EMITTANCE MEASUREMENTS IN LOW ENERGY STORAGE RINGS

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Abstract

The development of the next generation of ultra-low energy antiproton and ion facilities requires precise information about the beam emittance to guarantee optimum performance. In the Extra-Low ENergy Antiproton storage ring (ELENA) the transverse emittances will be measured by scraping. However, this diagnostic measurement faces several challenges: non-zero dispersion and systematic errors due to diffusion processes, such as intra-beam scattering, and the speed of the scraper with respect to the beam revolution frequency. In addition, the beam distribution will likely be non-Gaussian. Here, we present algorithms to efficiently address the emittance reconstruction in presence of the above effects, and present simulation results for the case of ELENA. We also discuss the feasibility of using alternative non-invasive techniques for profile and emittance measurements.

INTRODUCTION

ELENA is a low energy storage ring designed to increase the efficiency of the antimatter experiments at CERN [1]. Currently under construction, ELENA will accept antiprotons from the Antiproton Decelerator (AD) [2] and employ the use of an electron cooler to keep the beam under control while they are decelerated from a kinetic energy of 5.3 MeV to 100 keV. At these lower energies, fewer antiprotons will be lost to degrader foils at the end of the deceleration process and as a result the anti-hydrogen experiments will receive higher intensity beams.

In order to monitor the quality of the beam between deceleration and cooling phases, emittance measurements will be taken using a scraper. A scraper is a destructive diagnostics device, comprising a set of blades individually moving orthogonal to the beam, into the path of the beam at a low velocity compared to that of the beam. The scraper removes particles from the beam and measurements of the beam intensity as a function of the position of the scraper are taken. A fit to the intensity data is used to reconstruct the transverse beam profile and obtain emittance measurements. A scraper was chosen due to its simple operation with low intensity antiproton beams with the additional feature of being able to collimate the beam (to a specific size or intensity) if desired.

In the AD ring two pairs of horizontal and vertical tungsten scrapers are used to destructively measure the beam profile [3]. To simplify the algorithm the scrapers are located in a dispersive-free region. In ELENA there is no region with zero dispersion which complicates the data fitting and beam analysis process. The details of these challenges are discussed in the following section.

THEORY

Reconstruction for a Gaussian Beam

The working idea for the scraper is to sweep through the beam in a specific direction e.g. from the positive x-direction, to obtain a density distribution for that plane. If the scraper blades are aligned correctly, the measurement will only act in one plane and any particles with larger betatron amplitudes than the scraper edge are removed from the beam. The scraper blade moves slowly in comparison to the beam velocity to allow time for higher amplitude particles to be eliminated. Here, for simplicity, to illustrate the process let us consider a single scraper blade moving the x-plane (Fig. 1). However in ELENA the scraper consists of four scraper blades coming from the ±x and ±y directions.

Considering only a 2D Gaussian beam, an integration over the density distribution can be performed to reconstruct the beam profile. Combining this with parameters describing the beam and the accelerator’s optics at the scraper position, the emittance can be calculated. In the primary method presented here, we expand upon this technique to consider a beam with a non-zero momentum distribution in a dispersive region.

The momentum component is accounted for by including an additional energy term (dependent on the relative momentum offset and the rms relative momentum spread) in the integral and averaging over the momenta of the beam. The rms relative momentum spread of the beam can be taken as a free parameter or if known, used in the calculation.

In order to account for the dispersion in the Gaussian calculation, the upper limit on the energy integral must be changed from infinity to the maximum relative momentum offset which is dependent on the dispersion at the position of the scraper. An additional term, which depends on the relative momentum offset and also the dispersion at the
scraper, is added to the closed orbit term in the transverse integral.

