CORRECTION OF BETA-BEATING DUE TO BEAM-BEAM FOR THE LHC AND ITS IMPACT ON DYNAMIC APERTURE∗

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Abstract

Minimization of the β-beating at the two main interaction points of the LHC arising from the head-on and long-range beam-beam interactions can be performed by adjusting the strength of quadrupole or sextupole correctors. This compensation scheme is applied to the current LHC optics where the results show a significant reduction of the peak and RMS β-beating; and the impact on the dynamic aperture is computed. A proposal for a similar strategy to be adopted in the High Luminosity LHC is also discussed.

INTRODUCTION

When the bunches of two beams of a particle collider come into proximity, they interact electromagnetically and give rise to beam-beam effects [1, 2], such as tune shift and β-beating, which in turn may limit the performance of the machine. Control of this phenomena is important since it can lead, for example in the case of the LHC, to luminosity imbalance between the two main experiments [3]. The force exerted on a particle is, in the case of Gaussian transversal profiles, only dependant of the radial distance r from the center of the other bunch. The horizontal and vertical kicks are given [4] by

\[
\begin{align*}
\{ \Delta x' \} & = -\frac{2N r_0}{\gamma r^2} \left\{ \begin{array}{c} x \\ y \end{array} \right\} \left[ 1 - \exp \left( -\frac{r^2}{2\sigma^2} \right) \right], \\
\{ \Delta y' \} & = -\frac{2N r_0}{\gamma r^2} \left\{ \begin{array}{c} x \\ y \end{array} \right\} \left[ 1 - \exp \left( -\frac{r^2}{2\sigma^2} \right) \right],
\end{align*}
\]

where \( r^2 = x^2 + y^2 \), \( \sigma \) is the beam size at the interaction point (IP), assumed to be equal in the horizontal and vertical coordinates; \( \gamma \) is the relativistic Lorentz factor, \( r_0 \) the classical particle radius, and \( N \) the bunch population.

In the following section, analyses of the expansion of the latter equation are presented to guide the multipolar compensation of the beam-beam kick. The correction scheme is first applied to the LHC and then followed by a discussion for the High Luminosity LHC (HL-LHC).

BEAM-BEAM EXPANSION AND CORRECTION SCHEME

Head-on Beam-Beam

For small amplitudes, linearisation of Eq. (1) around the origin leads to the definition of the beam-beam parameter

\[
\xi = \frac{N r_0 \beta^*}{4\pi \gamma \sigma^2},
\]

as a measure of the induced tune shift. In this regime, the HO kick can be seen as a quadrupole error in the lattice, and due to its cylindrical symmetry, it is focusing or defocusing in both the horizontal and vertical planes, depending on the charges (opposite or equal).

Ideally, this effect would be corrected locally and close to its source. Nevertheless, since the HO has the same effect on both planes, and the fact that beams travel in opposite directions, common magnets are not considered for the correction scheme. Therefore, in order to compensate the HO beam-beam effect, variation of the strength of quadrupole magnets in the matching sections is implemented.

Long-Range Beam-Beam

Collisions of multi-bunch beams yield to parasitic long-range (LR) interactions. Assuming a horizontal crossing angle, the Taylor expansions at second order around a point in the horizontal plane at a distance equal to the beam separation \( d \), are

\[
\Delta x' = K_0 + (K_1 + K_1') \Delta x + (K_2 + K_2') (\Delta x)^2 - K_2 (\Delta y)^2, \\
\Delta y' = -K_1 \Delta y - 2K_2 \Delta x \Delta y,
\]

with

\[
\begin{align*}
K_0 &= N r_0 \left( \frac{1 - E_d}{\gamma d} \right), \\
K_1 &= N r_0 \left( \frac{1 - E_d}{\gamma d^2} \right), \\
K_1' &= \frac{2N r_0 E_d}{\gamma \sigma^2}, \\
K_2 &= N r_0 \left( \frac{1 - E_d}{\gamma d^3} - \frac{E_d}{2\sigma^2 d} \right), \\
K_2' &= \frac{2N r_0 E_d d}{\gamma \sigma^4},
\end{align*}
\]

and \( E_d \equiv \exp \left( -\frac{d^2}{2\sigma^2} \right) \). These functions are plotted in Fig. 1, where \( K_{2,a} \) and \( K_{2,b} \) are defined as the two monomials in \( K_2 \). For a large separation, the terms \( K_1' \) and \( K_2' \) can be neglected and the first- and second- order terms correspond to those of a pure quadrupole and sextupole; moreover, \( K_2 \) tends to \( K_{2,a} \) for large separations. This is consistent with the expansion in [5] for \( d \gg \sigma \). When the beams cross in the vertical plane, the expansions \( \Delta x' \) and \( \Delta y' \) are interchanged,

