ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPML099

PHASE EXTRACTION AND STABILIZATION FOR COHERENT PULSE STACKING*

Y. Xu†, L. Doolittle, Q. Du, G. Huang, W. Leemans, D. Li, R. Wilcox, Y. Yang, T. Zhou,
Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA
L. Yan, W. Huang, C. Tang, Tsinghua University, Beijing 100084, China
A. Galvanauskas, University of Michigan, Ann Arbor, MI 48109, USA
†also at Tsinghua University, Beijing 100084, China

Abstract

Coherent pulse stacking (CPS) is a new time-domain coherent addition technique that stacks several optical pulses into a single output pulse, enabling high pulse energy and high average power. We model the CPS as a digital filter in the Z domain, and implement two deterministic algorithms extracting the cavity phase from limited data where only the pulse intensity is available. In a 2-stage 15-pulse CPS system, each optical cavity is stabilized at an individually-prescribed round-trip phase with 0.7 deg and 2.1 deg RMS phase errors for Stage 1 and Stage 2 respectively. Optical cavity phase control with nm accuracy ensures 1.2% intensity stability of the stacked pulse over 12 hours.

INTRODUCTION

Development of advanced kW-class ultrafast lasers will have a significant impact on laser-driven particle accelerator systems [1]. However, inefficiency of thermal handling capability currently limits the repetition rate of high energy systems. A different laser technology is needed for high repetition rate. Fibers are superior in many ways, *i.e.,* demonstrated high average power, good heat removal efficiency, excellent beam quality, and stable alignment, but challenges such as small aperture and narrow bandwidth limit output energy and pulse width. Fortunately, we can increase energy and bandwidth by adding pulses temporally, spatially and spectrally [2].

CPS is a new time-domain coherent addition technique that stacks several optical pulses into a single output pulse [3]. The initial pulses of the tailored optical pulse burst enter the reflecting resonant cavity and interfere destructively at the cavity output port, thus storing optical energy inside the resonant cavity. Later, the final pulse in the burst produces a constructive interference with the previous intra-cavity pulses at the output port, so that all stored energy is extracted from the resonant cavity into a single output pulse.

The efficiency of the CPS system is related to the ability to control the cavity phase accumulated by optical pulses in each path so as to guarantee the constructive interference. Failure of maintaining the cavity phase matching translates into a decrease of the stacking efficiency and combined peak power.

* Work supported by the U.S. Department of Energy, Office of Science, Office of High Energy Physics, under Contract DE-AC02-05CH11231.
† yilunxu@lbl.gov; xu-yl13@mails.tsinghua.edu.cn

CAVITY PHASE EXTRACTION

Z-Domain Model of CPS

We have developed a control system model describing the CPS process in the Z domain, which gives a direct link to digital radio frequency (RF) engineering and provides solutions to deterministic optical phase measurement and scalable feedback control. If the round-trip length is L, the cavity round-trip optical phase shift is $\varphi = 2\pi L/\lambda_0$, where λ_0 is the optical wavelength. Z-transform is employed to describe the first-order physics of a front-mirror as a beam combiner/splitter, as shown in Fig. 1. The input and output pulse electric fields at both sides of the front-mirror can be described by a scattering matrix

$$\begin{bmatrix} \widetilde{O}_4 \\ \widetilde{O}_3 \end{bmatrix} = \begin{bmatrix} r & jt \\ jt & r \end{bmatrix} \begin{bmatrix} \widetilde{W}_1 \\ \widetilde{W}_2 \end{bmatrix}, \quad (1)$$

where input waves ($\widetilde{W}_1$, $\widetilde{W}_2$) and output waves ($\widetilde{O}_3$, $\widetilde{O}_4$) are all complex numbers. Here r and t are the reflection and transmission coefficients respectively, which are related by $r^2 + t^2 = 1$. We use the Z-transform formalism to express the delay line in the context of a pulsed laser,

$$\widetilde{W}_2 = z^{-1}\alpha e^{j\varphi}\widetilde{O}_3, \quad (2)$$

where α is the transmission loss coefficient. Here we call the round-trip phase φ the "cavity phase". To diagnose an optical cavity resonator, one has to derive the cavity phase from limited measurements provided by the corresponding photodiode, where only the pulse intensity is available.

