Abstract
The aim of the Jülich Electric Dipole moment Investigations (JEDI) collaboration is the measurement of the Electric Dipole Moment (EDM) of charged particles like protons or deuterons. There are two possible concepts under consideration for the realization of EDM measurement with deuterons; the “Frozen Spin” (FS) and “Quasi-Frozen Spin” (QFS) method. Both approaches are discussed and compared in this paper. Detailed spin- and beam dynamics simulations are performed to investigate the effect of various misalignments of ring elements and systematic effects. Furthermore the utilization of counter rotating beams is studied and checked for its validity.

INTRODUCTION
Permanent EDMs of subatomic particles violate parity $P$ and time reversal $T$ symmetry. Assuming the CPT theorem, this leads to $CP$ violation. The Standard Model (SM) predicts non-vanishing EDMs, their magnitudes, however, are expected to be not measurable with current experimental techniques. The discovery of a nucleon EDM larger than $10^{-31}$ e cm [1] would be a signal for new physics beyond the SM and could explain the matter-antimatter asymmetry observed in our Universe [2]. The measurement of an EDM is based on the observation of a spin precession in the presence of strong electric fields. So far, upper EDM limits of electrically neutral systems and charged leptons have been measured.

Charged-particle measurements have been proposed for EDM searches of protons and deuterons in storage rings [3], since the electric field also bends the particle’s trajectory. These experiments require a new class of high-precision storage rings to significantly improve the technology for polarized beam storage and measurement by orders of magnitude [4], as well as the knowledge of and control over the beam and spin itself.

The JEDI Collaboration is working on a series of feasibility studies at the Cooler Synchrotron COSY in Jülich and in parallel on the design study for a dedicated EDM storage ring [5,6].

LATTICE DESIGN
The FS and QFS concepts are investigated for a deuteron EDM storage ring. In the FS concept the spins are always aligned to the momentum vector [3] which can be achieved with combined static electromagnetic beam deflectors ($E \times B$ deflectors). The QFS concept is based on the fact that the anomalous magnetic moment of deuterons has a small value [7]. Magnetic and electric fields can be spatially separated with magnetic arcs and either pure electric field or combined straight electromagnetic field sections connecting the two arcs. The rotation of spin relative to the momentum in two parts of ring -- the arc and straight sections -- can compensate each other, leading a quasi-frozen spin rotation turn by turn. This concept has the advantage that curved $E \times B$ deflectors don’t have to be utilized. Two possible lattice options are considered (see Fig. 1) [8], where the elements with the combined electric and magnetic fields are used. The FS lattice has been designed based on deflectors with incorporated E and B fields with an electric field of 12 MV/m and a magnetic field of 0.46 T. The QFS lattice only used two magnetic arcs with a magnetic field of 1.5 T and two straight sections having combined straight elements with magnetic and electric fields. The straight elements have a horizontal electric field of 12 MV/m and a vertical magnetic field of 80 mT. They provide the compensation for the spin rotation in the arc and at the same time allow having straight electric plates without higher order field components. Despite the difference in the concepts, the basic parameters, namely the size of the rings, the length of the straight sections, the number of focusing periods, and the number of deflecting and focusing elements and sextupoles, all remain approximately the same.

Figure 1: FS lattices with incorporated electrostatic and magnetic fields (top) and QFS lattice with straight electrostatic deflectors (bottom).
In both options, the free dispersion straight sections are placed between the arcs. They are necessary to accommodate the polarimetry, the beam injection and extraction systems, and RF cavity [8].

**BEAM AND SPIN TRACKING**

**Simulation Program**

The COSY Toolbox (COTOBO) has been utilized to perform the simulations. It is based on a C++ based interface [9] for COSY INFINITY [10]. The usability of ROOT [11] enables a fast and easy way to analyze the simulation results. $E \times B$ deflectors are the basis of the FS concept. Consequently, these elements had to be added to COTOBO. COSY INFINITY [12] contains elements to simulate $E \times B$ deflectors which are called Wien Filter. The implementation of $E \times B$ deflectors is realized such that the simulation of fringe fields and misalignments is possible. In COSY INFINITY fringe fields are approximated by

$$F(z) = \frac{1}{1 + \exp(\sum_{n=0}^{\infty} A_n \cdot [z/2d]^{2n})} \quad (1)$$

where $A_n$ are Enge coefficients, $z$ is the coordinate on the field axis where the origin is the effective edge of the element, $d$ is the aperture and $2d$ is the full aperture of the element. The old default Enge coefficients of electric fields in COSY INFINITY were replaced by optimized coefficients [13].

**Simulation Results for CW-CCW Beams**

Detailed beam and spin tracking simulations have been performed to investigate the effect of systematic errors for the EDM build-up.

In the range of 1000 to 10000 particles have been tracked with transverse emittances of $\varepsilon_{x,y} = 1 \text{ mm mrad}$ (rms) and a momentum spread of $\Delta p/p = 10^{-4}$. Due to the sideways component of the spin with respect to the momentum vector, the vertical spin build-up for the QFS method will be reduced compared to FS. Simulation results indicate that this reduction is below 2% and can therefore be neglected.

