Abstract

Direct s-channel Higgs production in $e^+e^-$ collisions is of interest if the centre-of-mass energy spread can be reduced to be comparable to the width of the standard model Higgs boson. A monochromatization scheme could be employed in order to achieve the desired reduction, by introducing a non-zero horizontal dispersion of opposite sign for the two colliding beams at the interaction point. In high-energy high-luminosity circular colliders, beamstrahlung may increase the energy spread and bunch length. The horizontal emittance blow up due to beamstrahlung, a new effect not present in past monochromatization proposals, may degrade the performance, especially the luminosity. We study, for the FCC-ee at 62.5 GeV beam energy, how we can optimize the IP optics parameters ($\beta^*_x$, $D^*_x$) along with the number of particles per bunch so as to obtain maximum luminosity at a desired target value of the collision energy spread.

INTRODUCTION

Monochromatization is a method for reducing the centre-of-mass energy (c.m.) spread at $e^+e^-$ colliders [1]. It decreases of the effective collision energy spread $\sigma_{E^*}$ without reducing the inherent energy spread $\sigma_E$ of either of the two colliding beams, by introducing correlations between spatial position and particle energy at the interaction point (IP). In beam-optical terms, this can most easily be accomplished through a non-zero IP dispersion function of opposite sign for the two colliding beams. The IP dispersion is determined by the optical lattice [2]. Implementation of a monochromatization scheme with non-zero vertical dispersion was studied theoretically for several past colliders [1–8], but until now such a scheme was never applied at any operating collider.

The concept of monochromatic collision allows for an interesting option presently under study for the FCC-ee collider [9, 10], namely the possibility of direct Higgs production in the s channel, $e^+e^- \rightarrow H$, at a beam energy of 62.5 GeV [11, 12]. This scheme could result in a useful Higgs event rate and also provide the energy precision required to measure the width of the Higgs resonance [13, 14].

The FCC-ee design considers two horizontally separated rings for electrons and positrons. For such a double ring collider, where the two beams circulate in separate beam pipes with independently powered magnets, it will be possible to modify the dispersion function for the two beams independently. In particular, a horizontal dispersion can be generated at the IP with opposite sign for the two beams. For the FCC-ee, the impact of the monochromatization on the luminosity and energy spread needs to be analyzed including the effect of beamstrahlung.

MONOCHROMATIZATION PRINCIPLE

For a standard (std) collision at c.m. energy $w = E_{b^+} + E_{b^-} = 2E_b$, the relative rms c.m.-energy spread $\sigma_w/w$, is $\sqrt{2}$ times lower than the rms relative beam energy spread, $\sigma_\delta \equiv \sigma_{E^*}/E_b$ [1, 2], where $\epsilon$ denotes a single particle’s energy deviation from the average energy $E_b$. Namely, we have $\sigma_w = (\sigma_{E^*}^2 + \sigma_\delta^2)^{1/2} = \sqrt{2}\sigma_\delta E_b$, and, therefore, $\sigma_{w,\text{std}} = \sqrt{2}E_b\sigma_\delta$ or $\sigma_{w,\text{std}}/w = \sigma_\delta/\sqrt{2}$. In a monochromatic collision, particles with energy $(E_b + \epsilon)$ collide on average with particles of energy $(E_b - \epsilon)$ and the c.m. energy spread is reduced by the monochromatization factor $\lambda$, or $\sigma_{w,\lambda} = \sqrt{2}E_b\sigma_\delta/\lambda$, where for a non-zero horizontal IP dispersion, $D^*_x \equiv D^*_x = -D^*_x \neq 0$, $\lambda$ is

$$\lambda = \frac{D^*_x}{\epsilon_x \beta^*_x} + 1,$$

(1)

and $\beta^*_x(y)$ denotes the horizontal (vertical) beta function at the IP. For monochromatized collisions beams at FCC-ee, beamstrahlung affects the values of $\sigma_\delta$ and, especially, $\epsilon_x$. This must be taken into account when computing the luminosity and the true value of $\lambda$.

BEAMSTRAHLUNG EFFECT

Beamstrahlung (BS) [15–19] is the synchrotron radiation emitted during the collision in the electromagnetic field of the opposite beam. For short bunch lengths and small transverse beam sizes, the effective bending radius due to the field of the opposing bunch is exceptionally small compared to the typical arc bending radius.