Combining these factors for a Gaussian beam we obtained an expression for the relative remaining intensity as a function of the position of the scraper blade and the emittance:

\[
\frac{N_r}{N} = \frac{1}{2} \left(1 + \text{Erf} \left( \frac{A_0}{\sqrt{2\epsilon_{\text{rms}} d}} \right) \right) \left(1 + \text{Erf} \left( \frac{A_0}{\sqrt{2\epsilon_{\text{rms}} d}} \right) \right) - \frac{1}{2\sqrt{1+d^2}} e^{2\left(1+d^2\right)\epsilon_{\text{rms}}} 
\]

where \( A_0 = \frac{(x-x_{\text{co}})}{\sqrt{\beta}} \) and \( d = \frac{\sigma_{\delta D}}{\sqrt{\epsilon_{\text{rms}}}} \)

and \( \text{Erf} \) is the so-called error function. Making the substitution 'd' allows us to encapsulate all of the longitudinal phase space dependence into a single component and hence treat it as a free parameter if necessary.

**Beam Profiles**

Using the code BETACOOL [4] additional studies into the stability of the beam in ELENA during the cooling plateaus were performed [5]. Taking an initial ideal Gaussian beam distribution and applying heating and cooling effects (the electron cooler, rest gas and Intra-beam scattering) for the two cooling plateaus, a deviation in beam distribution was observed. The simulations showed a very dense core surrounded by a wider halo. This is explained as a result of the nature of the electron cooler – particles with smaller amplitudes are cooled more efficiently due to being at the centre of the electron beam and hence experiencing a greater friction force due to space charge effects of the electron beam. Further, more detailed studies into this effect are currently pending publication [6]– the results from which are used here.

![Figure 2: Gaussian and bi-Gaussian beam distributions.](image)

Analysis of the beam distributions shows we can recreate the transverse beam profiles as the sum of two Gaussian distributions to give an approximation of the core-tail effect. These bi-Gaussian beams were generated and run through the scraping process as well as the ideal Gaussian distributions (Fig. 2). Additionally, the integration performed for a Gaussian beam will be performed for a bi-Gaussian profile and used to reconstruct the emittance from the bi-Gaussian runs. Once these methods have been fully established, a study into reconstructing arbitrary beam profile shapes will be conducted.

**SIMULATIONS**

The simulations undertaken in this study were carried out using the Polymorphic Tracking Code (PTC) module in MAD-X [7]. The input beams were generated using Monte Carlo methods in a Python script with the parameters of Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Momentum</td>
<td>13.7</td>
<td>MeV c(^{-1})</td>
</tr>
<tr>
<td>Input Emittance, (\epsilon_x) &amp; (\epsilon_y)</td>
<td>1.2</td>
<td>mm mrad</td>
</tr>
<tr>
<td>Relative Momentum spread, (\delta_p)</td>
<td>0.001</td>
<td>-</td>
</tr>
<tr>
<td>Number of macroparticles</td>
<td>10,000</td>
<td>-</td>
</tr>
<tr>
<td>(\alpha_{x,y}) (Optics at injection)</td>
<td>1.068, 1.076</td>
<td>rads</td>
</tr>
<tr>
<td>(\beta_{x,y}) (Optics at injection)</td>
<td>4.510, 4.512</td>
<td>m</td>
</tr>
<tr>
<td>(\beta_{x,y}) (Optics at scraper)</td>
<td>0.6651, 2.909</td>
<td>m</td>
</tr>
<tr>
<td>(D_x) (Injection, Scraper)</td>
<td>1.452, 1.181</td>
<td>m</td>
</tr>
<tr>
<td>(\beta_{x,y}) (Optics at scraper)</td>
<td>0.6651, 2.909</td>
<td>m</td>
</tr>
</tbody>
</table>

Non-relativistic corrections were made during the beam generation process to circumvent the known error that MAD-X assumes a relativistic case. The beam of 10,000 macroparticles, representing 2.5x10\(^7\) antiprotons, was run through the MAD-X model of the ELENA lattice and the scraper was moved into the beam.

![Figure 3: Transverse beam profile at \(x_s = 4.6\) mm and at \(x_s = 2.4\) mm. The blade position is represented by the green vertical line.](image)

To simulate the correct velocity of 40 mm s\(^{-1}\), the particles made 360 revolutions for each 0.1 mm step in the x position of the scraper \((x_s)\). Every particle with a larger amplitude than the position of the edge of the scraper blade was removed from the simulation and the phase space coordinates and number of particles remaining after each 0.1 mm step were recorded. Figure 3 shows the reduction in size and intensity of the beam as the scraper moves to a smaller amplitude.