\[
\begin{align*}
\Delta x' &= -K_1 \Delta x - 2K_2 \Delta x \Delta y, \\
\Delta y' &= K_0 + (K_1 + K_1') \Delta y - K_2 (\Delta x)^2 + (K_2 + K_2') (\Delta y)^2,
\end{align*}
\]

and the beam separation \( d \) lies on the vertical plane.
By adjusting dedicated magnet strengths to counteract its multipole components $K_i$, it is possible, in principle, to counteract the effect on the optics due to the LR kick. For example, pairs of quadrupole magnets at both sides of the IP can be adjusted in a way such that the first order term of the LR beam-beam expansion is cancelled, or pairs of sextupole magnets are powered to compensate either $K_2$, or alternatively, $K_1$ by a process of feed-down.

Note that the correction has to be applied to both beams in the machine. The strengths of common magnets are transformed between the two counter-rotating beams according to the following expression

$$K_n \rightarrow (-1)^n K_n,$$

that is, it remains the same for sextupoles, decapoles, etc., and the sign reverses for quadrupoles, octupoles, etc. Moreover, the reference systems of the counter-rotating beams are connected by a $180^\circ$-rotation around the vertical axis, $x \rightarrow -x$, $y \rightarrow y$, and $s \rightarrow -s$. These considerations allow using the interaction region (IR) sextupoles for the compensation of the LR quadrupolar component for both beams via feed-down.

Due to the sign change between the two beams for octupole magnets it is not possible to directly compensate beam-beam octupolar components for both beams using such magnets; however, it can be shown that global octupole tune spread can be corrected at least partially [6].

**IMPLEMENTATION IN THE LHC**

Beta-beating induced by beam-beam interactions is studied for the LHC with the 2016 optics ($\beta^* = 40$ cm, half-crossing angle of $140 \mu\text{rad}$ at IP1 and IP5) for an intensity of $1.2 \times 10^{11}$ particles per bunch, $2.5 \mu\text{m}$ of normalized emittance, a collision energy of $6.5\text{ TeV}$ [7], and with zero-current octupoles. Compensation of the head-on and long-range effect are first analysed separately per IP, and then combined in a model that includes both effects at the two main experiments of the LHC. First experimental tests to measure $\beta$-beating from beam-beam can be found in [8].

The correction of the beating was performed by locally re-matching the optics (horizontal and vertical $\beta$ and $\alpha$ functions) at the beginning and at the end of each IR separately, as well as at the corresponding IP. This results in eight degrees of freedom per beam—since $\beta_{x,y}$ and $\alpha_{x,y}$ at the start of the IR are given as initial conditions—per IP, for which eight variables (four left-right pairs of magnets for each beam) are adjusted by a process involving a series of minimization routines available in MAD-X [9]. The tunes and chromaticities are then re-matched to $(64.31, 59.32)$ and 2 units, respectively, for all the cases presented in this work.

**Head-on Beam-Beam**

Head-on beam-beam interactions at IP1 and IP5 yield a peak $\beta$-beating in both planes of around $3\%$ when studied separately, and and up to $7\%$ ($3\%$) horizontally (vertically) when both IPs are taken into account simultaneously.

Partial compensation of this effect is achieved by re-matching the strengths of the four quadrupoles Q4 to Q7 in both IPs as described before. By applying this process the RMS $\beta$-beating can be lowered significantly: from $3.7\%/1.9\%$ down to $0.3\%/0.2\%$ for the case of HO at both IP1 and IP5, as seen in Table 1.

**Long-Range Beam-Beam**

With a bunch separation of $25\text{ ns}$, 16 long-range encounters occur at each side of the two IPs, taking place inside the inner triplet. Although every single LR collision has lower impact on the beam than a HO, their effect adds up.