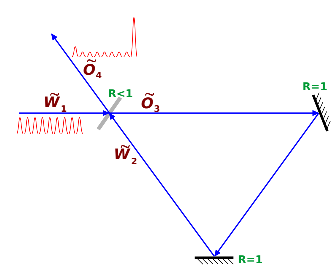

Figure 1: Physical model of CPS in the Z domain.

The system transfer function $H(z)$ is therefore:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{r - \alpha e^{j\varphi} z^{-1}}{1 - r\alpha e^{j\varphi} z^{-1}}, \quad (3)$$

where $H(z)$ is the linear mapping of the Z-transform of the input $X(z)$ to the Z-transform of the output $Y(z)$. The coherent pulse stacker acts as a digital filter which is characterized

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPML099

by the cavity phase φ and the front-mirror reflectivity r. The Z-domain model can be extended to cascaded cavities easily as

$$H_{\text{cascaded}}(z) = \prod_i H_i(z). \quad (4)$$

Direct Iteration Algorithm

For an ultrashort optical pulse train consisting of n pulses, we define the complex field of the k^{th} individual pulse as $A_k \cdot e^{j\psi_k}$, where A_k and ψ_k characterize the amplitude and phase. Instead of using the stack pulse train itself, a special phase probe pulse train can be injected together with the stack pulse train to diagnose the optical cavity fluctuation. Let us denote the input of the phase probe pulse train as $x(k) = {}^{\text{in}}A_k \cdot e^{j^{\text{in}}\psi_k}$, while the output as $y(k) = {}^{\text{out}}A_k \cdot e^{j^{\text{out}}\psi_k}$. Taking the Z-transform of input and output pulse trains respectively yields $X(z) = \sum_{k=1}^{n} {}^{\text{in}}A_k \cdot e^{j^{\text{in}}\psi_k} \cdot z^{-(k-1)}$ and $Y(z) = \sum_{k=1}^{n} {}^{\text{out}}A_k \cdot e^{j^{\text{out}}\psi_k} \cdot z^{-(k-1)}$. Each output pulse phase can be derived iteratively from

$$\begin{aligned} &\alpha^2 \cdot {}^{\text{in}}A_{k-1}^2 + \alpha^2 \cdot r^2 \cdot {}^{\text{out}}A_{k-1}^2 \\ &-2\alpha^2 r \cdot {}^{\text{in}}A_{k-1} \cdot {}^{\text{out}}A_{k-1} \cdot \cos({}^{\text{in}}\psi_{k-1} - {}^{\text{out}}\psi_{k-1}) \\ &= r^2 \cdot {}^{\text{in}}A_k^2 + {}^{\text{out}}A_k^2 - 2r \cdot {}^{\text{in}}A_k \cdot {}^{\text{out}}A_k \cdot \cos({}^{\text{in}}\psi_k - {}^{\text{out}}\psi_k). \end{aligned} \quad (5)$$

The cavity phase can be extracted from limited data with the formula:

$$\varphi = \arg\left[\frac{{}^{\text{out}}A_k \cdot e^{j^{\text{out}}\psi_k} - r \cdot {}^{\text{in}}A_k \cdot e^{j^{\text{in}}\psi_k}}{\alpha(r \cdot {}^{\text{out}}A_{k-1} \cdot e^{j^{\text{out}}\psi_{k-1}} - {}^{\text{in}}A_{k-1} \cdot e^{j^{\text{in}}\psi_{k-1}})}\right], \quad (6)$$

where arg is a function giving the argument of a complex number.

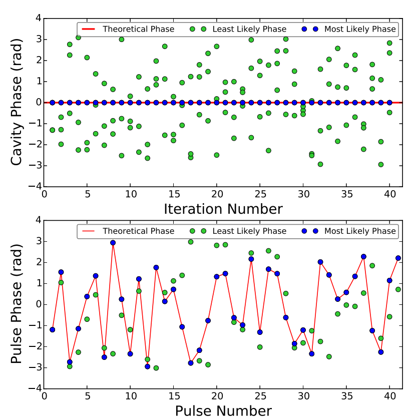

Figure 2: Cavity and pulse phases in direct iteration algorithm.

