MULTIPLE BUNCH HOM EVALUATION FOR eRHIC ERL LINAC CAVITIES

Chen Xu #,1, I. Ben-Zvi1,2, Michael M. Blaskiewicz1, Yue Hao1, Vadim Ptitsyn1,  
1 Brookhaven National Laboratory, Upton, New York 11973-5000, USA  
2 Stony Brook University, Stony Brook, New York 11794, USA

Abstract

The proposed ERL-based electron-ion collider eRHIC in BNL has multiple high current Superconducting Radiofrequency (SRF) 5-cell cavities. The HOM power generated when a single bunch traverses the cavity is estimated by the corresponding loss factor. Multiple recirculations through the ERL create a specific bunch pattern. In this case the loss factor can be different than the single bunch loss factor. The HOM power generation can be surveyed in the time and frequency domains. We estimate the average HOM power in the eRHIC 5-cell cavity with different ERL bunch patterns.

INTRODUCTION

The eRHIC accelerator was proposed at BNL to collide electron with protons and ions in order to explore internal structure of nucleons and nuclei and, especially, a role that gluons play in nucleon processes [1,2].

The proton and electron bunches will collide at a rate of 9.38 MHz and this frequency is defined by the present repetition rate of protons. The electrons are accelerated by a SRF (Superconducting Radiofrequency) Linac whose fundamental frequency is 647.2 MHz. In order to reach the final colliding energy, electrons will be accelerated by multiple recirculation through the SRF linac. Maximum 12 recirculation are considered with electrons gaining 1.665 GeV energy from each pass through the linac. The resulting bunch pattern also contains a gap, about 1 μsec duration, for ion clearing purpose. This gap is also used for placing single diagnostic bunches to monitor orbits and other beam parameters. The gap repeats itself at a rate of 78.2 kHz, which is the one turn revolution frequency. Therefore, both proton and electron bunch patterns at the collision point are defined by two frequencies, 78.2 kHz on top of the 9.38 MHz in the time domain. Within the 12.7 μsec period of the 78.2 kHz, there will be 110 bunches with repetition rate of 9.38 MHz and the gap. The bunch patterns from multiple recirculations overlap in the linac in figure 1.

In the longitudinal direction, the HOM (Higher order mode) power is proportional to the product of average current and bunch charge [3]. On the other hand, the power is also proportional to the cavity loss factor that is affected by the cavity design as well as the bunch length (or, in general, by bunch longitudinal distribution). Currently, we propose a 5-cell superconducting cavity which is designed to minimize the loss factor. This cavity has longitudinal loss factor of 3.06 V/pC when the bunch RMS length is 3 mm. However, this loss factor is obtained when single Gaussian bunch passes the cavity. By using the parameters in table 1, HOM generation for both nominal and ultimate designs are simply estimated by single bunch. The HOM generation of each 5 cell cavity is 2.9 kW and 8.109 kW respectively from single bunch operation.

In this study, we will estimate the HOM generation by multiple bunch patterns and calculate the loss factor for multiple bunch patterns.

The loss factor can be obtained by integrating the product of the bunch charge and the wake potential in time domain. It may also be obtained by integrating the bunch spectrum and cavity impedance. In the case of single bunch, those two algorithms can be expressed in equations:

\[
k = \frac{1}{Q^2} \int_0^L W(s) J(s) ds = \frac{1}{Q^2} \sum_{i=0}^N W_i I_i \Delta s
\]

\[
k = \frac{1}{Q^2} \int_0^L W(s) J(s) ds = \frac{1}{Q^2} \int_0^L \frac{1}{2\pi} \int_{-\infty}^{\infty} \Omega(k) \tilde{I}(k)e^{ikx} dk I(s) ds
\]

\[
= \frac{1}{2\pi Q^2} \int_{-\infty}^{\infty} \Omega(k) \tilde{I}(k)(-\tilde{I}(k)) dk = \frac{1}{NQ^2} \sum_{n=0}^N \Omega_n \tilde{I}_n \tilde{I}_n \Delta s
\]
The term \(I(k) I(k)^*\) is the PSD (power spectrum density) of the current resulting from multiple bunches.

**HOM GENERATION BY A BUNCH TRAIN**

Since the eRHIC linac will operate as an ERL, the cavity will see a series of accelerating and deaccelerating bunches in some given pattern in time domain. This series includes bunches with different energies that propagate collinearly at the same RF phase. All bunches travel practically at the speed of light, and the HOM power generation is independent of the bunch energy. Therefore, for the HOM perspective, one can treat the accelerating/deaccelerating bunches with multiple energies as one bunch train inserted into the ERL with the proper pattern.

We are studying the bunch pattern within the period of 9.38 MHz. A bunch pattern where the heads of accelerating and deaccelerating bunches are apart by half of the period of the fundamental frequency. Within the accelerating and deaccelerating trains, the bunch separation time is the integer times of period of 647.2 MHz. The accelerating and deaccelerating trains may shift with different RF cycles. Meanwhile, both bunch trains are still in an ERL mode. That means that the head of the accelerating and deaccelerating bunches are apart by \(N+0.5\) times of period of fundamental frequencies. Note that the total average current remains the same for the various patterns (although the peak current may change), while, the bunch frequency spectrum does change. A few selected bunch patterns are shown in Fig 2 with different RF shifts. In Fig.6, the plots show the charge distribution inside of period of 9.38 MHz. The period of 9.38 MHz is 69 times of that of 647.2 MHz.

