Space Charge Effects on the Third Order Coupled Resonance

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## Acknowledgments

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</tr>
</tbody>
</table>
Overview

The case for accelerators and projects

Space charge effect in nonlinear rings

One dimensional resonances

The coupled resonances

Experiment and simulations

Conclusion / Outlook
Motivation: the FAIR project

- SIS100 beam parameters:
  - Every ion from p to U
  - U\textsuperscript{28+} -ions for RIB production:
    - 5x10\textsuperscript{11} / cycle
    - Rep. rate: 0.5 Hz
    - Energy: 400–2715 MeV/u
Motivation: the FAIR project

GSI

Winfried A. Barth - TUPVA055 Further Investigations for a Superconducting cw-LINAC at GSI
Lars Bozyk - TUPVA056 Ionization Loss and Dynamic Vacuum in Heavy Ion Synchrotrons
Stephanie Deveaux - MOPIK127 FAIR Risk Management as a Proactive Steering Tool for the Large Scale Multi Project
Manuel Heilmann - MOPVA054 High Power RF Coupler for the CW-Linac Demonstrator at GSI
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Egbert Fischer - WEOCB2 Superconducting Magnets at FAIR
Michael Frey - THPIK015 Prototype Results of the ESR Barrier-Bucket System
Carl M. Kleffner - TUPVA058 The Status of the FAIR pLinac
Harald Klingbeil - THPIK016 Status of the SIS100 RF Systems
Sergio Mauro - THPIK017 Field Uniformity Preservation Strategies for the ESS DTL: Approach and Simulations
Carsten Omet - WEPVA029 SIS100 Tunnel Design and Status
David Ondreka - TUPVA059 Overcoming the Space Charge Limit: Development of an Electron Lens for SIS18
Thomas Reichert - MOPAB034 SIS-100 BPM System: Design and Realization
Stephan Reimann - THPB096 Automated Optimization of Beam Lines Using Evolutionary Algorithms
Anna Rubin - THPV003 Status of the Beam Dynamics Design of the New Post-Stripper DTL for GSI-FAIR
Mariusz Sapinski - TUPVA060 Upgrade of GSI Hades Beamline in Preparation for High Intensity Run
Marcus Schwickert - MOPAB035 Status of Beam Diagnostics for SIS100Bernd
Robert Schlei - TUPIK045 Closed Orbit Feedback for FAIR - Prototype Tests at SIS18
Peter J. Spiller - WEPVA030 FAIR SIS100 - Features and Status of Realisation
Ralph Jeffrey Steinhagen - TUPIK046 Beam-Based Feedbacks for FAIR - Prototyping at the SIS18
Markus Vossberg - TUPIK047 FAIR Control Centre (FCC) - Concepts and Interim Options for the Existing GSI Main Control Room
Natalya Winters - MOPIK128 Integrated Project Planning as a Central Steering Tool for the Large Scale Multi Project FAIR
Stepan Yaramyshov - TUPVA061 Beam Dynamics Study for the HIM&GSI Heavy Ion sc cw-Linac

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Alexander Andreev - THPAB097 Phase Calibration of Synchrotron RF Signals
Jens Harzheim - WEPVA047 Input Signal Generation for Barrier Bucket RF Systems at GSI
Erika Kazantsyeva - WEPAB026 SUSPSIK049 B RHo-Dependent Taylor Transfer Maps for Super-FRS Dipole Magnets
Benjamin Frederic Reichardt - THPIK048 Longitudinal Beam Stabilization at FAIR by Means of a Derivative Estimation
Thibault Ferrand - THPV041 Progress in the Bunch-to-Bucket Transfer Implementation for FAIR
Herbert De Gersem - THPIK018 Simulating Cross-Magnetization Effects in Combined-Function Accelerator Magnets
Kerstin Gross - THPA098 Test Setup for Automated Barrier Bucket Signal Generation
Nicola Schweizer - THPV042 Modular Robot for Visual Inspection of the Vacuum Beamline of a Particle Accelerator
William Stern - THPV004 Pushing the Space Charge Limit: Electron Lenses in High-Intensity Synchrotrons
Dinu Mihailescu Stoica - THPB010 On the Impact of Empty Buckets on the Fermi Cavity Control Loop Dynamics in High Intensity Hadron Synchrotrons

