FREQUENCY CHOICE STUDIES OF ERHIC CRAB CAVITY

Yue Hao†, Yun Luo and Vadim Ptitsyn, BNL, Upton, Long Island, NY
Ji Qiang, LBNL, Berkeley, CA

Abstract

Crab crossing scheme is essential collision scheme to achieve high luminosity for the future electron-ion collider (EIC). Since the ion beam is long when cooling is not present, the nonlinear dependence of the crabbing kick may present a challenge to the beam dynamics of the ion beam, hence a impact to the luminosity lifetime. In this paper, we present the initial result of the weak-strong and strong-strong beam-beam tracking with the crab crossing scheme. The result provides beam dynamics guidance in choosing the proper frequency the crab cavity for the future EIC.

INTRODUCTION

The future electron ion collider (EIC) aims on achieving high luminosity. Therefore, all designs adopt crossing angles between the two beams, which allows fast beam separation, smaller beta function at interaction point (IP). The crossing angle leads to the geometric luminosity loss. The figure of merit to characterize the loss is the 'Piwinski Angle' \( \theta_P \):

\[
\theta_P = \frac{\sigma_z}{\sigma_x} \theta_c
\]

where \( \sigma_z \) and \( \sigma_x \) are the rms longitudinal/transverse bunch size and \( \theta_c \) is the half crossing angle.

To prevent the geometric beam loss we adopt the crab crossing scheme to recover the luminosity loss using crab cavities. The crab cavity exerts a kick on the beam that create a sinusoidal transverse tilt at IP

\[
x_c = \frac{\theta_c}{k_c} \sin(k_c z)
\]

where \( \theta_c \) is the half crossing angle and \( k_c = f_c/c \) is the wave number of the crab cavity.

In EIC, the bunch length of the ion beam is much longer than that of the electron beam. Therefore, the frequency of the ion beam crab cavity has to be considered carefully. The same frequency can be used safely for the electron. The transverse deviation the ion at \( z_i \) and the electron at \( z_e \) reads

\[
\Delta x(z_i, z_e) \sim \frac{\theta_c}{k_c} \sin \left( \frac{z_i k_c}{2} \right) - \frac{\theta_c z_i}{2}
\]

The mismatch will cause luminosity loss and potential beam dynamics problem due to synchro-betatron resonance. At very low frequency, \( k_c \rightarrow 0 \), the crab kick fully compensate the geometric loss due to the crossing angle and the deviation \( \Delta x \) vanishes. However, the cavity with very low frequency has large surface area and not feasible to be manufactured. Therefore we need to find the proper frequency of the crab cavity to satisfy both the beam dynamics requirement and hardware feasibility.

In this paper we present the studies to frequency choice of the crab cavity for the eRHIC ring-ring scheme [1] without cooling. In this case, the bunch length of the ion beam is longer, and requires more careful study. The parameter of the ring-ring scheme is listed in Table 1. We consider 4 possible frequencies \( n = 1 \) to \( n = 4 \), which corresponds to 112, 225, 338, 450 MHz. Figure 1 shows the horizontal grid lines corresponds to the rms bunch length of ion beam, the horizontal grid lines are at \( -\sigma_x \) and \( \sigma_x \).

LUMINOSITY CONSIDERATION

We use the luminosity degradation parameter \( H_L \) as one figure of merit to determine the proper frequency of the crab cavity.

\[
H_L = \frac{L_{crab-crossing}}{L_{head-on}}
\]