The strength of the beamstrahlung is characterized by $\Upsilon$, defined as [18, 19] $\Upsilon \equiv B/B_c = (2/3)\omega_\omega_c/E_b$, with $B_c = m_p c^3/(e\hbar)$, the Schwinger critical field, $\omega_c$ the electron or positron energy spread as defined by Sands [20], and $E_b$ the electron energy before radiation. For the collision of Gaussian bunches with rms sizes $\sigma_x^*$, $\sigma_x$ and $\sigma_\delta$, and bunch population $N_b$, the peak and average values of $\Upsilon$ are given by [19] $\Upsilon_{\text{max}} = 2r_\gamma^2\gamma N_b/((\alpha \sigma_x^*(\sigma_\delta^* + \sigma_x^*))$ and $\Upsilon_{\text{ave}} \approx (5/6)r_\gamma^2\gamma N_b/((\alpha \sigma_x(\sigma_\delta^* + \sigma_x^*))$, where $\alpha$ denotes the fine structure constant, $\alpha \approx 1/137$, $\gamma$ the relativistic Lorentz factor, and $r_\gamma$ the classical electron radius, $r_\gamma \approx 2.8 \times 10^{-15}$ m. All proposed high-energy circular colliders operate in a parameter region where $\Upsilon \ll 1$ and $\sigma_x^* \gg \sigma_\delta^*$.
In the following, we indicate the usual equilibrium parameters, determined by the arc synchrotron radiation (SR) alone, with a subindex “SR”, and the total values in collision, including the effect of beamstrahlung, with a subindex “tot”. In [21] we derived two coupled nonlinear equations for the longitudinal and transverse plane, which must be solved self-consistently for the two unknowns \( \varepsilon_{x_{\text{tot}}} \) and \( \sigma_{\delta_{\text{tot}}} \). In the case of monochromatization, with \( D_x^s \sigma_{\delta_{\text{tot}}} \gg \beta_x^s \varepsilon_x \), these equations partially decouple and simplify to [21]

\[
\varepsilon_{x_{\text{tot}}} \approx \varepsilon_{x_{\text{SR}}} + \frac{2B}{D_x^s \beta_x^s \sigma_{\delta_{\text{tot}}}^5} \tag{2}
\]

\[
\sigma_{\delta_{\text{tot}}}^2 = \sigma_{\delta_{\text{SR}}}^2 + \frac{B}{D_x^s^3 \sigma_{\delta_{\text{tot}}}^5} \tag{3}
\]

with

\[
B \approx 50 \frac{\text{IP}_T \varepsilon_{\text{SR}}}{T_{\text{rev}}} \frac{\varepsilon^5_e N_b^3 y^2}{(\alpha C C/(2\pi Q_s))^2}. \tag{4}
\]

In the approximation above, we assumed a separate function optics, for which the horizontal radiation damping time equals two times the longitudinal \( (\tau_{\text{SR}} = 2\tau_{E,\text{SR}}) \).

After solving Eq. (3) for \( \sigma_{\delta_{\text{tot}}} \), the emittance follows from Eq. (2), and the bunch length from

\[
\sigma_{z_{\text{tot}}} = \frac{\alpha_C C}{2\pi Q_s} \sigma_{\delta_{\text{tot}}}, \tag{5}
\]

with \( \alpha_C \) the momentum compaction factor, \( C \) the circumference, and \( Q_s \) the synchrotron tune.

### APPLICATIONS TO FCC-ee

In a classical monochromatization scheme, with fixed emittance, energy spread, IP beta function, and only adding opposite IP dispersion for the two beams the resulting luminosity \( L \) scales as \( \lambda^{-1} \). However, for the FCC-ee, the self-consistent values of emittance \( \varepsilon_x, \sigma_\delta \) and \( \sigma_z \) must be used for computing the true values of \( \lambda \) and \( L \).

Table 1 presents the nominal FCC-ee parameters for (non-monochromatic) collisions at 45.6 GeV and 80 GeV [10], with an interpolated head-on collision scheme at 62.5 GeV, a “baseline mono-chromatization scheme” at this same energy (obtained by adding IP dispersion to the former), and an optimized monochromatization where the bunch charge and IP beta functions have been re-optimized (plus the value of the IP dispersion in proportion to \( \sqrt{\beta_x^s} \)) — see the following discussion. The total bunch length and emittance for the two cases of \( D_x^s \neq 0 \) (the second and third column at 62.5 GeV) in Table 1 were obtained from Eqs. (2), (3), and (5). Table 1 shows horizontal emittance and beam sizes first without (“SR”) and then including the effect of beamstrahlung (“tot”) [21].