IAP-Frankfurt

Ali Mohammad Almomani - TUPVA064 Updated Cavities Design for the FAIR pLinac
Markus Baschke - TUPAB147 The Final RF-Design of the 36 MHz-HSI-RFQ-Upgrade at GSI
Daniel Koser - THPIK021 SUSPSIK091 Structural Mechanical Analysis of 4-Rod RFQ Structures in View of a Newly Revised CW RFQ for the HLI at GSI

ITEP

Sergey Markovich Polozov - TUPAB013 Beam Dynamics Study and Electrodynamic Simulations of the CW RFQ

G. Franchetti
## Intensity Requirements in SIS18 for FAIR

<table>
<thead>
<tr>
<th>FAIR stage</th>
<th>Today</th>
<th>Stage 0 (Existing Facility after upgrade)</th>
<th>Stage 1 (Existing Facility supplies Super FRS, CR, [HESR])</th>
<th>Stage 2 (SIS100 Booster)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Ion</td>
<td>(\text{U}^{73+})</td>
<td>(\text{U}^{73+})</td>
<td>(\text{U}^{73+})</td>
<td>(\text{U}^{28+})</td>
</tr>
<tr>
<td>Maximum Energy</td>
<td>1 GeV/u</td>
<td>1 GeV/u</td>
<td>1 GeV/u</td>
<td>0.2 GeV/u</td>
</tr>
<tr>
<td>Maximum Intensity</td>
<td>(4 \times 10^9)</td>
<td>(2 \times 10^{10})</td>
<td>(2 \times 10^{10})</td>
<td>(1.5 \times 10^{11})</td>
</tr>
<tr>
<td>Repetition Rate</td>
<td>0.3 - 1 Hz</td>
<td>1 Hz</td>
<td>1 Hz</td>
<td>2.7 Hz</td>
</tr>
</tbody>
</table>

### SIS100:
- 5x10^{11} \(\text{U}^{28+}\) ions per cycle
- 3x10^{11} \(\text{U}^{28+}\) ions per second
Accelerator case

High intensity bunch stored for many turns
High intensity bunch stored for many turns

Particles subject to
Space charge

- Space charge tune-shift
- Amplitude dependent detuning
- Structure resonances
  - Collective effects
  - Impedances

Particles are subject to the nonlinear motion

- Error and structure resonances
- Dynamic aperture
- Chromatic effects
Single particle nonlinear dynamics

Error / Structure Resonances
SIS18

Dynamic Aperture: LHC

Lattice induced nonlinear resonances

\[ n_x Q_{x0} + n_y Q_{y0} = m \]

G. Guignard, CERN 78-11, (1978); A. Bazzani et al., CERN94-02 (1994).

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Space charge vs. magnets force

Example in a focusing quadrupole

Space charge tune-spread

Gaussian distribution

\[ \Delta Q_x \sim -0.2 \]
\[ \Delta Q_y \sim -0.3 \]
The space charge limit

Tolerable space charge tune-shift in order not to overlap with resonances

If resonances are too many, or the incoherent tune-shift is too large there is always a resonance overlapping

What happens if space charge tune-spread overlaps a resonance?
Example: Coasting beam and 1D resonance

PIC simulation

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Above the resonance: Large stable 3rd order islands are created
The quest of the incoherent effects of space charge

2002 2007

study of the space charge on 1D 4th order resonance

1 week

study of the space charge on 1D 3rd order resonance: proof of principle

12 days (as users)
1D third order resonance

Resonance
3 $Q_x = 13$
Bunched beam at high intensity

Large emittance growth

The bunch is shorter!

\[ \Delta Q_x = -0.04/ -0.045 \]

Space charge and resonances

If tails extend beyond acceptance, slow beam loss take place.

Pipe

Slow diffusion

If tails extend beyond acceptance, slow beam loss take place.

