization of such model shifts. The scope of this paper is to simulate the effect of

different model errors on the closed orbit correction in order to draw hints for the robustness requirements of the feedback controller. The controller action and dynamic aspects are not included yet.

If  $\mathbf{R}$  represents the actual machine model and  $\mathbf{R}'$  is the assumed model used to calculate the corrector strengths for an initial perturbed orbit  $\Delta y_0$ , the residual orbit  $r_1$  after one iteration can be written as

$$\begin{aligned} r_1 &= \Delta y_0 - \mathbf{R}\theta \\ r_1 &= \Delta y_0 - \mathbf{R}\mathbf{R}'^{-1} \Delta y_0 \\ r_1 &= (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}) \Delta y_0 \end{aligned} \quad (3)$$

The residual after  $n$  iterations becomes

$$r_n = (\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1})^n \Delta y_0 \quad (4)$$

The first iteration residual  $r_1$  bears a direct relation to the model mismatch and gives a hint of the correctability and stability criteria. If any of the Eigenvalues of the matrix  $(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}) > 1$ , repeated orbit corrections will lead to the instability. The large deviation of only one eigenmode of ORM can fulfill this condition. On the other hand, the larger the value of  $r_1$ , the higher the number of iterations required to converge the matrix product given in Eq. (4) even if all Eigenvalues of  $(\mathbf{I} - \mathbf{R}\mathbf{R}'^{-1}) \leq 1$ . In this paper, the effect of following three kinds of model mismatch on the first iteration residual has been presented for a comparison; (a) On-ramp systematic ORM change and non-systematic tune shifts, (b) Intensity dependent tune shifts and (c) Beta beating. The ratio of first iteration residual to the initial perturbed orbit is defined as

$$\delta_1 = \frac{r_{1\text{RMS}}}{\Delta y_{0\text{RMS}}} \quad (5)$$

## ON-RAMP SPATIAL MODEL CHANGES

A peculiar behavior of SIS18 is the transition from a triplet to doublet quadrupole configuration during the ramp [4]. This is in connection to incorporate the larger beam size at the beginning of the ramp because of multi-turn injection in the horizontal plane. Figure 1 (top) shows a typical variation of quadrupole strengths over a ramp of 10 T/s generated by the accelerator control software with a time step of 1 ms. The quadrupole strengths are varied in a way to keep the transverse tune almost constant over the ramp. Such a lattice transition causes a systematic change in the ORM by varying the beta functions and phase advances at the locations of BPMs and correctors in Eq. (1). The length of the ramps from cycle to cycle is also variable ( $\approx 100$  ms to 500 ms) depending upon user requirements.

Figure 1 (bottom) compares the orbit response matrix variation over the two ramps (5 T/s and 10 T/s) by plotting

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<sup>†</sup> s.h.mirza@gsi.de

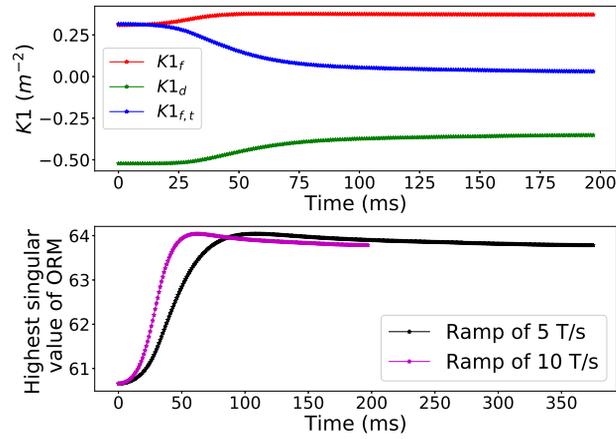


Figure 1: Top: Triplet to doublet quadrupole transition over the ramp.  $K1_f$ ,  $K1_d$  and  $K1_{f,t}$  are the normalized strengths of the doublet focusing, doublet defocusing and triplet focusing quadrupole families of SIS18, respectively. Bottom: Variation of vertical ORM over ramp.

