Abstract

Trapped magnetic flux is known to degrade the quality factor of superconducting cavities by increasing the surface losses ascribed to the residual resistance. In Nb$_3$Sn cavities, which consist of a thin layer of Nb$_3$Sn coated on a bulk niobium substrate, the bimetallic interface results in a thermal current being generated in the presence of a thermal gradient, which will in turn generate flux that can be trapped. In this paper we quantify the impact of trapped flux, from either ambient fields or thermal gradients, on the performance of the cavity. We discover that the sensitivity to trapped flux, a measure of the increase in residual resistance as a function of the amount of flux trapped, is a function of the accelerating gradient. A theoretical framework to explain this phenomenon is proposed, and the impact on the requirements for operating a Nb$_3$Sn cavity in a cryomodule are considered.

INTRODUCTION

Niobium cavities coated with a layer of Nb$_3$Sn are a promising high-efficiency alternative to more conventional niobium for SRF applications [1–5]. In particular, the lower BCS resistance of Nb$_3$Sn allows operation of 1.3 GHz cavities at a bath temperature of 4.2 K, permitting the use of cryo-coolers or liquid helium without active pumping. To allow this to happen, however, the other components of the surface resistance, vis-à-vis residual resistance, must be minimised.

By far the greatest contributor to the surface resistance from sources other than BCS are contributions from trapped magnetic flux. Quantised flux vortices, trapped in the superconductor during the transition through the superconducting transition temperature $T_c$, oscillate under the application of the RF electromagnetic field. Due to viscous drag (and, as will be described in detail, other loss mechanisms), the oscillation of these vortices dissipates energy in the superconductor, resulting in a decreased efficiency.

In niobium cavities, the increase in residual resistance is proportional to the amount of flux trapped [6]. The explanation for this is simple: the losses are linearly proportional to the number of flux cores trapped in the superconductor. The amount of residual resistance gained per unit of flux trapped is dubbed the sensitivity to trapped flux, often quoted in n$\Omega$/mG. In niobium, this value has largely been found to be independent of the applied RF field – with the exception of large grain cavities – but in Nb$_3$Sn, it demonstrates a linear dependence on the RF field.

In this paper we demonstrate that this result, reaffirmed here, is consistent with a weak collective flux pinning scenario, in which a loss term from the presence of many weak pinning sites is introduced into the flux vortex equations of motion. Using a simplified form of the Bardeen-Stephen vortex motion equation, this can be demonstrated analytically.

EXPERIMENTAL METHOD

An ILC-style single-cell 1.3 GHz cavity, niobium coated with Nb$_3$Sn, was utilised for this experiment. The cavity was tested in a vertical cryostat, using an experimental arrangement described by the diagram in Fig. 1. A Helmholtz coiled mounted over the cavity allowed the application of a near-constant magnetic field along the beam axis of the cavity, while a heater located at the base of the cryostat allowed the application of a thermal gradient across the cavity during the cool-down through $T_c$. The latter is necessary for the

![Figure 1: Diagram illustrating the experimental setup of the single-cell cavity. Liquid helium is introduced in a control fashion at the base of the cryostat, which, in the absence of power from the heater unit, allows cooling in an almost-uniform gradient. The Helmholtz coil allows the application of an external magnetic field, whilst the use of the heater develops a temperature gradient across the cavity, measured by the Cernox sensors mounted on the equator and irises.](image-url)
generation of thermoelectric currents from the metal bilayer interface of Nb/Nb$_3$Sn, which will result in the generation of a thermally-induced magnetic field. This measurement was used to demonstrate that the application of a thermal gradient during cool-down is equivalent to the application of an external magnetic field.

RESULTS

The equivalence of a thermal gradient to an externally applied magnetic field during cool-down is demonstrated in Fig. 2. In both cases, a linear increase in the $\Delta T$ across the cavity (measured in K/m) or an equivalent increase in the external magnetic field results in an increase in the residual resistance. This linear dependence allows us to quote an equivalence factor between the two sources for this cavity, which was found to be $(6.2 \pm 0.3)$ mG/(K/m).