Bare tune

Periodic crossing of a resonance

Lattice error Resonance or Space Charge Structure Resonance
Resonance strength:
• Island size
• Tunes around fixed-points

Determined by
\[ n[q_{x0} + \Delta Q_x(X)] = N \]

Halo size is determined by the outer position of islands

Longitudinal motion and space charge drives islands away too fast

“scattering regime”
The difficulty of the coupled dynamics

\[ Q_x + 2Q_y = 11 \]
The difficulty of the coupled dynamics

Near the resonance $Q_x + 2Q_y = 11 \quad Q_x = 4.27, \quad Q_y = 3.3575$

Orbits become fuzzy

Very difficult to understand what is going on.
The quest of the incoherent effects of space charge

- **2002**: Study of the space charge on 1D 4th order resonance
- **2007**: Study of the space charge on 1D 3rd order resonance: proof of principle
- **2012**: Study of the space charge on 2D 3rd order resonance
- **2014**: Resonance compensation with space charge
- **2015**: Experimental discovery of the fixed-lines

- **PS**: 1 week 12 days (as users)
- **SIS18**: 1 week
- **SPS**: ~2 days
- **IOTA**: 4 days
The 2012 CERN-PS measurement campaign

[Diagram showing a map of losses as a function of horizontal and vertical tune with labels for specific tunes and loss values.]
**TABLE I. Beam and machine parameters.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intensity $N_p$ [$10^{10}$ p]</td>
<td>55</td>
</tr>
<tr>
<td>Normalized horizontal rms emittance $\varepsilon_x^{\text{nr}}$ [mm mrad]</td>
<td>3.6</td>
</tr>
<tr>
<td>Normalized vertical rms emittance $\varepsilon_y^{\text{nr}}$ [mm mrad]</td>
<td>2.2</td>
</tr>
<tr>
<td>Rms bunch length $\sigma_t$ [ns]</td>
<td>33</td>
</tr>
<tr>
<td>Rms momentum spread $\frac{\Delta p}{p}$ [$10^{-3}$]</td>
<td>0.95</td>
</tr>
<tr>
<td>Horizontal maximum tune spread $\Delta Q_{x,\text{max}}$</td>
<td>-0.05</td>
</tr>
<tr>
<td>Vertical maximum tune spread $\Delta Q_{y,\text{max}}$</td>
<td>-0.071</td>
</tr>
<tr>
<td>Sextupole current $I_{\text{SX}}$ [A]</td>
<td>2</td>
</tr>
<tr>
<td>Harmonic number $h$</td>
<td>8</td>
</tr>
<tr>
<td>RF voltage $V_{\text{RF}}$ [kV]</td>
<td>20.5</td>
</tr>
<tr>
<td>Horizontal linear chromaticity $\xi_x$</td>
<td>-0.83</td>
</tr>
<tr>
<td>Vertical linear chromaticity $\xi_y$</td>
<td>-1.12</td>
</tr>
<tr>
<td>Energy of stored beam [GeV]</td>
<td>2</td>
</tr>
<tr>
<td>Turns stored</td>
<td>497646</td>
</tr>
<tr>
<td>Storage time [s]</td>
<td>1.1</td>
</tr>
<tr>
<td>Relativistic $\beta$</td>
<td>0.948</td>
</tr>
<tr>
<td>Relativistic $\gamma$</td>
<td>3.14</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>$1163^{-1}$</td>
</tr>
<tr>
<td>Horizontal flying w. (SS68 at 422.8 m) $\beta_x$ [m]</td>
<td>12.40</td>
</tr>
<tr>
<td>Vertical flying w. (SS64 at 397.7 m) $\beta_y$ [m]</td>
<td>21.75</td>
</tr>
</tbody>
</table>

*The tune spread is calculated according to Ref. [18].*  

$\xi_{x,y} = \frac{Q_{x,y}}{Q_{x,y}} = \frac{\Delta Q_{x,y}}{Q_{x,y}}$  

$\Delta Q_{x,y} = \frac{\Delta p}{p}$
PS campaign results

$Q_{x0} = 6.104$

- $\varepsilon_{xf}/\varepsilon_{xi}$
- $\varepsilon_{yf}/\varepsilon_{yi}$
- $I_f/I_i$
Comparison with simulations
Beam Profiles for $Q_{x0} = 6.104$
Experiment-Code
Beam Profile benchmarking

\[ Q_{x0} = 6.104 \]
Resonance condition: discussion

No space charge

Distance of the resonance

\[ \Delta r_0 = Q_{x0} + 2Q_{y0} - 19 \]