the highest singular value of each ORM as a signature of the matrix. One can see that different ramps traverse different paths for the ORM variation requiring an understanding how should COFB take this change into account. A dipole ramp of 10 T/s was selected for simulations and the vertical orbit correction was performed at all time steps of the ramp using only the initial ORM  $R_{t=0}$  (corresponding to injection settings). MADX [5] was used for the generation of 1000 perturbed closed orbits at each time step of the ramp using the random combinations of transverse misalignments of quadrupoles with Gaussian probability distribution ( $\sigma = 0.3$  mm cut at  $3\sigma \approx 1$  mm). As a result the RMS values of the perturbed orbits also had a Gaussian distribution with mean = 12.5 mm and  $\sigma = 7.5$  mm. Corrector settings were calculated using all the singular values of  $R_{t=0}$  for each perturbed orbit. Residual orbit percentage ( $\delta_1\%$ ) over the ramp has been plotted in Fig. 2 (top) in blue color. The residuals also have a Gaussian distribution but with a significantly smaller standard deviation represented as error bars.  $\delta_1\%$  increases directly with model mismatch up to a maximum of  $6 \pm 2\%$ . For a typical orbit distortion of 12 mm the value of  $r_1$  for such a model mismatch is less than 1 mm which shows that on-ramp systematic model drift is not the bottle neck for the robustness requirements of the COFB.

In addition, tune shifts away from model tune ( $\Delta Q_y \approx 0.01$  and  $\Delta Q_x \approx 0.02$ ) during the ramp have also been observed in SIS18 [6] which is regarded as a non-systematic model error in this contribution. Tune shifts of comparable magnitudes have also been reported for electron beams during fast ramps e.g. at ELSA [7]. The exact reasons for such a tune shift is not trivial to determine because there may be many factors inter-playing together during the ramp e.g. output current of the power supplies not following the control curve, errors in the calibration of current to magnetic field of the magnets and eddy currents in the vacuum chambers or magnets. All these effects can result into the quadrupole field

gradient errors during the ramp and consequently can affect the tune. The eddy currents in magnet cores are thought to be

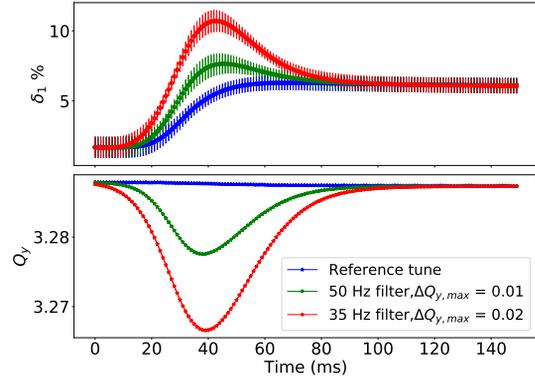


Figure 2: Top: Orbit correction over a ramp of 10 T/s using the first ORM of the ramp. For reference tune (blue), for tune shift of 0.01 (green) and for tune shift of 0.02 (red). Bottom: Simulated tune patterns.

the primary cause of quadrupole gradient errors. Therefore the on-ramp tune-shift is simulated by application of low pass filtering. Two low pass 1<sup>st</sup> order butterworth type filters of cut-off frequencies 50 Hz and 35 Hz were applied to the quadrupole strengths in order to produce a vertical tune shift of  $\Delta Q_y = 0.01$  and 0.02 as shown in Fig. 2 (bottom). The tune corresponding to unfiltered strengths is also plotted as a reference (blue). The contribution of such tunes shifts in the residual orbit is depicted in Fig. 2 (top). Non-systematic tune shifts add an additional residual orbit on top of that produced by systematic model drift.

## INTENSITY DEPENDENT TUNE SHIFT

Intensity dependent coherent tune shifts have been measured experimentally in SIS18 [8]. Such a tune shift is modelled as image charge effect of the vacuum chambers around the beam. Image charges of opposite sign pull the beam outward like a defocusing force causing a decrease in coherent tune. The image charge force is a non-linear function of the beam's transverse position [9] depending upon the boundary but can be linearized for small oscillations and for simple geometries (circular or elliptical) as performed in [10] and given as,

$$F_y^{image} \propto y, \quad (6)$$

where  $F_y^{image}$  is the defocusing force in the y-direction.

