Abstract
In High Energy Accelerator Research Organization, SuperKEKB project is progressing toward upgrade. This project aims improvement luminosity \(8 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}\) which is 40 times of the performance of the KEKB accelerator. In Phase 1 of SuperKEKB, a performance test as storage ring was carried out. Understanding of ring impedance/wake is an important subject in phase I. Measurement of head tail damping using Turn-by-Turn monitor was also performed to evaluate impedance/wake. Betatron motion is excited by kicker and its damping is measured for several sets of bunch current and chromaticity in both HER and LER. The wake field was calculated from the decrement of betatron amplitude. We present the wake field which is cross-checked with tune shift based on the current dependence.

INTRODUCTION
Phase 1 of SuperKEKB was operated from 1 February to 28 June 2016. In Phase 1, interaction region including final quads is not installed. Performance tests as a storage ring were performed. Impedance evaluation, optics study, vacuum conditioning, finding of problems and countermeasures caused by high current operation were important subjects. The operation of Phase 2 is scheduled for about 5 months from 2018. Phase 2 aims at the luminosity \(10^{34} \text{ cm}^{-2}\text{s}^{-1}\) achieved at KEKB.

Understanding of wake/impedance is important problems in recent high intensity accelerator design. Head-tail damping caused by wake is sometimes used to suppress instabilities. In Phase 1, measurements of current dependent tune shift and head-tail damping using Turn by Turn monitor (TbT monitor) were performed to evaluate wake.

BETATRON TUNESHIFT
Betatron tune shift is expressed by the following equation:

\[
2\pi \Delta \nu = -i \frac{N_{\text{rc}}}{\sqrt{\beta \gamma}} \sigma Z_{\text{eff}}.
\]

\[
Z_{\text{eff}} = \frac{\sigma}{\sqrt{\beta \gamma}} \int_{-\infty}^{\infty} Z(\omega) e^{-\frac{i(\omega - \omega_0)^2 \sigma^2}{\epsilon^2}} d\omega.
\]

\(Z_{\text{eff}}\) is the effective impedance of dipole mode. \(\beta\) is averaged beta, \(\beta \sim L/2\pi v \sim 1\text{m}\). The real part and imaginary part of \(\Delta \nu\) correspond to tune shift, growth or damping, respectively. The effective impedance/wake can be evaluated by measurement of tune shift and head-tail damping. The parameters shown in Table 1 are used.

EXPERIMENT
Head-tail Damping
Measurements using a TbT monitor were performed in both horizontal and vertical directions in LER and HER. A bunch is kicked by a kicker. We measured damping of the betatron motion by changing bunch current \(I\) and chromaticity \(\xi\) [1]. Figure 1 shows the damping per current in the chromaticity 3.1, and figure 2 shows the damping per chromaticity at 0.5 mA of the current. From the figure 1 and 2, it can be seen that the damping becomes faster as either the current or the chromaticity increases.

<table>
<thead>
<tr>
<th>Table 1: SuperKEKB Parameters</th>
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<tbody>
<tr>
<td><strong>LER</strong></td>
</tr>
<tr>
<td>Circumference, L (m)</td>
</tr>
<tr>
<td>Energy, E (GeV)</td>
</tr>
<tr>
<td>Bunch pop., N (10^{10})</td>
</tr>
<tr>
<td>Bunch length, (\sigma) (mm)</td>
</tr>
<tr>
<td>Hor. Tune, (v_x)</td>
</tr>
<tr>
<td>Vert. Tune, (v_y)</td>
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<tr>
<td>Syn. Tune, (v_s)</td>
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</table>

Figure 1: Horizontal amplitude data in LER. The center of oscillation was shifted and plotted. As the current increases, damping becomes faster.

Figure 2: Horizontal amplitude data in LER. The center of oscillation was shifted and plotted. As the chromaticity increases, damping becomes faster.
The betatron oscillation data obtained from one monitor is 4096 turns. Fast Fourier transform (FFT) was applied every 256 turns. From the obtained amplitude plot using FFT, the damping rate \( T_0/\tau \) was determined by fitting \( \exp(-T_0/\tau) \), where \( T_0 \) and \( \tau \) are revolution time and damping time in second. The figure 3 shows the result of FFT of the betatron oscillation.

![Figure 3: Evolution of FFT amplitude of the betatron oscillation data where current is 0.5 mA and chromaticity is 3.1.](image)

Figures 4-7 show the horizontal and vertical damping rates in LER and HER. The horizontal axis represents the product of the current and the chromaticity, and the vertical axis represents the damping rate per turn, \( T_0/\tau \). The difference in color of the plot represents the difference of chromaticity. As the current and the chromaticity become larger, the damping becomes faster.