Resonance condition

\[ \Delta r_0 = 0 \]

With space charge

Distance from the resonance for one particle at amplitudes \( X, Y \)

\[ \Delta r = \Delta r_0 + \Delta Q_{sc,x}(X, Y) + 2\Delta Q_{sc,y}(X, Y) \]

Resonance condition

\[ \Delta r = 0 \]

\( \Delta r_0 \) may be different from zero
Resonant particles

\[ Q_{x_0} = 6.104 \quad \Rightarrow \quad \Delta r_0 = 0.056 \]

\[ \Delta r_0 = 0.056 \]

“resonant detuning cancel \( \Delta r_0 \) and makes particle resonant”
Comparison with simulations without chromaticity

\[ Q_{x_0} = 6.104 \]
Comparison with simulations without chromaticity

\[ Q_{x0} = 6.104 \]

Something seems wrong!!

The \(-x\)-profile does not exhibit an halo that Extended up to \(X_h\)
Comparison with simulations including chromaticity

\[ Q_{x_0} = 6.104 \]

No halo in x, but only core growth

Only \(~ 5.5\sigma\), but a detuning analysis predicts \(9\sigma\)!
Comparison with simulations including chromaticity

$Q_{x_0} = 6.104$

Only $\sim 5.5\sigma$, but a detuning analysis predicts $9\sigma$ !

No halo in $x$, but only core growth

How do we understand the puzzle? Something is missing!
Missing: the coupled dynamics on the resonance

DANGER!
Fixed-lines

F. Schmidt PhD thesis, and others

G. Franchetti and F. Schmidt

G. Franchetti and F. Schmidt
http://arxiv.org/abs/1504.04389
SPS campaign on May 2015

Experiment organized by F. Schmidt

Fixed-lines do exist
Does fixed-lines play a role with space charge?

Longitudinal motion is kept frozen, so to retrieve Poincare’ section orbits.

\[ \frac{z}{\sigma_z} = 0 \quad \text{Full S.C.} \]

\[ \frac{z}{\sigma_z} = \frac{1}{2} \quad \text{Smaller S.C.} \]
Largest resonant orbits at $\frac{z}{\sigma_z} = 0$

No doubt they have a structure of fixed-lines.
Largest resonant orbits at $z/\sigma_z = 1/2$

No chromaticity

two larger resonant orbits: now the amplitude of the orbits is smaller, but still have the structure of fixed-lines
Periodic crossing of fixed-lines

Scattering process

Extended paper to appear in PRAB
Periodic crossing of fixed-lines

Scattering process

Extended paper to appear in PRAB

4 kicks per synchrotron oscillation

Resonance crossing kicks are coupled in x-y
Prediction of the halo size: the adiabatic limit

For adiabatic synchrotron motion all particles trapped are transported to the “same” fixed-line.

The sizes of this fixed-line characterize the halo/core formation.

\[ x = \sqrt{\beta_x a_x} \cos(-2t - \alpha + \pi M) \]
\[ y = \sqrt{\beta_y a_y} \cos(t) \]
Halo asymmetry explained with fixed lines
Conclusion / Outlook

- A successful experiment-code benchmarking of the beam dynamics on the 3rd order coupled resonance is carried out for the full PS structure.

- Outstanding asymmetric halo is formed well retrieved by the simulations.

- Thinking in terms of resonance detuning leads paradoxes.

- The “fixed-lines” or tori are the new objects that explain the dynamics of diffusion in a high intensity bunch subject to a coupled resonance.

- “Fixed lines” are experimentally measured in the SPS.

- Simulations show that the periodic crossing of the fixed-lines causes the asymmetric halo as result of fixed lines geometry.