This approximation holds for SIS18 where the measured orbit distortions are within 25% of the vacuum pipe size (e.g. the effective vertical dimension of SIS18 vacuum chamber  $\approx 80$  mm [6]) and has been used to simulate the effect of image charge tune shift on the orbit correction. The drift regions in SIS18 were replaced with weak defocusing quadrupoles of strength  $K1_{defoc,imag}$  (Fig. 3 (top)) in y-plane and the same quadrupole strength was added to the strengths of already present quadrupole families resulting in a weak defocusing force throughout the synchrotron. However, such a simulation is only possible for one plane at a time.

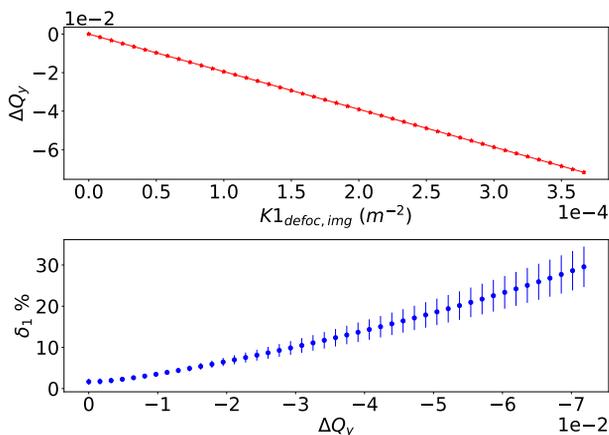


Figure 3: Top: Simulated image charge tune shift. Bottom: Orbit correction with ORM of zero tune shift.

Strength of distributed quadrupoles was varied over a range of (+0 to  $+3.6 \times 10^{-4} m^{-2}$  in x-plane) to produce a maximum tune shift of -0.07 in y-plane (higher than experimental value of -0.05 [8] to account for higher intensities in future). Figure 3 (top) shows the resultant linear variation of the tune. Orbit correction was performed for 1000 randomly generated orbits (as discussed in previous section) at each intensity using the ORM corresponding to low intensity ( $\Delta Q_y = 0$ ) and  $\delta_1$  % has been plotted in Fig. 3 (bottom) with error bars showing the  $1\sigma$  of the Gaussian distribution of residuals. Even for a linear approximation of image charge force, a significant residual orbit (mean value  $\approx 20\%$ ) can be seen up to a  $\Delta Q_y = 0.05$ . Moreover, Orbit correction attempts at injection energies at high intensities would require to take this effect into account.

## BETA BEATING

Beta beating is another source of non-systematic model error resulting from the addition of spurious focusing coming from orbit distortions in higher order sextupolar fields of dipoles and field errors in quadrupoles. Dedicated measurement of beta beating is carried out in fixed lattice machines before orbit correction, but this can not be expected at SIS18 due to its flexible range of operation settings. Simulations here demonstrate the effect of beta beating on the orbit correction, if the nature of beta beating is not known or when the orbit response matrix is not measured.

A peak-peak beta beating [11] of up to  $\approx 50\%$  was produced in the simulation by varying the strength of only one quadrupole relative to others (a scenario of localized error). Orbit correction was performed using the ORM corresponding to zero beta beating for the calculation of corrector settings for all models of non-zero beta beating. 1000 random orbits were corrected for each beta beating value. The residual orbit  $\delta_1$  % is plotted in Fig. 4 (bottom) where  $1\sigma$  of the Gaussian distribution of residuals (error bars) also increases significantly with beta beating. The corresponding tune shift has also been plotted for comparison in Fig. 4 (top).