When studying the effect on the two main interaction points separately, the strengths of sextupoles MCSSX for IP1 and MCSX for IP5 are involved in the matching; the use of skew or normal sextupoles is due to the vertical and horizontal crossing angles, respectively, in the IPs. The original strengths of these magnets is zero, and their resulting

<table>
<thead>
<tr>
<th>Beam-beam</th>
<th>H/V RMS $\beta$-beating [%]</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Before</td>
</tr>
<tr>
<td>HO at IP1</td>
<td>2.30 / 1.81</td>
</tr>
<tr>
<td>at IP5</td>
<td>2.22 / 1.93</td>
</tr>
<tr>
<td>at IP1 and IP5</td>
<td>3.67 / 1.91</td>
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<tr>
<td>LR at IP1</td>
<td>2.13 / 2.28</td>
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<tr>
<td>at IP5</td>
<td>2.18 / 2.26</td>
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<tr>
<td>at IP1 and IP5</td>
<td>2.69 / 3.84</td>
</tr>
<tr>
<td>HO and LR at IP1</td>
<td>0.18 / 4.08</td>
</tr>
<tr>
<td>at IP5</td>
<td>4.49 / 0.42</td>
</tr>
<tr>
<td>at IP1 and IP5</td>
<td>4.39 / 3.79</td>
</tr>
</tbody>
</table>

Table 1: RMS $\beta$-Beating due to HO and LR Beam-Beam at IP1 and/or IP5 in the LHC Before and After Correction

Figure 1: Dipolar (top), quadrupolar (center), and sextupolar (bottom) terms in the LR kick multipolar expansion.
magnitudes are listed in Table 2. After the correction is implemented (either for beam 1 or beam 2), the RMS β-beating reaches almost zero for these two cases; an average between the strengths found for each beam has to be implemented in a simultaneous correction.

Combined Head-on and Long-Range Effects

Finally, when a more realistic model with HO and LR beam-beam interactions at the two main IPs is studied, the implementation of a joint correction scheme for both effects shows a reduction of the RMS β-beating to less than 0.15% in both planes, proving to be effective, as seen in Table 1 and Fig. 2. The peak β-beating is lowered from 7% horizontally and 6% vertically to around 0.9% in both planes, once the correction is implemented. In this case, the quadrupoles Q4 to Q6 at both IPs, and MCSSX (at IP1) and MCSX (at IP5) are used in a global compensation of the two effects. Correction reduces tunes by 0.01 and increases chromaticities by about two units. Tunes and chromaticity are then matched to nominal values. An identical process is followed to correct the β-beating in the opposite beam and similar results are obtained.

Since the correction scheme makes use of the adjustment of non-linear optics elements, its implication in terms of the single-particle stability of the beam has to be addressed. Sextupole strengths coming from the matching process with the same (opposite) sign than the sextupolar term of the LR Sextupole strengths coming from the matching process with the single-particle stability of the beam has to be addressed.

The DA is computed for the corrected lattice for beam 1 via particle tracking for 10^6 turns with the SixDesk environment for SixTrack [10, 11], with the beam settings described at the beginning of the previous section. The results in Fig. 3 do not exhibit a significant reduction with respect to the case prior the implementation of the correction, and it remains above 5.5σ for all angles. The DA has yet to be verified for the opposite beam to ensure its simultaneous applicability.

HL-LHC

Beta-beating of almost 15% is observed [2] in the presence of head-on and long-range beam-beam interactions in both IP1 and IP5 of the nominal HL-LHC (β^* = 15 cm) at collision energy (7 TeV) with an intensity of 2.2 × 10^{11} particles per bunch and 2.5 μm of normalized emittance. In this case, the working point is (62.3, 60.32) and the chromaticities are re-matched to 3 units. Contrary to LHC, HL-LHC is equipped with decapoles in the triplet region, which will allow to locally compensate the octupolar component of the long ranges via feed-down from the decapoles.

CONCLUSION

Beam-beam interactions can limit the machine operation and performance, and can even yield to luminosity imbalance due to β-beating between the two main experiments in the case of the LHC; hence, depending on their magnitude, compensation of their effect might be crucial. Aspects of machine protection have to be studied in more detail, since they affect particles with large-amplitude oscillations.

In particular, the β-beating induced by such interactions can be corrected, at least partially, by the proper adjustment of the strengths of the magnets around the IPs where such effects originate. The presented scheme has been applied successfully to the current LHC optics, showing a reduction of the RMS β-beating to less than one-percent, and with a negligible impact on dynamic aperture.

Further studies are needed to optimize the correction scheme described here, and its application to the HL-LHC—for which the use of higher order multipoles is envisioned—is on-going.

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Figure 3: Dynamic aperture in the LHC before and after correction of HO and LR beam-beam at IP1 and IP5 (2 units of chromaticity, I_{oct} = 0 A).

Figure 2: Beta-beating due to HO and LR beam-beam at IP1 and IP5 in the LHC before and after correction.
REFERENCES