- Particle seems to diffuse to “one” fixed-line → adiabatic limit.

- The doors are open for massive studies of all coupled resonances and space charge.

- Strategies to mitigate particle diffusion.
Open problems:

- Estimating the diffusion time
- Mitigation strategies:
  1) Resonance compensation
  2) E-lenses?
- Coherent vs. incoherent…
Simulations: the effect of chromaticity

How do we understand the puzzle?

Something is missing!
Resonant orbits

Each of these dots identify a resonant orbit
The difficulty of the coupled dynamics

Near the resonance 3 \( Q_x = 13 \quad Q_x = 4.335, \quad Q_y = 3.27 \)

\[ y = y' = 0 \]

\[ y \neq y' \neq 0 \]

X – Y coupling
Modes of oscillation

$2^{nd}$, even

$3^{nd}$, even

$4^{th}$, even

$2^{nd}$, odd

$3^{nd}$, odd

$4^{th}$, odd
**Space charge vs. magnet force**

**Example in a focusing quadrupole**

**Quadrupole forces**

\[ F_x = kx \]
\[ F_y = -ky \]

**Space charge forces**

\[ F_x = K \frac{x}{r^2} \left(1 - e^{-\frac{1}{2} \frac{x^2+y^2}{\sigma^2}}\right) \]
\[ F_y = K \frac{y}{r^2} \left(1 - e^{-\frac{1}{2} \frac{x^2+y^2}{\sigma^2}}\right) \]
For a Gaussian distribution

\[
\Delta Q_x = -\frac{R^2 K}{Q_x 2} \frac{1}{\sqrt{\bar{\epsilon}_x \langle \beta_x \rangle_s} \left( \sqrt{\bar{\epsilon}_x \langle \beta_x \rangle_s} + \sqrt{\bar{\epsilon}_y \langle \beta_y \rangle_s} \right)}
\]
Lattice induced nonlinear resonances

\[ n_x Q_x + n_y Q_y = m \]

G. Guignard, CERN 78-11, (1978); A. Bazzani et al., CERN94-02 (1994).

Resonant dynamics

A combination of optics, and Magnets strength

Resonance driving terms

Magnets nonlinearities drives resonances

\[ \kappa = \frac{1}{2\pi(2R)^{(N/2)}} \left| n_x \right| \left| n_z \right| \int_0^{2\pi} d\theta \beta_x |n_x|/2 \beta_z |n_z|/2 \times \]

\[ \times \exp \left\{ i \left[ n_{x \mu} x + n_{z \mu} z - (n_{x \mu} x + n_{z \mu} z - p) \theta \right] \right\} \begin{cases} (-1)^{(|n_z|+2)/2} K_z^{(N-1)} & \text{for } n_z \text{ even} \\ (-1)^{(|n_z|-1)/2} K_x^{(N-1)} & \text{for } n_z \text{ odd} \end{cases} \]
Including the chromaticity

\[ \Delta_r = \Delta_{r0} + \Delta Q_{sc,x}(X, Y) + 2\Delta Q_{sc,y}(X, Y) + Q'_x \frac{\delta p}{p} + 2Q'_y \frac{\delta p}{p} \]

**Bare tunes**

\[ \Delta_{r0} = 0.056 \]

**Effect of space charge**

AMPLITUDE DEPENDENT

- Incoherent tune-shift
  
  \[ \Delta Q_{x, max} \simeq -0.05, \]
  
  \[ \Delta Q_{y, max} \simeq -0.071 \]

\[ \mathcal{D}_{r, sc} \simeq -0.19 \]

**Effect of chromaticity**

AMPLITUDE INDEPENDENT

Consider a test particle with maximum \( \frac{dp}{p} \)

\[ +/- 0.037 \]
Resonant Particle including chromaticity

\[ Q_{x0} = 6.104 \]

\[ \Delta r \]

\[ Y / \sigma_y \, X / \sigma_x \]

Expected halo @ 9σ
Position of the islands (1D resonances)

\[ Q_x = Q_{x0} + \Delta Q_x \]
Position of the islands (1D resonances)

Approximate location of the fixed-point

$3Q_x = 13$