STUDIES OF BEAM LIFETIME AT HEPS*

S.K. Tian †, G. Xu, Y. Jiao
Key Laboratory of Particle Acceleration Physics and Technology, Institute of High Energy Physics, Chinese Academy of Sciences, 100049 Beijing, China

Abstract
The High Energy Photon Source (HEPS), a kilometre scale storage ring light source, with a beam energy of 6 GeV and transverse emittances of a few tens of pm.rad, is to be built in Beijing and now is under design. In addition to the brilliance of the emitted radiation, the beam lifetime is another important quality criterion for a synchrotron radiation source. In order to prepare and perform an experiment, constant beam position and intensity without interruption may sometimes be more important than high brilliance or photon flux. The electron storage ring’s beam lifetime consists of two components: the residual gas scattering lifetime and Touschek lifetime. The residual gas lifetime is comprised of the elastic and inelastic scattering on nuclei (single Coulomb scattering), Touschek scattering involves scattering of particles within the bunch. In this paper we calculates only the elastic scattering on nuclei (single Coulomb scattering), inelastic scattering on nuclei (bremsstrahlung) of the residual gas lifetime is comprised of the elastic and inelastic scattering lifetime component and Touschek lifetime.

HEPS RING PARAMETERS
High Energy Photon Source (HEPS), which proposed to be built in Beijing, was a storage ring light source with beam energy of 6 GeV. Extensive efforts have been made on the lattice design and relevant studies of this project. A hybrid 7BA design for the HEPS has been made, this design with a 59.4 pm·rad natural emittance, ~3% MA and dynamic aperture (DA) ~2.5 mm in x and 3.5 mm in y plane provide a basic for the further studies to be based on [1]. The main parameters were listed in Table 1.

Table 1: HEPS Lattice Design Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy $E_0$</td>
<td>6 GeV</td>
</tr>
<tr>
<td>Beam current $I_0$</td>
<td>200 mA</td>
</tr>
<tr>
<td>Circumference</td>
<td>1295.6 m</td>
</tr>
<tr>
<td>Natural emittance $\varepsilon_0$</td>
<td>59.4 pm.rad</td>
</tr>
<tr>
<td>Working point $\nu_0/\nu_b$</td>
<td>116.16/41.12</td>
</tr>
<tr>
<td>Natural chromaticity (H/V)</td>
<td>-214/-133</td>
</tr>
<tr>
<td>No. of super-periods</td>
<td>48</td>
</tr>
<tr>
<td>ID section length $L_{ID}$</td>
<td>6 m</td>
</tr>
<tr>
<td>RMS energy spread</td>
<td>7.97×10^{-4}</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>3.74×10^{-5}</td>
</tr>
<tr>
<td>Energy loss per turn</td>
<td>1.995 MeV</td>
</tr>
</tbody>
</table>

More details of this design and related studies are shown in Ref. [1]. The lifetime study were mainly based on this design and will be reported below.

DEFINITION OF THE BEAM LIFETIME
The lifetime $\tau$ of a beam containing $N$ particles is defined through its relative loss rate at a given time [2]:

$$1/\tau \equiv -\dot{N}/N,$$  \hspace{1cm} (1)

What sometimes causes confusion is that the beam lifetime is not constant. Generally, the lifetime is not the time interval after which the beam has reached 1/e of its initial current. This would only be true for a purely exponential decay given by:

$$\dot{N}(t) = -a_i N(t)$$

$$N(t) = N(0)e^{-a_i t}$$  \hspace{1cm} (2)

$$\tau_{1/2}(0) = -N(0)/\dot{N}(0) = 1/a_i,$$

as would be the case for residual gas scattering under always constant conditions. In reality, the residual gas pressure depends on the stored beam current, and the observed decay of current does not follow an exponential law. For electron–electron scattering (Touschek effect) the following relations hold:

$$\dot{N}(t) = -a_z N^2(t)$$

$$N(t) = N(0)/(1 + N(0) a_z t)$$  \hspace{1cm} (3)

$$\tau_{1/2}(0) = -N(0)/\dot{N}(0) = 1/a_z N(0),$$

Here, the inverse of the relative loss rate corresponds to a “half-life” time, but its value is by no means constant. Furthermore, the condition $N \sim N^2$ is violated if the bunch volume depends on current, as it usually does. It is certainly not meaningful to relate the lifetime values of Eq. (2) and Eq. (3) by a factor of ln 2. The only precise statement is the loss rate at a given time:

$$\dot{N}(t) = -a_i N(t) - a_z N^2(t)$$

$$N(t) = N(0) \frac{\exp(-t/\tau_{1/2})}{1+(1-\exp(-t/\tau_{1/2})) \cdot \tau_{1/2}/\tau_{1/2}}$$  \hspace{1cm} (4)