Observing that cosine is an even function, generally we will find two solutions of ${}^{out}\psi_k$ from Eq. (5). According to Eq. (6), every iteration will generate a solution set consisting of four values of the cavity phase. Fortunately, we can compare the solutions of the cavity phase provided by different iterations. The most likely value of the cavity phase is supposed to be one of the four candidates in every iteration, while the other three candidates (the least likely cavity phases) are extraneous solutions [4]. As shown in Fig. 2, the most likely cavity phase points have the shortest distance to each other than the least likely cavity phase points.

Template Vector Algorithm

Since each pulse in a burst is affected by a different number of round-trips, there will be differing intensity functions as the cavity phase is tuned over 2π. Therefore, it is possible to identify the cavity phase by a unique combination of pulse intensities [5]. Intensities of N phase probe pulses (N is 6 here) at the cavity output port can be represented as a vector: $\vec{o}(\varphi) = [O_1, O_2, \cdots, O_N]$. The cavity phase can be computed simply and quickly by a dot-product of the N-long optical vector measurement with a known complex vector, which we call a "template vector", according to

$$\vec{o}(\varphi) \cdot \vec{v} = e^{j\varphi}, \quad (7)$$

where $\vec{v}$ is the template vector which requires an initial calibration beforehand, and φ is the cavity phase. As shown in Fig. 3, the FPGA (field-programmable gate array) does the dot product operation in real part and imaginary part to acquire the in-phase component (I) and the quadrature component (Q) respectively, followed by a CORDIC (COordinate Rotation DIgital Computer) module extracting the cavity phase information. A cascaded integrator-comb (CIC) filter is included in the control feedback loop as moving-average digital processing for noise suppression. Once optical matrix calculations are finished and the cavity phase is obtained, a PI loop is implemented to drive the piezo to lock the optical cavity at an intended phase.

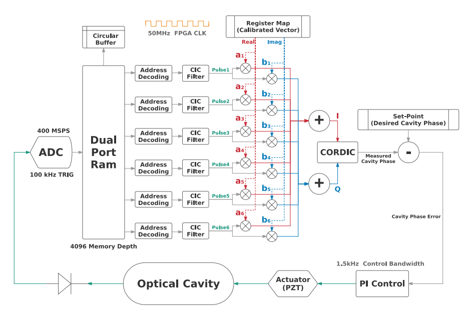

Figure 3: Digital processing chain of template vector algorithm.

To calibrate the template vector, one can vary the cavity phase over 2π while observing the output from a phase probe pulse train. Rewriting (7) in matrix form yields the complex template vector in the calibration procedure:

$$\begin{bmatrix} a_1 + b_1 j \\ a_2 + b_2 j \\ \vdots \\ a_N + b_N j \end{bmatrix} = \begin{bmatrix} O_{1,1} & O_{1,2} & \dots & O_{1,N} \\ O_{2,1} & O_{2,2} & \dots & O_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ O_{M,1} & O_{M,2} & \dots & O_{M,N} \end{bmatrix} \backslash \begin{bmatrix} e^{j2\pi\frac{1}{M}} \\ e^{j2\pi\frac{2}{M}} \\ \vdots \\ e^{j2\pi\frac{M}{M}} \end{bmatrix}, \quad (8)$$

where M is the scanning resolution in one whole cycle of the cavity phase, and a and b are real and imaginary parts

ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-THPML099

of the template vector. The complex template vector is the least-squares solution to the above linear matrix equation.

CAVITY PHASE STABILIZATION

We have demonstrated the CPS system in a two-stage configuration (two short cavities in Stage 1 and one long cavity in Stage 2). Fifteen equal-amplitude pulses (5×3) were stacked into one single output pulse in multiplexed 2+1 cavities. The enhancement factor and the stacking efficiency were 11.0 and 76%, compared to the theoretical limits of 12.0 and 80% respectively.

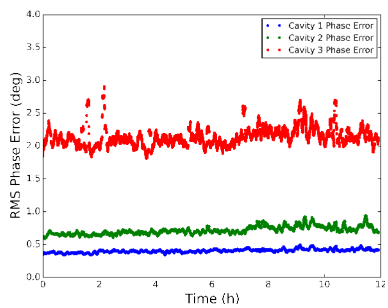

Figure 4: Cavity phase errors over 12 hours.

