CHARACTERIZING THE NONLINEAR PERFORMANCE OF A DLSR WITH THE EFFECTIVE ACCEPTANCE OF THE BARE LATTICE*

Y. Jiao†, Z. Duan, G. Xu, Key Laboratory of Particle Acceleration Physics and Technology, Institute of High Energy Physics, Beijing, China

Abstract

In a diffraction-limited storage ring (DLSR) light source, associated with the strong focusing and sextupoles, the detuning terms are large and integer and half integer resonances can be reached at small momentum deviation and transverse amplitudes. We propose to use the effective ring acceptances of the bare lattice to characterize the nonlinear performance of the actual ring, by considering the limiting effects of integer and half integer resonances on beam dynamics. Such a concept is believed to be very useful in lattice design of a DLSR light source. In this paper, we will discuss the reasoning, verification, and application range of this definition.

MOTIVATION OF PROPOSING THE EFFECTIVE RING ACCEPTANCE

In storage ring light sources, strong quadrupoles are usually used to attain a lowest possible beam emittance, which however, will induce large natural chromaticities. Consequently, strong sextupoles are needed to compensate for the chromaticities. The strong fields, together with the machine imperfections, will cause detuning effects (tune shifts with amplitude and momentum deviation) and excite resonances from low to high order, leading to orbit diffusion and even unstable motions. Among these resonances, integer and half integer resonances are induced by linear field errors, whose widths depend on the setting of the tunes and the level of the linear field errors but are independent of the betatron amplitudes of particles [1]. Different from the integer and half integer resonances, the higher order resonances are driven by nonlinear fields (e.g., sextupole fields) or nonlinear field imperfections, and their strengths are generally weak neighboring the ideal particle trajectory, but grow rapidly with increasing amplitude. Therefore the higher order resonances usually have strong impact on dynamics only for large amplitudes.

In the third generation light sources (TGLSs) widely built around the world, experiences indicated that through suitably arranging the sextupoles along the ring and delicately tuning the sextupole strengths and the nominal tunes, the sextupole-induced aberrations can be greatly cancelled or minimized, resulting in small resonance driving terms and detuning terms. The betatron tunes can be kept far enough away from the integer and half integer resonances even for large betatron amplitudes or momentum deviations, and the beam dynamics is usually dominated by higher order resonances.

However, in a diffraction-limited storage ring (DLSR) [2] with natural emittance one or two orders of magnitude lower than available in a TGLS, the situation tends to be quite different. To reach an ultralow emittance, the double-bend achromat or triple-bend achromat lattices that are commonly used in TGLS designs are no longer desirable; and instead, multi-bend achromat (MBA) lattices are usually used in DLSR designs. Furthermore, novel design philosophies (e.g., the so-called ‘hybrid’ MBA [3]) and small-aperture magnets are adopted to make the DLSR design more compact and cost effective. On the other hand, since even stronger focusing is used, the natural chromaticities and sextupole strengths in a DLSR are much larger than those in a TGLS. Furthermore, scientists usually push the emittance down to be close to its lowest limit so as to achieve a highest possible brightness. In such a design, even with the most advanced analytical (e.g., Lie algebra, global or local nonlinearity-cancellation approaches) and numerical optimization techniques (e.g., multi-objective genetic algorithm, frequency map analysis), it is still hard to simultaneously reduce the resonance driving terms and detuning terms to a sufficiently small level. Consequently, the resonances near the nominal tunes are reached for small betatron amplitudes or momentum deviations. The impact of integer and half integer resonances on dynamics cannot be avoided. Crossing of these resonances may cause unstable motion and particle loss.

It is believed that the integer resonances, when excited, are always fatal to dynamics and can never be crossed. The half integer resonances are less fatal, but are also dangerous. It was reported that in several TGLSs it is feasible to approach or even cross the half integer resonances without beam loss (see, e.g., [4]), with the state-of-art optics correction technique. In a DLSR, since the linear optics is generally pushed to its extreme, the nonlinear dynamics is more sensitive to machine imperfections. Statistical numerical studies based on the HEPS design [5] indicated that the probability of MA reduction due to crossing of half integer resonances is closely correlated with the level of beta beats at the nominal tunes. To reach a small MA reduction probability of about 1%, the rms amplitude of beta beats should be kept below 1.5% horizontally and 2.5% vertically.

Considering the limiting effects of integer and half integer resonances in a DLSR, we propose the ‘effective’ ring acceptance of the bare lattice (without taking any error into account), such as the ‘effective’ dynamic aperture (DA) and the ‘effective’ momentum acceptance (MA) or the ‘effective’ local MA (LMA). Within the effective DA or MA, it is required not only the motion remains stable after tracking over a few thousand turns (traditional definition of the DA or MA), but also the tune footprint is

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† jiaoyi@ihep.ac.cn

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bounded by the integer and half integer resonances nearest to the nominal tunes of the storage ring. In the following, taking a 60-pm lattice of the HEPS storage ring as an example, we will show that the ‘effective’ DA and MA of the bare lattice can give a reasonable estimation of the ring acceptance of an actual DLSR with practical errors. Then we will discuss the application of such a concept.

