Abstract

The installation of Phase 1 of CLARA, the UK’s new FEL test facility, is currently underway at Daresbury Laboratory. When completed, it will be able to deliver 45 MeV electron beams to the pre-existing VELA beamline, which runs parallel to CLARA. Phase 1 consists of a 10 Hz photocathode gun, a 2 m long S-band travelling wave linac, a spectrometer line, and associated optics and diagnostics. A detailed study into the beam dynamics of the lattice is presented, with a focus towards the effects of space charge and coherent synchrotron radiation on the electron bunch. Simulations disagree with predictions from a one-dimensional model of coherent radiation, and this disagreement is believed to be due to a violation of the Derbenev criterion.

INTRODUCTION

Free electron lasers (FELs) currently offer the best temporal and spatial resolution for sub-nanometer scale dynamic systems. In order to provide such resolution whilst ensuring stability, the dynamics of the electron beam must be finely controlled up to the lasing section. In doing so, it is necessary to understand nonlinear collective effects such as space charge and coherent synchrotron radiation (CSR). Due to the high charge densities required in FELs, the magnitude of the forces arising from these effects can be large, especially so for CSR.

The latest version of General Particle Tracer (GPT) includes a new 1-dimensional CSR model which takes into account the beam’s transverse size, alongside its pre-existing 3-dimensional PIC space charge model [1,2]. This provides an attractive opportunity to model both of these effects in concert in the context of both the current CLARA beam line (Phase 1). These effects are likely to be significant given the relatively low energy and short bunch length expected in CLARA Phase 1. As this model is newly developed, it requires benchmarking against experimental data to ensure its validity. The simulations presented in this paper will provide evidence as to whether or not such an experiment conducted at CLARA would be able to produce a measurable amount of CSR emission.

THEORY

Whilst analytical expressions for the effects of CSR on an electron bunch are based on numerous approximations and assumptions that do not hold strictly true in the context of a real accelerator, they provide a first approximation for the expected magnitude of CSR effects. The effects of the CSR interaction are a longitudinal redistribution of the beam energy, a net loss in the beam energy, and an increase in the transverse emittance. By looking for these indicators, the presence and magnitude of the CSR interaction on the beamline can be inferred.

A one-dimensional model for the electric field produced along an electron bunch was developed by Saldin et al [3]. This model holds as a good approximation for an electron bunch passing through a bending magnet provided that the bunch dimensions fit the Derbenev criterion [4]:

\[
\sigma_z \left( \frac{1}{R\sigma_z^3} \right)^{1/3} < 1, \tag{1}
\]

where \(\sigma_z\) is the longitudinal beam size, \(\sigma_x\) is the transverse beam size in the bending plane, and \(R\) is the bending radius of the bunch trajectory. Further to this restriction on the bunch dimensions, the one-dimensional approach makes the assumption that the bunch initial electron beam is monoenergetic, and that transverse particle velocities are negligible. Given these assumptions, we can express the longitudinal energy modulation of an electron bunch travelling through a sufficiently long dipole (meaning significantly longer than the bunch length) as [3]:

\[
\frac{dE(z,s)}{dt} = -\frac{e^2}{4\pi e_0(3R^2)^{1/3}} \int_{z-\Delta z(s)}^{z} \frac{1}{(z-z')^{1/3}} \frac{d\lambda(z')}{dz'} dz', \tag{2}
\]

where \(E\) is the energy of a given witness charge, \(z\) refers to the longitudinal coordinate of the witness within the bunch, \(s\) refers to the average distance travelled through the dipole by the electron bunch, \(\lambda(z)\) is the longitudinal charge distribution, \(\Delta z(s) = \frac{s^3}{24R^2}\) is the slippage length, and all other symbols are constants with the usual meaning. The lower integration limit in Eq. (2) enforces the fact that particles only radiate when inside the dipole. The slippage length is the difference in path length between the curved trajectory taken by the bunch and the straight trajectory from the dipole entrance to the bunch’s position which is taken by the radiation. In this instance, the slippage length does not account for the velocity difference between the radiation and the beam, which is negligible at this energy.
When the dipole is very long relative to the bunch length, the lower bound of the integral in Eq. (2) can be taken to be \(-\infty\), in what is the so-called “steady state” limit. Borland shows in [5] that for a Gaussian longitudinal density distribution in this state, the fractional change in average energy and the change in the fractional energy spread are described by:

\[
\frac{\Delta \langle E \rangle}{E_0} = -0.3505 \frac{r e Q l_d}{e \gamma (R^2 \sigma_z^2)^{3/2}},
\]

\[
\Delta \delta E = 0.1833 r_e^2 Q^2 l_d^2 + e^2 \gamma^2 (R^2 \sigma_z^2 \gamma) - \frac{3}{2} \frac{0.3505 r_e Q l_d}{e \gamma (R^2 \sigma_z^2)^{3/2}},
\]

where \(\delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}\) is the fractional energy spread of the bunch, \(r_e\) is the classical electron radius, \(Q\) is the bunch charge, \(l_d\) is the length of the dipole, and \(\gamma\) is the Lorentz factor (see Table 1 for values).