A linear fit of the trapped flux as a function of the trapped magnetic field (and, through use of the equivalence factor stated previously, the $\Delta T$ as a function of the same) determines the sensitivity to trapped flux. By performing the measurement at different RF fields, the sensitivity as a function of the applied RF field can be determined. This measurement is shown in Fig. 3. As can be seen, the sensitivity appears to follow a linear trend between the measured values of 5-45 mT, describe the relation

$$\frac{dB_{\text{trapped}}}{dR} \left[ \frac{dT}{dx} \right]^{-1} = \frac{dB_{\text{trapped}}}{dT}. \quad (1)$$

A THEORETICAL FRAMEWORK USING WEAK FLUX PINNING

The observance of a non-constant sensitivity to trapped flux indicates the presence of non-ohmic losses, which simple viscous drag opposing vortex motion cannot explain. Instead, we propose a model in which the superconductor is populated with a high density of weak pinning sites, with a pinning strength $f_p$ that can be overcome with a sufficiently strong (and within the means of that sustainable by the cavity) RF magnetic field. This model is shown pictorially in Fig. 4.

In this scenario, the vortex line is displaced from its neutral position by the application of the RF field, whose strength overcomes the weak pinning centres near the RF surface. The force applied by the Lorentz force is propagated along the vortex line by the elastic line tension, which causes the oscillation to extend further into the bulk, out of the influence of the RF field. Deeper into the bulk, the force transmitted by the line tension is weakened by the elastic nature of the vortex line and so is less able to overcome the pinning force, until deep enough into the bulk the flux line is pinned in the same manner considered in a strong pinning scenario, unaffected by the actions of the RF field at the surface.

As the vortex line reaches its peak displacement under the RF field, and the Lorentz force starts decreasing, the same pinning centres also oppose the restoration of the vortex line to its original shape. This results in a shape being drawn out by the vortex line in a manner seen in Fig. 5, with two parabolas of opposite curvatures describing the shape of the vortex as it changes direction of travel under the influence of the RF field.

It is this manner of motion that gives rise to a sensitivity dependent upon the magnitude of the RF field, $B_{r,f}$. Solving the Bardeen-Stephen equation of motion [7], neglecting the viscous, effective mass and Magnus terms while maintaining the contribution from the Lorentz, elastic, and pinning forces, results in a sensitivity given by

$$\frac{R_0}{B_{\text{trapped}}} = AB_{r,f}, \quad (2)$$

where
Figure 4: A representation of the weak collective flux pinning model. A vortex line, oscillating under the influence of the RF field, dissipates power into the superconductor due to the force imposed by a high density of weak pinning sites.

Figure 5: Motion of a flux line under the influence of the RF field in a weak collective flux pinning scenario. During the initial phase, the vortex line is displaced from neutral (blue) until it reaches the peak of its oscillation. As the Lorentz force starts decreasing and the vortex moves in the opposite direction (red), the impact of the weak pinning sites changes the sign of the curvature near the tip of the vortex.

\[ A = \frac{4}{3} \frac{f \lambda^2 \mu_0}{B_c^2 \xi} \left( \frac{j_o}{j_c} \right)^{3/2}, \]  

(3)

with \( f \) being the RF frequency, \( \lambda \) the superconducting penetration depth, \( B_c \) the thermodynamic critical field, and \( \xi \) the superconducting coherence length. The terms \( j_o \) and \( j_c \) are the de-pairing current (at which point the Cooper pairs are broken) and de-pinning current (at which point the flux line is de-pinned), respectively. Using Ginzburg-Landau theory, the former can be written as

\[ j_o = \frac{4}{3 \sqrt{6} \mu_0} \frac{B_c}{\lambda}. \]  

(4)

Although this analytical solution does not consider the impact of viscous forces (which are not negligible), it demonstrates that a linear dependence of the sensitivity to trapped flux is consistent with a weak collective flux pinning scenario. Efforts are currently underway to re-introduce the viscous term using numerical solutions, to allow direct comparison to experiment.

CONCLUSION

The sensitivity to trapped flux of Nb_3Sn cavities has been shown to be a function of the applied RF field, which is consistent with a weak collective flux pinning scenario of trapped vortex motion. This phenomenon places a constraint on the requirements for operating an Nb_3Sn cavity in a cryomodule, specifically in terms of the amount of trapped flux that can be sustained before the additional losses from the residual resistance prohibit operation at target fields. From our data, it appears that a target of 100 mK/m of thermal gradient across the cavity, in 1-2 mG of ambient magnetic field, is sufficient to guarantee a quality factor of \( > 10^{10} \) at 4.2 K and 16 MV/m in an ILC-style 1.3 GHz cavity. Although quite demanding, these operating parameters are not unreasonable, and have been obtained in contemporary cryomodules.

REFERENCES