Table 1: Related Parameters of eRHIC Ring-Ring Scheme

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Ion</th>
<th>Electron</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crossing angle (mrad)</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>Crab cavity frequency (MHz)</td>
<td>4n × 28.15</td>
<td></td>
</tr>
<tr>
<td>Beam size (mm) at IP, horizontal</td>
<td>0.123</td>
<td>0.123</td>
</tr>
<tr>
<td>Transverse tune, horizontal</td>
<td>0.31</td>
<td>0.08</td>
</tr>
<tr>
<td>( \beta^*_x ) (m)</td>
<td>0.94</td>
<td>0.62</td>
</tr>
<tr>
<td>Longitudinal bunch length (cm)</td>
<td>7</td>
<td>0.43</td>
</tr>
<tr>
<td>Synchrotron tune</td>
<td>0.01</td>
<td>0.069</td>
</tr>
<tr>
<td>Piwinsky angle (rad)</td>
<td>6.3</td>
<td>0.4</td>
</tr>
<tr>
<td>Beam-Beam parameter, horizontal</td>
<td>0.014</td>
<td>0.093</td>
</tr>
</tbody>
</table>

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yhao@bnl.gov
The $L_{\text{head-on}}$ is the luminosity of head-on collision, which also includes the hourglass effect. For ideal crabbing scheme, or the $k_c z_i \ll 1$, the parameter $H_L$ is 1.

Figure 2 shows the luminosity degradation $H_L$ as function of the crab cavity frequency. The solid lines are from the luminosity integration, while the red dots are the luminosity calculated from strong-strong simulation after 10000 turns. In the calculation of the blue line, we assume that both ion and electron beam has the same crab cavity frequency. In this case, the luminosity loss is less than 10% if the crab cavity frequency is less or equal than 338 MHz. The yellow line in figure 2 shows the luminosity when only the ion ring has crab cavity. The short electron bunch without crab crossing will only cause additional 3% luminosity loss. With balance the manufacturing difficulty and the geometric luminosity loss, the 338 MHz cavity is a wise candidate.

**WEAK-STRONG STUDIES**

We use week-strong code Simtrack [2] to study the crab crossing beam dynamics. The electron beam is assumed to have ‘perfect’ crab crossing and set as strong beam, while the ion beam is set as the weak beam with different the crab cavity frequencies.

The weak-strong studies, from figure 3 and 4, show that the luminosity and the rms beamsize of the ion beam do not degrade with in 1M turns, for all 4 crab cavity frequencies considered. These results indicate that the tail created by the sinusoidal RF wave does not affect the beam dynamics of the ion bunch and the frequency of the crab cavity can be chosen solely from the geometric luminosity consideration.

For the 338MHz crab frequency, we can group the macro-particles by different initial longitudinal location to analyze the difference of the beam dynamics. Figure 5 shows the frequency spectrum of the beam centroid of 3 longitudinal amplitude groups: $[0, \sigma_z]$, $[2\sigma_z, 3\sigma_z]$ and $[4\sigma_z, 5\sigma_z]$. The first group has sufficiently small longitudinal deviations and transverse offset at IP due to the nonlinear crab kick. The synchrotron line in frequency spectrum is very weak. The other two groups, in the contrary, shows strong synchrotron lines, which indicates the synchro-betatron coupling. However, the coupling effect does not cause the beamsize increase and the luminosity loss.

**STRONG-STRONG STUDIES**

In the weak-strong studies, the dynamics of electron beam is 'frozen'. Therefore the strong-strong study is required to include dynamics of the both beam. Especially, the electron will have a horizontal position shift during collision with the tail of the ion beam with horizontal crab crossing. Figure 6 shows the electron beam centroid evolution under different crab cavity frequencies. The tilted tail of the ion beam will give dipole kick to the electron beam and creates transverse offsets.

We use BeamBeam3D [3] to include the dynamics of both beams. The number of macro-particle used in the simulations are 0.5 and 2 millions for electron and proton beam.
The choice of the crab cavity frequency involves both the geometric luminosity calculation and the beam dynamics study. From the former study, 338 MHz is the candidate that balances the simpler manufacturing and luminosity loss. There is still discrepancy between the weak-strong and strong-strong beam dynamics study. More simulations are needed to evaluate importance of the electron beam orbit shift during interaction, and to justify the frequency choice.

REFERENCES

