Abstract

The High-Energy Storage Ring (HESR) is part of the future Facility for Antiproton and Ion Research (FAIR) in Darmstadt (Germany). The HESR is designed for antiprotons with a momentum range from 1.5 GeV/c to 15 GeV/c, but will as well be suitable to provide heavy ion beams with a momentum range from approximately 0.6 GeV/c to 5.8 GeV/c. To guarantee smooth operation it is crucial to verify and to optimize the design with the help of beam-dynamics simulations.

Within recent studies [1] calculations based on a variant of the Lyapunov exponent were carried out to estimate the dynamic aperture. The studies could reproduce expected influences as reduced aperture due to tune resonances and tune shifts due to coupling. Thus, they can be extended to investigate the dynamic behaviour of the beam and identify the main restrictions to the dynamic aperture near the chosen betatron tune. As an example the influence of a coupling multipole component of the bending dipole is shown. Furthermore, ongoing measurements of the magnetic fields of the already produced bending dipoles and quadrupoles deliver a more precise insight to the harmonic content of these elements. The existing simulations can be updated by including the new measurement results. A major issue is the high uncertainty of the currently used measurement method for the dipoles. To address this problem a new device [2] to measure the harmonic content of the dipoles has been developed and is near completion.

INTRODUCTION

The basis for the studies presented in this paper are dynamic aperture calculations similar to the calculations described in [1]: The starting point is the linear lattice description of the HESR and the measured B-fields of the dipole and quadrupole magnets as input. In the following all studies are performed with the so-called injection lattice for antiprotons with a kinetic energy of \( T = 3 \) GeV. The measured transverse B-field is expressed through the multipole components \( a_n \) and \( b_n \), as defined in

\[
B_y + iB_x = \sum_{n=1}^{\infty} \frac{b_n + i\tilde{a}_n}{10^4} B_N \left( \frac{x + iy}{r_0} \right)^{n-1},
\]

with reference radius \( r_0 \) and where the main field component \( B_N \) is chosen such, that \( b_N = 10^4 \) in the upright 2N-pole. The measured values are randomized such that each element within the simulation has slightly different multipoles. Now a tracking series with the PTC module of MAD-X [3] is performed, where the minimal radius is searched at which the motion changes from stable to chaotic. This radius is considered to be the dynamic aperture. To discriminate between stable and chaotic motion at each starting point two trackings are performed, where the second tracking starts with a small initial distance from the first one. From the evolution of the distance \( d(n) \) over the number \( n \) of turns, a Lyapunov indicator \( \lambda \) as introduced by [4] is calculated:

\[
\lambda(n) := \frac{\tilde{d}_{n/2} - \tilde{d}_{0,n/2}}{d_{n/4,3n/4}},
\]

where \( \tilde{d}_{i,j} = \sum_{n=0}^{j-i} \frac{d(n+i)}{j-i+1} \) denotes the average distance of the tracked particles between turns \( i \) and \( j \). If \( \lambda(n) < 1 \) \( \forall n \in (100, 1000) \), the initial coordinate is considered to lead to stable motion, otherwise it is considered chaotic. The tracking can be repeated with a different random seed to get an estimate for the statistic uncertainties created by the varying multipole components.

SIMULATION STUDIES

As already explained in [1] tune scans are performed, where the dynamic aperture for different combinations of the horizontal tune \( Q_x \) and vertical tune \( Q_y \) is estimated. In Fig. 1 each colored square represents one tune combination for which a dynamic aperture estimation has been performed. In the proximity of tune resonances (black lines) the dynamic aperture is reduced as expected. The influence of different resonances on the dynamic aperture varies, if a different random seed is chosen, but the expected resonance pattern can be seen in all simulations.