Given the resonance width of the standard model Higgs boson of 4.2 MeV and the much larger natural rms energy spread of the electron and positron beams at 62.5 GeV of about 40 MeV, the monochromatization factor should be large, at least \( \lambda \sim 5 \) [22]. Requesting \( \lambda \sim 10 \), to have some margin, while considering the emittance and energy-spread values due to arc synchrotron radiation alone, from Table 1, the necessary value of the IP dispersion is given by \( D_x^s \beta_x^s \approx 10^{-2} \) m. Using this value, the baseline monochromatization scheme in the second 62.5 GeV column of Table 1 was obtained from the first column. The smaller \( \beta_x^s \) can be made, the smaller the horizontal beam size becomes, and the lower the luminosity loss compared with a zero-dispersion collision. Since the horizontal beam size with monochromatization, dominated by the dispersion, is much larger than the corresponding beam size for a standard collision scheme, the effects of beamstrahlung are small in the longitudinal plane [21].

### Table 1: Baseline beam parameters for FCC-ee crab-waist (CW) collisions at the Z pole and at the WW threshold [10], compared with newly proposed parameters for operation on the Higgs resonance (beam energy \( E_e = 62.5 \text{ GeV} \)) both in a simple head-on (h.-o.) collision scheme, and with “baseline” or “optimized” monochromatization (m.c.), always considering \( n_{\text{IP}} = 2 \) identical IPs. The parameter \( \theta_c \) is the full crossing angle, and \( \xi_{x(y)} \) the horizontal (vertical) beam-beam parameter.

<table>
<thead>
<tr>
<th>Scheme</th>
<th>CW</th>
<th>h.-o.</th>
<th>m.c.</th>
<th>m.c.</th>
<th>CW</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>basel.</td>
<td>opt’d</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E_e ) [GeV]</td>
<td>45.6</td>
<td>62.5</td>
<td>62.5</td>
<td>62.5</td>
<td>80</td>
</tr>
<tr>
<td>( I_b ) [mA]</td>
<td>1450.3</td>
<td>408</td>
<td>408</td>
<td>408</td>
<td>152</td>
</tr>
<tr>
<td>( N_b ) [10^{10}]</td>
<td>3.2</td>
<td>1.05</td>
<td>3.3</td>
<td>1.11</td>
<td>6.0</td>
</tr>
<tr>
<td>( n_{\text{IP}} ) [1]</td>
<td>91.5</td>
<td>81</td>
<td>25.8</td>
<td>7.7</td>
<td>5.3</td>
</tr>
<tr>
<td>( D_x^s ) [mm]</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( \beta_x^s ) [mm]</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( \sigma_x ) [mm]</td>
<td>9.5</td>
<td>9.2</td>
<td>132</td>
<td>186</td>
<td>16</td>
</tr>
<tr>
<td>( \sigma_x ) [( \mu m )]</td>
<td>9.5</td>
<td>9.2</td>
<td>144</td>
<td>189</td>
<td>16</td>
</tr>
<tr>
<td>( \sigma_y ) [( \mu m )]</td>
<td>45</td>
<td>45</td>
<td>45</td>
<td>32</td>
<td>45</td>
</tr>
<tr>
<td>( \sigma_z ) [nm]</td>
<td>0.09</td>
<td>0.17</td>
<td>0.17</td>
<td>0.17</td>
<td>0.26</td>
</tr>
<tr>
<td>( \varepsilon_{x,\text{SR}} ) [nm]</td>
<td>0.09</td>
<td>0.17</td>
<td>0.21</td>
<td>0.70</td>
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<td>( \varepsilon_{x,\text{SR}} ) [( \mu m )]</td>
<td>1.6</td>
<td>1.8</td>
<td>1.8</td>
<td>1.8</td>
<td>2.0</td>
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<td>( \varepsilon_{x,\text{SR}} ) [( \mu m )]</td>
<td>3.8</td>
<td>1.8</td>
<td>1.8</td>
<td>1.8</td>
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<tr>
<td>( \sigma_{\delta,\text{SR}} ) [%]</td>
<td>0.04</td>
<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
<td>0.07</td>
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<tr>
<td>( \sigma_{\delta,\text{SR}} ) [%]</td>
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<td>0.06</td>
<td>0.06</td>
<td>0.06</td>
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</tr>
<tr>
<td>( \theta_c ) [mrad]</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>( C ) [km]</td>
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<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>( \alpha_C ) [10^{-6}]</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>( \tau_{E,\text{rev}} ) [s]</td>
<td>1320</td>
<td>509</td>
<td>509</td>
<td>509</td>
<td>243</td>
</tr>
<tr>
<td>( Q_s ) [( 10^7 )]</td>
<td>0.025</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.04</td>
</tr>
<tr>
<td>( T_{\text{max}} ) [10^{-11}]</td>
<td>1.7</td>
<td>0.8</td>
<td>0.3</td>
<td>0.85</td>
<td>4.0</td>
</tr>
<tr>
<td>( \theta_c ) [mrad]</td>
<td>30</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>( \xi_x ) [10^{-2}]</td>
<td>5</td>
<td>12</td>
<td>1</td>
<td>22.2</td>
<td>7</td>
</tr>
<tr>
<td>( \xi_y ) [10^{-2}]</td>
<td>13</td>
<td>15</td>
<td>4</td>
<td>6.76</td>
<td>16</td>
</tr>
<tr>
<td>( \lambda ) [1]</td>
<td>1</td>
<td>1</td>
<td>9.2</td>
<td>5.08</td>
<td>1</td>
</tr>
<tr>
<td>( L ) [pb^{-1} s^{-1}]</td>
<td>0.9</td>
<td>0.2</td>
<td>0.10</td>
<td>0.37</td>
<td>0.2</td>
</tr>
<tr>
<td>( \sigma_w ) [MeV]</td>
<td>58</td>
<td>53</td>
<td>5.8</td>
<td>10.4</td>
<td>113</td>
</tr>
</tbody>
</table>
In order to profit from the larger horizontal beam size we scan the bunch charge $N_b$ (along with the number of bunches $n_b$), and the IP beta function (along with the IP dispersion) until we find the maximum luminosity for a selected value of $\lambda$. Specifically, searching for an optimal point in solution space, we first reduce $\beta_x^*$ from the nominal value of 2 mm to 1 mm, which is permitted by the present bunch population of $N_b = 1.1 \times 10^{11}$. This solution is the optimized monochromatization scheme of Table 1.