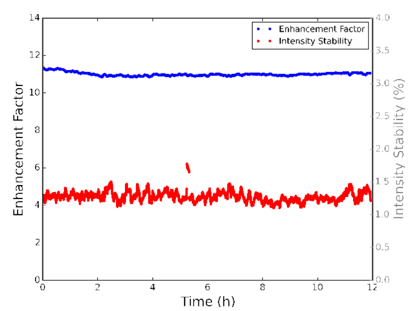

Figure 5: Enhancement factor and stacked pulse intensity stability over 12 hours.

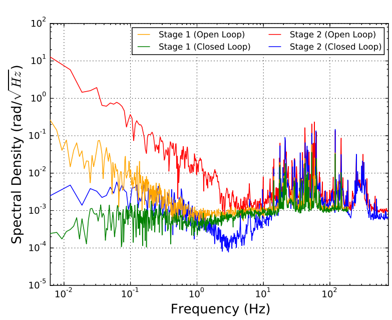

Figure 6: Noise spectrum of the cavity phase.

The cavity phase should be stabilized within a fraction of optical wavelength against thermal drift, acoustic perturbation and mechanical vibration by proper feedback control of a piezo-driven mirror for each cavity. The stacking system was controlled by the FPGA for 12 hours, while logging the cavity phase error in degrees and the stacked pulse intensity. As shown in Fig. 4, the phases of Cavity 1 and Cavity 2 in Stage 1 were maintained within 0.4 deg (RMS) and 0.7 deg (RMS), while the phase of Cavity 3 in Stage 2 was stabilized at 2.1 deg (RMS) phase error over 12 hours. Fig. 5 shows the peak-power enhancement factor versus time, during which the long-term intensity stability of the single output pulse was kept within 1.2% (RMS).

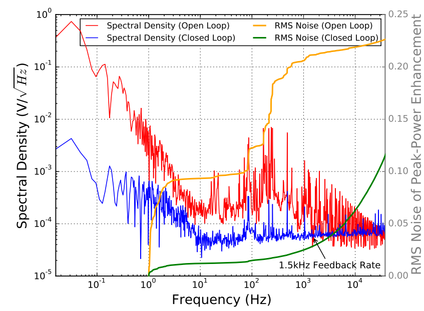

Figure 7: Noise spectrum of the stacked pulse.

Since the cavity phase is controlled to ensure the stability of the stacked pulse, we measured the noise spectrum of the cavity phase and the stacked pulse. Fig. 6 shows the noise spectrum of the cavity phase sampled at 1.5 kHz rate over 3 minutes. Fig. 7 shows the noise spectrum of the single stacked pulse sampled at 100 kHz rate over 1 minute. The closed loop suppressed the in-band noise significantly compared to the open loop.

CONCLUSION

In summary, we have assembled a Z-domain model describing the optical interference process, and two deterministic algorithms extracting the cavity phase from limited data where only the pulse intensity is available. The optical cavity phase control on FPGA with nm accuracy has been demonstrated over 12 hours, and ensures the multiplexed 2+1 cavities (15-pulse) stacking at a 1.2% level intensity stability. Based on the stabilization of the optical cavity phase, CPS in the fiber amplifier system is a promising technique combining high average power and high repetition rate.

REFERENCES

[1] W. Leemans, and E. Esarey, "Laser-driven plasma-wave electron accelerators", *Phys. Today*, vol. 62, no. 3, pp. 44–49, 2009.

[2] A. Klenke *et al.*, "Coherent beam combination of ultrafast fiber lasers", *IEEE J. Sel. Top. Quant.*, vol. 24, no. 5, p. 0902709, 2018.

[3] T. Zhou *et al.*, "Coherent pulse stacking amplification using low-finesse Gires-Tournois interferometers", *Opt. Express*, vol. 23, no. 6, pp. 7442–7462, 2015.

[4] Y. Xu *et al.*, "Extracting cavity and pulse phases from limited data for coherent pulse stacking", *Chin. Opt. Lett.*, vol. 16, no. 4, p. 040701, 2018.

[5] Y. Xu *et al.*, "FPGA-based optical cavity phase stabilization for coherent pulse stacking", *IEEE J. Quantum Elect.*, vol. 54, no. 1, p. 1600111, 2018.