**VERIFICATION OF THE EFFECTIVE RING ACCEPTANCE**

The High Energy Photon Source (HEPS), is a 6-GeV, kilometre-scale storage ring light source to be built in Beijing, China. After several iterations in the past few years, we obtained a lattice (see Ref. [6] and references therein) for the storage ring that consists of 48 hybrid 7BA s, and has a circumference of 1295.6 m and a natural emittance of ~ 60 pm (denoted as 60-pm lattice hereafter).

To verify the effectiveness of the proposed concept, taking this 60-pm lattice design as an example, we do comparisons between the ‘effective’ ring acceptance of the bare lattice and the ring acceptance in the presence of practical errors.

In each 7BA of the 60-pm lattice, six sextupoles and two octupoles are used and grouped into 4 families. To optimize the nonlinear dynamics, the nominal tunes are scanned by varying the quadrupole strengths, and for each set of fractional tunes a grid scan of the multipole strengths is performed, with the aim to find an optimal set of multipole strengths as well as optimal values of the nominal tunes of the ring. In the multipole strength scan, the sextupoles are grouped in three families (SF, SD1 and SD2) and octupoles in one family. Since two sextupole families are required to correct the chromaticity to (+0.5, +0.5), only two free knobs are left for nonlinear optimization, which make it possible to do the grid scan in a reasonable time.

With the resulting setting of multipole strengths and only the bare lattice, the chromatic curve, i.e., variation of tunes with respect to momentum deviation, is shown in Fig. 1. The ring acceptances at the center of the 6-m straight section, projected in the (x, y) and (x, δ) planes, are shown in Fig. 2.

The results are obtained from numerical tracking with the AT program. In the simulation, the momentum deviation is assumed to be constant and the RF system and synchrotron radiation effects are not considered. Although optimized, the amplitude-dependent detuning terms and higher order chromaticities are still very large. Note that that when using only the bare lattice, particle motions remain stable as crossing the integer and half integer resonances.

Nevertheless, we think that when these resonances are more excited in the presence of errors, crossing of these resonances probably causes beam loss and obvious DA and MA reduction. Thus, we calculate the corresponding effective DA and LMA, with the results shown in Figs. 3 and 4, respectively.

![Figure 1: Chromatic curve for the 60-pm bare lattice.](image)

![Figure 2: Ring acceptances at the center of the 6-m straight section, for the 60-pm bare lattice. The colors, from blue to red, represent the stability of particle motion, from regular to irregular.](image)

![Figure 3: Effective DA and the corresponding frequency map for the 60-pm bare lattice. The black curve represent the traditionally defined DA of the bare lattice.](image)

![Figure 4: Effective LMA for the 60-pm bare lattice.](image)
The comparison between the LMA with errors and the effective LMA of the bare lattice is also made, with the results shown in Fig. 6. One can see a reasonable agreement between the effective DA and LMA of the bare lattice and those with errors. It indicates that the effective DA and LMA of the bare lattice can give a reasonable estimation of the actual ring acceptance.

**APPLICATION OF THE EFFECTIVE RING ACCEPTANCE**

In our view, the concept of ‘effective’ ring acceptance of a bare lattice can be applied to the following two scenarios,

**Preliminary Lattice Design Stage**

In the preliminary lattice design stage, one usually has only the bare lattice in hand. The detailed error study might not be done yet and lattice calibration process has not been systematically simulated.

In such a case, the effective DA and MA of the bare lattice can provide a quick and reasonable estimation of the nonlinear performance of the actual machine, and can serve as a better standard to the assessment of nonlinear dynamics, compared to the DA and MA of the bare lattice calculated in the traditional way.

**Global Optimization of the Lattice Design**

Using the effective DA and MA in the global optimization of the lattice design (see, e.g., Ref. [7]) can greatly save computing time and resonances.

To precisely evaluate the actual ring acceptance in the presence of errors, it needs to generate a large enough number of ensembles of random errors, simulate the lattice calibration process, and then perform number tracking to calculate the DAs and LMAs. An alternative way (more simple but less precise) is to generate a large number of ensembles of errors but with such small rms amplitudes that the deviations of ring parameters are in the ranges that are expected to be achieved after lattice calibration in a practical machine. Let us assume the number of error ensembles is 100. Even in the second approach, one has to repeat the DA and MA calculation for 100 times, the DA and MA calculation may take 100 times longer time than calculate the effective DA and MA of the bare lattice.

This will make great difference when optimizing the lattice with stochastic optimization methods. In such an optimization, a population is generated first, evolved generation by generation, until good convergence is reached. Let us consider a case where the population size is 2000, the number of generations is 100, and one time of DA and MA evaluation takes 1 s on a computing cluster. If using the effective acceptance of the bare lattice to form the objective, it needs about 2.3 days to finish the optimization; while if using actual DA and MA (by considering 100 sets of random errors) it will need more than 6 months!

One concern about the effective ring acceptance of the bare lattice is that it assumes both integer and half integer resonances are dangerous and cannot be safely crossed. Concentrating too much on these resonances, especially the half integer resonances, may result in suboptimal solutions. It is lucky that we obtain multi (but not ‘infinite’) instead of only one ‘optimal’ result from the stochastic optimization, and we can then do further error modelling and lattice calibration simulation for these obtained solutions, and finally choose one robust design. In this way, this problem can be resolved to some extent. One can avoid to get a ‘far from best’ solution, and can separate the lattice optimization and the lattice evaluation process.

**REFERENCES**