The data of Figs. 1 and 2 can be combined and the dependencies $L(e_{\text{tot}}(T,S))$ and $\lambda(e_{\text{tot}}(T,S))$ are analyzed simultaneously. This allows us to determine the maximum achievable luminosity for a given value of $\lambda$. The result is displayed in Fig. 3. As our main result, for the requested minimum value of $\lambda \approx 5$ (5.08) we obtain a luminosity of $L = 3.74 \times 10^{35}$ cm$^{-2}$s$^{-1}$ at $T = 0.3$ and $S = 1.4$, which corresponds to $\beta_x^* = 1.96$ m and $D_x^*$ = 0.308 m, and to a bunch population of $N_b = 1.1 \times 10^{11}$. This solution is the optimized monochromatization scheme of Table 1.

Figure 3: Optimal luminosity as a function of $\lambda$.

CONCLUSIONS

We have derived FCC-ee IP beam parameters which would result in a factor 5–10 monochromatization at high luminosity. Accounting for the increase of the horizontal emittance due to beamstrahlung and non-zero IP dispersion, for a baseline monochromatization scheme a luminosity of about $10^{35}$ cm$^{-2}$s$^{-1}$ can be reached on the Higgs resonance with an effective collision energy spread below 6 MeV. Indeed, beamstrahlung results in a significant horizontal blow up and a concomitant degradation of the monochromatization. Nevertheless, by raising the optical functions in the horizontal plane ($D_x^*$, and $\beta_x^*$ in proportion to $D_y^2$), the effect of beamstrahlung can be controlled. For each value of $D_x^*$, the luminosity is optimized by increasing the bunch charge while lowering the number of bunches, keeping the total beam current constant. For the minimum required value of $\lambda \approx 5$ (about 10 MeV rms collision energy spread) our analytical calculation suggests that the luminosity can be raised to about $4 \times 10^{35}$ cm$^{-2}$s$^{-1}$.

Considering 200 scheduled physics days per year, and a “Hübner factor” (an empirical factor relating peak and average luminosity) of 0.6, typical for PEP-II and KEKB with top-up injection, the expected annual luminosity at 62.5 GeV becomes about 2 ab$^{-1}$ for the baseline monochromatization and 7 ab$^{-1}$ for the optimized monochromatization scheme. For a c.m. energy spread around 5 MeV, commensurate with the natural width of the Higgs boson, the cross section of $e^+e^- \rightarrow H$ is about 290 ab [24]. Assuming this cross-section value, the monochromatized FCC-ee would produce between 500 and 2000 s-channel Higgs bosons per IP per year.

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Figure 1: Self-consistent monochromatization factor $\lambda$ in the $(S,T)$-plane including beamstrahlung effects.

Figure 2: Self-consistent luminosity in the $(S,T)$-plane including beamstrahlung effects.
REFERENCES