Since lifetime is observed and measured for relatively short time intervals just the loss rates for $t \approx 0$ may be added. Total beam life time $\tau$ related to different processes:

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i}$$  \hspace{1cm} (5)
BEAM LIFETIME

Touschek Lifetime

The high bunch densities in low emittance electron storage rings lead to an enhanced rate of elastic collisions between electrons within the bunch. This Coulomb scattering of charged particles in a stored beam causes an exchange of energies between transverse and longitudinal motions and is referred to as a Touschek effect. When the small transverse momentum of the particle is transferred into a large longitudinal momentum due to scattering, the energy change may be large resulting in bunch particles scattered outside the RF bucket or the momentum aperture of the lattice and be lost. The average Touschek lifetime is determined by the bunch volume, the bunch current, and the energy acceptance. For an actual beam, the variation of the beam envelope, i.e. the derivatives of the horizontal and vertical amplitude functions and dispersion, should be taken into account in evaluating the particle loss rate variation along the lattice, the formula for the Touschek lifetime is given by [3]:

\[
\frac{1}{\tau_{\text{Touschek}}} = \left( \frac{r_0^2 c N_b}{8 \pi \gamma^2 \sigma} \right) F(\delta_m, B_1, B_2) \ldots \left( \frac{1}{\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_\delta^2 D_x D_y + \beta \delta m}} \right)
\]

(6)

Where \( \sigma_x, \sigma_y, \sigma_\delta \) are the bunch average dimensions, \( \gamma \) is the relativistic factor, \( \delta_m \) is the momentum acceptance, \( N_b \) is number of charged particles per bunch, \( r_0 = 2.82 \times 10^{-15} \) is the classical radius of the electron, \( c \) is the speed of light. \( F \) is a function of the \( \beta \) amplitude functions, dispersion and momentum acceptance of the ring.

The Touschek lifetime depends on the momentum acceptance in the ring, and thus we have calculated the momentum acceptance due to first order optics as a function of location in HEPS: In tracking, at a given position \( s \) a beam particle is given a relative (positive) momentum kick \( \delta_m \), and it undergoes betatron oscillation. The largest value of \( \delta_m \) for which the particle survives defines the positive momentum aperture at position \( s \). Then the same is done for a negative momentum kick. Based on the ideal linear lattice with 3.5% RF energy acceptance \( (V_{\text{RF}} \approx 2.65 \text{MV}) \), the typical value of local momentum acceptance for HEPS is obtained by using ELEGANT [4] or AT.

Figure 1 shows the typical value of momentum acceptance for HEPS ideal linear lattice is \( \delta_m \approx 4\% \). Using the \( \delta_m \) as shown in Fig.1, Touschek lifetimes were calculated for the low charge mode (200mA with 648 buckets) and high charge mode (200mA with 60 buckets). Note that some calculations are based on the IBS determined steady-state beam sizes and bunch length with the harmonic Landau cavities (\( \sigma \approx 32 \text{mm} \)). The results were shown in Table 2.

Table 2: Touschek Lifetime for Different Bucket Currents

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \sigma_[h] )</th>
<th>( \tau_[h] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare Lattice</td>
<td>8.1</td>
<td>0.75</td>
</tr>
<tr>
<td>Bare Lattice with IBS</td>
<td>9.6</td>
<td>1.39</td>
</tr>
<tr>
<td>Bare Lattice with LC</td>
<td>47.4</td>
<td>4.39</td>
</tr>
<tr>
<td>Bare Lattice with LC and IBS</td>
<td>49.5</td>
<td>5.60</td>
</tr>
</tbody>
</table>

We also do some work on momentum acceptance affected by magnetic field error, misalignment effect [5]. In this study, we assumed 30–50μm displacement and 100μrad rotation r.m.s error for each magnet element. For each magnet, a random relative error was added to the original field, the scale for different type magnets list as blow:
- Quadrupole magnets: 0.02%;
- Sextupole magnets: 0.1%;
- Bending magnets: 0.1%;

After orbit and optics correction, the local momentum acceptance which are displayed in Fig.2 are used in Touschek lifetime calculation and the results are shown in Fig.3. Note that the Touschek lifetime calculation with errors is 36 hours for low charge mode and 4 hours for high charge mode as we set the tenth-lowest set as the reference standard. It will eventually probably have longer lifetime than shown.
Residual Gas Scattering Lifetime

The beam chamber in a storage ring does not have a perfect vacuum and the particles in the beam can scatter off the atoms of the residual gas. There are essentially two dominant processes in electron beam-gas interactions: the elastic scattering on nuclei and the Bremsstrahlung on nuclei.