A second obvious feature that can be seen in Fig. 1 is the gap without stable solution for beam motion near the coupling resonance \( Q_x = Q_y \). The tune combinations in the gap cannot be reached, as the phase spaces in this region are coupled, this effect is described in [5]. The width of the gap varies as well depending on the random seed. The source of the coupling, its influence on the width \( \Delta Q \) of the gap and the resulting dynamic aperture at a possible tune combination for operation of the HESR, are described in the following.
The current measurement method has a rather poor reproducibility, an all dipoles in the simulations now have the same skew quadrupole component.

To investigate it in the simulation, $a_2$ is excluded from the randomization but chosen by the user instead. So all dipoles in the simulations now have the same $a_2$. All the skew markers are slightly shifted to the right. The emphasized marker belongs to the investigated $a_2$ component.

**NON-VANISHING SKEW QUADRUPOLE COMPONENT**

The measurements of the multipole content of the dipoles, as shown in Fig. 2, indicate, that there might\(^1\) be a strong skew quadrupole component $a_2$. This component is the origin of the coupling. As no dedicated skew Quadrupole elements are foreseen in the HESR, this effect cannot be corrected. To investigate it in the simulation, $a_2$ is excluded from the randomization but chosen by the user instead. So all dipoles in the simulations now have the same $a_2$. All other multipole components are still randomized. With the matching algorithms of MAD-X along with PTC, it was tried to reach a tune combination as near to the coupling resonance as possible. The nearest reachable tune combinations above and below the gap define its width $\Delta Q$. The result for different magnitudes of $a_n$ is shown in Fig. 3. The width of the gap increases linearly with $a_2$ and its value for $a_2 = 0$ is compatible with zero: $\Delta Q = m \cdot a_2$. So this skew quadrupole component can be considered as the only source of coupling present in the simulation.

Having identified the source of the non-linearities, the next step is to investigate the influence of $a_2$ to the dynamic aperture. A fixed tune combination\(^2\) ($Q_x = 7.61$ and $Q_y = 7.64$) is taken and the dynamic aperture is estimated for different magnitudes of $a_2$. The result is plotted in Fig. 4. In the region $0.4 < a_2 < 0.7$ (red background in Fig. 4) the dynamic aperture shrinks the available acceptance. But the value is still above $17.6 \times 10^{-6}$ mrad, the acceptance\(^3\) of the accumulation lattice [6]. So the calculations reveal that even with an $a_2$ in this region the HESR can be operated without limitations.

\(^1\) The current measurement method has a rather poor reproducibility, an improved setup to measure the harmonic content has been developed and is the subject of the next section.

\(^2\) This combination might be a good working point during operation, as preceding dynamic aperture calculations revealed.

\(^3\) Please note that the acceptance of the HESR is usually given as $15.6 \times 10^{-6}$ mrad based on an aperture of $39.5$ mm, including a reserve for orbit displacements of $5$ mm. In contrast the dynamic aperture calculations are given in relation to the acceptance based on the full aperture of $44.5$ mm.
IMPROVED MEASUREMENT OF THE DIPOLES

As stated before, the main uncertainty to the measurement of multipole components of the dipole is the reproducibility of the measurement itself. As a working point close to the coupling resonance is desired for operation, and the influence of the coupling in this region strongly depends on the actual multipole values the knowledge of these values is essential. Thus a new device for the measurement of the multipoles is developed. The main part consists of eight 3D Hall probes that are placed equidistantly on a disc that is mounted rotatable on a carriage (cf. Fig. 5). The carriage can be moved through the aperture of the magnets. Compared to conventional setups using rotating or vibrating wire techniques, this setup has the advantage that it can be used for measurements in strongly bent magnets, such as the HESR dipoles. With the result of the new measurement new simulations will be performed.

CONCLUSION

The tracking simulations along with the analysis based on a Lyaponov indicator can be used to study influences on the dynamic aperture. The analysis revealed that the skew quadrupole component $a_2$ is responsible for coupling, as expected. Nevertheless it does not decrease the usable acceptance to values that do not permit operation of the HESR at the investigated working point. To have a more reliable simulation basis a new measurement device is developed.

REFERENCES