To simplify, the accelerating and deaccelerating bunches are 647.2MHz apart. Let us study the spectrum of different multiple bunches. Fig 3 shows the Power Spectral Density (PSD) of the current with a bunch pattern from Fig 6A. The spectrum is obtained with an assumption that the bunch trains repeat at the frequency of 9.38MHz in a CW (continuous wave) mode. The PSD are a set of delta functions with separation of 78.2 KHz.

The HOM generations of the nominal and ultimate operation designs are estimated. The ultimate design will have higher HOM power generation than the nominal design, because the current and bunch charge are higher. Since two designs have different number of turns to accelerate (and decelerate) the electrons bunches, we will vary the bunch patterns for both case and estimate the HOM power generation.

**NOMINAL DESIGN**

By varying the shift period \(N\), the beam spectrum changes, including the amplitude of the different frequencies. Therefore, the HOM power generation depends on the shift parameter \(N\), though the impedance of the cavity remains the same. Each bunch generates HOM power which is related to the wake field as seen by this electron bunch, and is different for the various bunches in the train. The wake field depends on the bunch entrance time into the cavity. The time averaged HOM power generation can be larger or smaller than what would be predicted by the single bunch loss factor. Note the single bunch loss factor is calculated when a bunch enters into an empty cavity. By integrating the current PSD with the cavity impedance, we can obtain the

![Figure 2. The CW bunch pattern of nominal design.](image)

![Figure 3. The PSD of the bunch pattern.](image)

![Figure 4. A) The effective loss factor of multi-bunch pattern (blue) with different RF cycle shifts for nominal design of eRHIC. B) The single bunch loss factor (red) for comparison.](image)
effective loss factor of CW multi-bunches. The loss factors are plotted as a function of shift $N$ in Fig. 4. Note, the maximum shift should be limited to avoid cross talk between to macro bunches.

From figure above we can see there is a minimal HOM generation when $N=4$. This means that the HOM power generation will be the smallest for this bunch pattern. At $N=4$ the effective loss factor is $2.221\text{V/pC}$. Compared with single bunch loss factor, the loss factor reduces by 26.3%. For the charge 2.8 nC and the average beam current 340mA one can obtain the average HOM loss equal to 2.17 kW instead of 2.913 kW from single bunch estimation.

**ULTIMATE DESIGN**

As mentioned early, in the ultimate linac-ring eRHIC design the electron bunches takes 5 turns to gain 8.3 GeV energy, but the average current is 500 mA and bunch charge is 5.3 nC. Therefore, the HOM power generation is larger than the previous case. We will apply the similar method to this case. The difference is that there are 5 bunches inside of accelerating and decelerating trains. The loss factors are plotted as a function of shift $N$ in Fig. 5.

![Figure 5](image1.png)

Figure 5. A). The effective loss factor of multi-bunches (blue) with different RF cycle shifts for Ultimate design of eRHIC. B). the single bunch loss factor (red) for comparison.

The integrated loss factor reaches minimal when $N=5$, and this means that the heads of accelerating and decelerating trains are apart by 7.72ns which is 5.5 times the period of 647.2MHz. The reduction is more than 30%. By using the charge and average current from table 1, the HOM power generation is 5.48kW instead of 8.10kW from single bunch estimation.

Now we consider the ultimate design case with higher energies. In order to obtain the highest colliding electron energy, electron bunch take 12 turns to boost their energy up to 20GeV. In this case, the average current that passes through the Linac is 500mA. Instead of the 5 turns previously, 12 bunches with 9.38MHz will repeat the ERL patterns. Similarly, we plot the effective loss factor as a function of RF shift cycles in Fig. 6.

![Figure 6](image2.png)

Figure 6. A). The effective loss factor of multi-bunches (blue) with different RF cycle shifts for the Ultimate design of eRHIC. B). the single bunch loss factor (red) for comparison.

It would be instructive to compare the PSD spectrum of bunch patterns with different shifts. Here, we chose two cases when $N=3$ (where we obtain the minimal effective loss factor) and $N=14$ (where the effective loss factor is more than that of single bunch). We compared their PSD spectrums which are plotted in Fig 7. Note the Y axis is normalized and used an arbitrary unit. In Fig 7, it is clear that the shift would not change frequencies where the peak spikes occur, but change the amplitude of distributions among different frequencies. The amplitude reduction of odd harmonics component of 3 RF cycles shift is larger than that of 14 RF cycles. That explains why the effective loss factor is smaller in the 3 RF cycles shift case.

![Figure 7](image3.png)

Figure 7. The current PSD spectrum for two cases: 3 RF shifts (red) and 14 RF shifts (blue). The frequency ranges up to 30GHz. Y axis is normalized and in an arbitrary unit.

**CONCLUSION**

The bunch pattern modifies the average HOM Power generation. The effective loss factor can be considerably different from single bunch estimation. When designing an energy-recovery linac one has the freedom to select a bunch pattern which minimizes the average HOM power generation. This study estimates the effective loss factor of different bunch patterns for the eRHIC project. Certain bunch patterns can reduce the averaged HOM power by more than 30% of the single bunch estimation.

**ACKNOWLEDGMENTS**

The authors would like to thank Yongfeng Gao, Wencan Xu and Binping Xiao, from BNL, H. Wang at JLab and Zenghai Li at SLAC for useful discussions.
REFERENCES