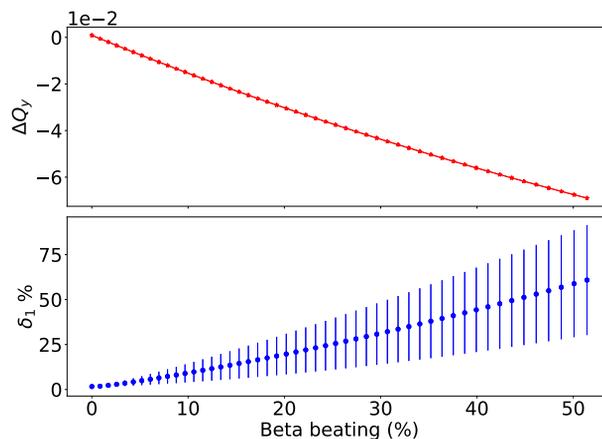


Figure 4: Top: Tune shift caused by beta beating. Bottom: Orbit correction with ORM of zero beta beating.

## DISCUSSION AND CONCLUSION

The correction during ramp using the ORM of injection settings leaves a maximum of  $6 \pm 2$  % residual after first iteration. This shows that on-ramp model drift can be taken into account by considering only a few (2-3) ORMs over the whole ramp. The variation of ORM without significant change in the tune does not change the relative strength of the eigenmodes and correction with wrong model has a similar effect as to apply wrong gain to all modes which can only reduce the controller bandwidth as suggested by Eq. (4). On-ramp tune shifts leave an extra 5% residual but they are expected to be less important for slower ramps. A measurement is planned to study the behavior of on-ramp tune shift versus ramp rate in the next beam time. Tune shift simulated by quadrupole gradient errors of all families is an example of errors that preserve the symmetry of the ORM. Image charge effect is an extension of such errors uniformly distributed throughout the synchrotron and tune shift has a significant effect on the relative strengths of the eigenmodes (singular values). Eigenmodes closest to the tune frequency are the most sensitive to the tune variation and become unstable even if other modes are correctable. Image charge effects will become more important in future for the high intensity beams planned for FAIR. Beta beating has contributions both from localized variation of beta function at BPMs and correctors and global change in tune. A comparison of Figs. 3 and 4 shows that for comparable tune shifts, the residual orbit  $\delta_1$  % is larger in cases of beta beating. Thus, the source of tune shift is also important in addition to its magnitude for the uncertainty modeling of the ORM. The effect of all these model errors are simulated separately but in reality they will coexist and their combinations can enhance the residual and decrease the instability thresholds.

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## REFERENCES

- [1] M. Sands, "The Physics of Electron Storage Rings: An Introduction", in *Conf. Proc. C6906161 257-411 SLAC-R121, SLAC-121*, Nov 1970.
- [2] S. H. Mirza *et al.*, "Investigation of spatial process model for the closed orbit feedback system at the SIS18 synchrotron at GSI", in *Proc. ICALEPCS'17*, Barcelona, Spain, Oct. 2017, paper THMPA02.
- [3] S. Gayadeen, "Synchrotron Electron Beam Control", Ph.D. thesis, St. Hugh's College, University of Oxford, UK, 2014.
- [4] Oliver Bruning and Stephan Myers, "Challenges And Goals For Accelerators In The XXI Century", World Scientific, 26 Feb 2016, pages 288-289.
- [5] <https://madx.web.cern.ch/madx/>
- [6] R. Singh, "Tune measurement at GSI SIS-18: Methods and Applications" PhD Thesis, TU Darmstadt 2014.
- [7] M. Eberhardt *et al.*, "Measurement and correction of the longitudinal and transverse tune during the fast energy ramp at ELSA", in *Proc. IPAC'10*, Kyoto, Japan, 2010, paper MOPD085.
- [8] R. Singh *et al.*, "Interpretation of transverse tune spectra in a heavy-ion synchrotron at high intensities", *Physical Review Special Topics - Accelerators and Beams* R 16, 034201 (2013)
- [9] B. Zotter, "The Q-shift of off-center particle beams in elliptical vacuum chambers", *Nuclear Instruments and Methods* 129 (1975) 377-395.
- [10] K. Schindl, "Space charge", CERN, CH-1211, Geneva 23, 1999.
- [11] R. Tomas *et al.*, "Procedures and accuracy estimates for beta-beat correction in the LHC", in *Proc. EPAC'06*, Edinburgh, Scotland, 2006, paper WEPCH047.