As an example, the simulation results for vertical spin build-up are shown in case of deflector rotations in Fig. 2 and vertical quadrupole shifts in Fig. 3 [14]. The magnitude of the EDM signal is described by the dimensionless scaling factor $\eta$, with $|d_{edm}| \approx \eta \cdot 5.3 \cdot 10^{-15} \text{ e cm}$.

These simulations indicate that the largest difference for artificial spin build-up between FS and QFS are unwanted longitudinal fields due to misalignments of ring elements that occur if $E \times B$ deflectors are rotated around the radial axis (see Fig. 2). Due to the orientation of the spin vectors parallel aligned to the beam motion, longitudinal fields do not disturb the spin motion in the FS concept in comparison to the QFS. The sideways component of the spin creates an additional artificial spin build-up, that can be derived from the Thomas-BMT equation [15] and reads

Figure 2: Vertical spin build-up for FS (top) and QFS (bottom) and different magnitudes $\eta$ of EDM signal and Gaussian distributed rotations of $E \times B$ deflectors (RMS values) around the radial axis. Each simulation has different randomly generated misalignments.

Figure 3: Vertical spin build-up for FS (top) and QFS (bottom) and different magnitudes of EDM and Gaussian distributed vertical shifts of quadrupoles (RMS values). Each simulation has different randomly generated misalignments.
\[ \frac{ds_{\text{EDM}}}{dt} = \frac{e}{m} \left[ (a + \frac{1}{\gamma}) \left( \begin{array}{c} -s_y B \\ 0 \\ s_x B \end{array} \right) - \frac{ay}{\gamma + 1} \left( \begin{array}{c} -s_y B \\ 0 \\ s_x B \end{array} \right) \right] \]

(2)

where \( e, m \) are the mass and charge and \( a \) is the anomalous magnetic moment of the particle, \( \beta, \gamma \), the kinematic Lorentz factors, \( s_x, s_y \), the sideways spin components and \( B \) the longitudinal magnetic field.

Therefore, rotations of \( E \times B \) deflectors around the radial axis are more sensitive for the investigated QFS lattice compared to the presented FS lattice. Concerning unwanted radial magnetic fields the sensitivity for FS and QFS is comparable as can be seen from Fig. 3 for shifts of quadrupoles in vertical direction [14].

Smaller contributions of non-frozen spin motion also show up in the FS method. Due to the motion of the particles in transverse and longitudinal phase space -- even in the ideal case without misalignments or fringe field of ring elements -- only the reference particle is perfectly frozen. In general, also the fringe field distribution for electric and magnetic fields have a different shape. Without any extra efforts of chopping the end regions of the elements (see Fig. 1) a sideways component of the spin will also appear in the fringe field region for FS.

**Simulation Results for CW-CCW Beams**

Counter-rotating beams, clockwise (CW) and counter-clockwise (CCW), enable the extraction of the magnitude \( \eta \) of an EDM signal for the simulated misalignments of both kinds of rings. However, a small systematic contribution for rotations of deflectors around the radial axis remains utilizing the CW-CCW method. Furthermore it was assumed that the polarity change of the magnetic field is perfect. Thus errors regarding the field reversal must be simulated. One CW-CCW simulation is exemplary discussed [14], which includes the CW-CCW simulation results of rotation misalignments of \( E \times B \) deflectors around the vertical axis of the FS lattice. If the influence of this misalignment on the spin motion is negligible the spin build-up for both directions seems to be the same (see Fig. 4, upper plot). In Figure 4 (lower plot) the scaling factor \( \eta \) is extracted and determined correctly within the analysis errors. The resulting values of \( \eta_0 \) confirm the above assumption.

**CONCLUSION**

For a perfect ring without misalignments the FS and QFS methods are proved and work equally well. However, unavoidable misalignments of ring elements prevent a measurement of the EDM with highest sensitivity. To counteract systematic errors CW-CCW beams are utilized. Assuming a perfect reversal of magnetic fields without influencing the electric field strength in \( E \times B \) deflectors of a deuteron EDM storage ring, this procedure is a certified measure to handle errors caused by rotations of the deflectors. Furthermore, this method suppresses the misleading effect by shifts of quadrupoles.

A possibility to minimize the systematic error in QFS storage rings would be to decrease the sideways spin component. E.g. a QFS lattice should be modified in such a way, that alternating pairs of magnetic deflectors and straight \( E \times B \) deflectors are used, leading to a polygon structure of the arc sections.

It is unavoidable to investigate different types of lattices (e.g. weak focusing structure) to make final conclusions for the sensitivity of EDM measurements with FS and QFS methods. In addition, 3D field maps have to be integrated in the simulation code and electric field plate and fringe field distribution of \( E \times B \) deflectors optimized.

**ACKNOWLEDGEMENT**

The authors would like to thank all members of the JEDI and srEDM collaborations for fruitful discussions. We are especially very grateful to M. Berz and K. Makino for supervising the upgrades of COSY Infinity, M. Rosenthal for developing COTOBO and Yu. Senichev for providing the two ring lattices.

This work has been financially supported by an ERC Advanced-Grant (srEDM # 694390) of the European Union.
REFERENCES