### SIMULATIONS

#### Simulation Settings

Simulations featured in this paper were carried out using the CLARA Phase 1 beamline (see Fig. 1). The beam distribution was extracted at the entrance and exit of dipole-1 in order to determine the change in energy spread and bunch length over the dipole. Two sets of simulations were conducted, one with no CSR calculations included, and another with the new CSR model. GPT’s 3-dimensional mesh space charge routine was included in all simulations, using a grid of 16 x 16 x 16 mesh lines [1,2].

The initial particle distribution consists of 2,097,152 (27 x 3) macroparticles, and was defined to represent the emission of a 250 pC electron bunch from a cathode surface. An option within GPT’s space charge routine allows for an appropriate boundary condition to be imposed at the photocathode, which mimics the effect of an image charge on the conducting surface. The initial velocity distribution is isotropic; the initial distribution is mono-energetic, with particle momenta distributed evenly over a half-sphere. The temporal profile of the laser pulse used in simulations was a Gaussian with \(\sigma_t = 340\) fs, which is the expected laser pulse length in CLARA Phase 1. This is expected to change to a flat-top profile of a few ps length in later phases of the CLARA installation. A specification of the initial distribution is given in Table 1.

The settings for various beamline elements are summarised in Table 1. The field strength for the solenoid was set so as to minimise the transverse beam emittance upon exiting linac 1. Quadrupoles were matched to minimise the beam’s transverse size in the bending plane. The off-crest phase of linac-1 was scanned to find the minimum bunch length achieved through magnetic compression within dipole-1. In order to keep the beam momentum constant, the peak field of linac-1 was scaled by a factor of \(1/\cos \phi\), where \(\phi\) is the off-crest phase.

#### Linac-1 Phase Scan

The phase of linac-1 was scanned around the maximal compression phase, between \(+21^\circ\) and \(+27^\circ\) (with positive defined as the head of the bunch gaining more energy than the tail). It is expected from Eq. (3) and Eq. (4) that a peak in the change in energy spread and the energy loss over the dipole will coincide with the minimum bunch length. This behaviour is seen in Fig. 2b. The percentage change in the mean energy seen in GPT simulations does not exhibit the same behaviour (see Fig. 2a, with no clear relationship seen between the energy loss and the bunch length.)
The bunch dimensions at the dipole entrance fulfil the criterion set out in Eq. (1), with an r.m.s transverse size of 0.398 mm and an r.m.s bunch length of 3.24 ps. The mean beam momentum at the dipole entrance is 24.9 MeV/c. The r.m.s bunch length decreases over the length of the dipole down to 0.140 ps. As the variation in the bunch length is not accounted for in Eq. (3) and Eq. (4), an approximation was made using a stepwise approach for the change in bunch length. This method samples the beam at 8 equally spaced positions within dipole-1, and assumes that beam parameters remain constant over the spacing. A drop in the beam energy of -0.02% is expected, alongside an expected 5.70% increase in the percentage energy spread. As can be seen in Fig. 2a, the percentage energy loss observed in GPT simulations is larger than that calculated from Eq. (3). The change in the percentage energy spread seen in GPT simulations is far lower than predicted by Eq. (4). It is worth noting however that the electron bunch ceases to fulfill the Derbenev criterion, Eq. (1), around halfway through dipole-1 and this is the most likely explanation for the discrepancy between GPT simulations and Borland’s model [5].

Another significant departure from the model used in the calculation of Eq. (3) and Eq. (4) is the beam’s charge density profile. As shown in Fig. 3, the beam’s longitudinal profile at the entrance of dipole-1 resembles a skewed Gaussian.

Figure 4 shows the results of a numerical integration of Eq. (2) using a skewed Gaussian fit of the beam’s charge distribution at dipole-1’s entrance. Again, there are limitations to the analytical model applied here, namely that the exit transients have been ignored (i.e., steady state) and the bunch length has been treated as constant over the dipole. From Fig. 4 it is clear that the changing bunch length must be factored into the model for it to be viable for comparison. The change in energy spread calculated from the numerical integration was negligible.

**SUMMARY**

Simulations into the expected effects of CSR using GPT’s new 3-dimensional CSR model on the CLARA phase 1 beam line have been presented. Results have been shown to differ significantly from simpler analytical models, with the simulated energy loss being around twice that predicted by the one-dimensional model. Furthermore the change in the energy spread seen in GPT simulations was around three orders of magnitude lower than predicted by the one-dimensional model. These discrepancies arise from a combination of a changing bunch length over the dipole, a non-Gaussian longitudinal bunch profile, and a violation of the Derbenev criterion. The expected change in the beam’s energy spread of 0.015% will be difficult to detect in CLARA. One option to increase the CSR-induced energy spread of the beam is to use further dipoles along the CLARA Phase 1 beamline, which form a transfer line to the parallel VELA beamline.
REFERENCES