![Figure 4: Horizontal damping rate as function of current-chromaticity product in LER.](image)

![Figure 5: Vertical damping rate as function of current-chromaticity product in LER.](image)

![Figure 6: Horizontal damping rate as function of current-chromaticity product in HER.](image)

![Figure 7: Vertical damping rate as function of current-chromaticity product in HER.](image)

The plots were fitted with a linear function:

\[
\frac{T_0}{\tau} = aI + b.
\]

Table 2 shows the slope(\(a\)) and offset(\(b\)).

<table>
<thead>
<tr>
<th>Ring</th>
<th>(a) [A]</th>
<th>(b) [A]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LER Horizontal</td>
<td>0.157</td>
<td>(4.79 \times 10^{-4})</td>
</tr>
<tr>
<td>LER Vertical</td>
<td>0.160</td>
<td>(4.32 \times 10^{-4})</td>
</tr>
<tr>
<td>HER Horizontal</td>
<td>0.110</td>
<td>(4.70 \times 10^{-4})</td>
</tr>
<tr>
<td>HER Vertical</td>
<td>0.081</td>
<td>(6.48 \times 10^{-4})</td>
</tr>
</tbody>
</table>

**Tune Shift**

Tune is measured for changing the bunch current in LER and HER. Figures 8 and 9 shows the tune as a function of the bunch current. The plots were fitted with a linear function:

\[
v = v' I + v_0.
\]

The slopes \(v'\) and \(v_0\), in units of mA\(^{-1}\), are also shown in the figure.
ANALYSIS

The real and imaginary part of effective impedance are determined given by the tune shift and head-tail damping rate using Eq. (1), where the real part is divided by chromaticity. Table 3 shows the effective impedance. We assume a broad band resonator (Q=1) as impedance/wake model [2]:

\[ W_m(z) = W \frac{\omega_R}{\alpha} e^{\frac{\alpha z}{\epsilon}} \sin \frac{\omega z}{\epsilon}, \]  

\[ Z(\omega) = \frac{WQ}{1+iQ(\frac{\omega_R-\omega}{\omega R})}. \]  

The wake strength (W) and resonator frequency (\( \omega_R \)) are determined from the real and imaginary parts of the effective impedance.

Imaginary part in Eq. (1), \( \text{Im}[\Delta \nu] = \frac{T_0}{(2\pi \tau)} \), is zero for zero chromaticity, because of \( Z(-\omega) = -Z^*(\omega) \). \( \text{Im}[\Delta \nu] \) is linear for chromaticity when \( \omega_R \sigma/c \gg 1 \) and \( \omega_R \ll \omega_R \). \( \text{Re}[\Delta \nu] \) does not depend on the chromaticity.

The tune shift, which is linearly for IW, is represented by:

\[ (\text{Re}[\Delta \nu] + i\text{Im}[\Delta \nu]) \approx \frac{Lr_B}{\beta \pi^2 \epsilon^3 c} (iW \times A(\omega_R) + iW\xi \times B(\omega_R)). \]

\[ \frac{i\text{Im}[\Delta \nu]/\xi}{\text{Re}[\Delta \nu]} = \frac{B(\omega_R)}{A(\omega_R)}, \]

where \( A = \text{Im}[Z_{\text{eff}}]/W \), \( B = -\text{Re}[Z_{\text{eff}}]/(W\xi) \).

CONCLUSION

In Phase 1 of the SuperKEKB project, the damping rate of betatron oscillation and the tune shift were measured. By using these data, transverse effective impedance was evaluated. The analysis results using the broad band Resonator Model are shown in Table 3, where the unit of \( Z \) is \( \text{s/m}^2 \). Effective impedance was estimated element-by-element by solving an electro-magnetic field. The value of LER horizontal was \( 2.17 \times 10^{-6} \text{ s/m}^2 \).

Tune shift is around 0.001 for I=1mA. Synchrotron tune is 0.025. The threshold of the LER current is about 10 times the design value. It also confirmed by simulation.

In the colliding operation, the impedance increases because collimators are set closed to beam.

Table 3: Impedance and Wake in SuperKEKB phase 1 Commissioning.

<table>
<thead>
<tr>
<th>Ring</th>
<th>( \text{Re}[Z_{\text{eff}}]/\xi )</th>
<th>( \text{Im}[Z_{\text{eff}}] )</th>
<th>( W/[m^2] )</th>
<th>( \omega_R \sigma/c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LER H</td>
<td>9.0</td>
<td>3.4</td>
<td>2.5</td>
<td>1.5</td>
</tr>
<tr>
<td>LER V</td>
<td>9.1</td>
<td>5.7</td>
<td>8.7</td>
<td>2.8</td>
</tr>
<tr>
<td>HER H</td>
<td>9.2</td>
<td>3.9</td>
<td>4.7</td>
<td>1.8</td>
</tr>
<tr>
<td>HER V</td>
<td>6.8</td>
<td>13.0</td>
<td>21.0</td>
<td>5.0</td>
</tr>
</tbody>
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REFERENCES