**ANALYSIS**

The analysis section of the study was performed in MATLAB. A plot of the ratio of particles remaining and total number of particles, \(N_r/N\), against the position of the scraper, \(x_s\), shows a cumulative distribution of the beam in...
the transverse x plane. Using Eq. (1), a fit to the data could be found. In order to make a fit to the data, $\epsilon_x$ was taken as a free parameter whilst the known values (including the optics parameters and dispersion, $D_x$, at the scraper position) were entered manually.

![Graph showing cumulative beam distributions based on scraper position vs. particles remaining.](image)

**Figure 4:** Cumulative beam distributions based on scraper position vs. particles remaining. The left figure shows the equation drawn against the data with $\epsilon_x = 1.2$ mm mrad, and the right figure shows the resultant fit from the MATLAB fitting algorithm, resulting in $\epsilon_x = 0.805$ mm mrad.

As can be seen in Fig. 4, the results from the simulation match well with the equation, however the accuracy of the emittance reconstructed when using the fitting code does not appear to find the correct value, although it is within the same order of magnitude. The reasons for this discrepancy could be related to the momentum offset of the beam being slightly different from the input parameters at this point, or some variation in the closed orbit of the beam. An investigation into the source of this error is currently underway. Several simulations with varying input emittances show a linear relation between the input emittance and reconstructed emittance suggesting a systematic error.

Simulations scraping through the y-axis have also been performed, with similar results. This suggests that the error is not due to momentum spread effects as $D_y = 0$ throughout the ring.

![Graph showing Bi-Gaussian/Gaussian beam comparison.](image)

**Figure 5:** Bi-Gaussian/Gaussian beam comparison.

To compare the Gaussian and bi-Gaussian cases, the cumulative beam distributions were plotted together (Fig. 5).

As we would expect, a clear difference between the two cases can be seen with the bi-Gaussian profile dropping off slowly at higher $x$, and with a sharp gradient towards the densely populated core of the beam.

**CONCLUSIONS & PROSPECTS**

Simulations and fitting for a Gaussian beam profile have been performed. The results from the fitting algorithm can reconstruct the emittance value to within the correct order of magnitude, however some discrepancy between the input and the reconstructed value remains.

During actual usage of the scraper, the parameter ‘d’ will be taken as a variable if there is an uncertainty in the RMS momentum spread of the beam or dispersion at the point of the scraper, and the exact value of $x_{co}$ may also be taken as unknown during commissioning.

In order to accommodate for a more realistic beam distribution, a bi-Gaussian beam was run through the simulation. The resultant beam profile looks as we would expect, and an equation to reconstruct the emittance of this beam will be derived. There are other mathematical distributions that could more accurately describe the beam, such as a Lorentzian distribution and reconstruction algorithms for these profile types will be investigated. A potential system for attempting to fit several mathematical models to the scraper data and choosing the fit with the lowest $\chi^2$ will be considered, however a more advanced algorithm, employing the data collected from two scrapers acting over independent runs, but in the same plane, is currently being investigated. This algorithm would be capable of reconstructing the emittance from any arbitrary beam distribution and would be an ideal solution for this particular device.

Further investigation into potential errors will be carried out. A tilt in the scraper blades would lead to the scraping process acting in more than one plane, the impact of such an error on the quality of the emittance measurement will be studied. Error tolerance in the velocity of the scraper blade will also be established.

Preliminary studies performed with GEANT4 to investigate the interaction between the antiprotons and the scraper blade, and its dependence on the beam energy have been carried out separately at CERN. The results from these studies will be incorporated into the algorithm simulations to ensure additional transmission and momentum effects are accounted for.

A long term plan for future upgrades to ELENA is currently being considered. A gas jet monitor initially developed for USR has the qualities required for non-invasive beam profile and emittance measurements, due to the similarities between the two machines.

**ACKNOWLEDGEMENTS**

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**REFERENCES**