Elastic Scattering

Elastic scattering on nuclei of the residual gas leads to an angular kick for the betatron motion. If the induced amplitude exceeds the transverse acceptance (physical or dynamic aperture) of the ring, the particle gets lost. The beam lifetime due to elastic scattering for the residual gas pressure $P$ is given by [6]:

$$\tau_{\text{elastic}} = \frac{1}{2\pi r_0^2 c N_A} \frac{P}{R} \int \frac{\beta_y}{\bar{A}} \rho_{x,y} \left( \sum_i Z_i (Z_i + 1) N_i r_{pi} \right),$$

where $N_A$ is Avogadro’s number, $R = 8.314 \, J/(K \cdot mol)$ is the universal gas constant, $T(K)$ is the residual gas temperature, $P(\text{Pa})$ is pressure, $n$ is the residual gas partial components number, $r_{pi}$ is partial fraction of the residual gas components, $Z_i$ is atomic number, $N_i$ is number of atoms per molecule, $A_{\text{min}}$ is the transverse acceptance (the minimum value of dynamic aperture and physical aperture) which was shown in Fig.4.

![Figure 4: Dynamic aperture and physical aperture at the centre of the long straight section.](image)

Inelastic Scattering

Inelastic scattering (Bremsstrahlung) is an effect of deceleration and photon emission due to beam interaction with the residual gas atoms. The electron gets lost if its relative momentum deviation exceeds the limiting momentum half-aperture of the ring, this limiting momentum half-aperture could be due to the RF bucket momentum half-height or to physical or dynamical aperture considerations.

The lifetime due to bremsstrahlung is given by [6]:

$$\tau_{\text{inelastic}} = \frac{4 \pi r_0^2 c N_A}{137 R} L(\delta_m) \frac{P}{T} \left[ \int \frac{\beta_y}{\bar{A}} \rho_{x,y} \right] \left( \sum_i \frac{183}{Z_i^{1/3}} Z_i (Z_i + \xi_i) r_{pi} N_i \right),$$

where $\xi_i$ is the radioactivity of nuclei.

$$\xi_i = \ln \left( \frac{1440 Z_i^{2/3}}{\ln (183 Z_i^{1/3})} \right),$$

$$L(\delta_m) = 4/3 \cdot \left( \ln \left( 1/\delta_m \right) - 5/8 \right)$$

With $\delta_m$ is the momentum acceptance as explained in section Touschek lifetime, other variables are explained in section elastic scattering.

The presence of the residual gas, quantified by its pressure, causes the electron hit the wall or be ejected out of the bunch and get lost. We assumed the pressure in the vacuum chamber $P = 1 \, \text{nTorr}$, the residual gas temperature $T = 293.15K$, the nature of gases present in the HEPS storage ring and their partial pressure as follow, then the lifetime due to elastic scattering and inelastic scattering is given in the Table 3:

Table 3: Different Cases of Residual Gas Composition for Beam Lifetime

<table>
<thead>
<tr>
<th>Case</th>
<th>H$_2$</th>
<th>H$_2$O</th>
<th>CH$_4$</th>
<th>CO</th>
<th>CO$_2$</th>
<th>N$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case1</td>
<td>0.65</td>
<td>0.02</td>
<td>0.07</td>
<td>0.22</td>
<td>0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>Case2</td>
<td>0.48</td>
<td>0.24</td>
<td>0.00</td>
<td>0.10</td>
<td>0.05</td>
<td>0.13</td>
</tr>
</tbody>
</table>

The dependences of the elastic scattering lifetimes on the residual gas pressure are given in Fig.5 for residual gas composition case 2.

![Figure 5: Residual gas lifetime vs pressure for residual gas composition case 2.](image)

CONCLUSION

Beam lifetime studies are performed on the HEPS 7BA ideal linear lattice. For residual gas composition case 2, the integrated beam lifetime is at the level of 32.5 hours for low charge mode and 5.3 hours for high charge mode with the 10% coupling of the horizontal and vertical oscillations, including harmonic cavity and intra-beam scattering effects. Future lifetime studies will include more detailed error setting and impedance effects, etc.
REFERENCES


