STUDY OF THIRD HARMONIC CAVITY FOR TAIWAN PHOTON SOURCE

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Abstract

Taiwan Photon Source (TPS) is a modern light source with 3 GeV electron energy and low emittance. The bunch length is about 3 mm at designed beam current of 500 mA and operating gap voltage of 3.2 MV. The short bunch length results in short Touschek lifetime and high parasitic loss of insertion device (ID). Some of the undulators are operated in vacuum at TPS, therefore the head load become an important issue. To install higher harmonic cavity is a solution for improving the Touschek lifetime and the heat load by lengthening the bunch length. The effect of installing 3rd harmonic cavity for TPS is investigated. The expected maximum elongation factor for bunch lengthening, as well as the effect on the Touschek lifetime and heat load of ID are presented in this paper.

INTRODUCTION

Taiwan Photon Source (TPS) is a third generation light source with 3 GeV electron energy and low emittance located in NSRRC, Taiwan. The designed maximum brightness can reach $10^{21}$ photons/s/0.1%BW/mm²/mrad at 10 keV with conventional undulators, in-vacuum undulators superconducting undulators and wigglers [1]. To reach the designed high brightness, a 24 cell double-bend achromat (DBA) lattice is used to provide 1.6 nm-rad emittance. Table 1 shows the TPS parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Energy</td>
<td>3 GeV</td>
</tr>
<tr>
<td>RF frequency</td>
<td>499.65 MHz</td>
</tr>
<tr>
<td>Beam current</td>
<td>500 mA</td>
</tr>
<tr>
<td>Circumference</td>
<td>518.4 MΩ</td>
</tr>
<tr>
<td>Harmonic number</td>
<td>864</td>
</tr>
<tr>
<td>RF Voltage</td>
<td>3.2 MV</td>
</tr>
<tr>
<td>Energy loss per turn</td>
<td>853 keV</td>
</tr>
<tr>
<td>Momentum compaction</td>
<td>2.4E-4</td>
</tr>
<tr>
<td>Longitudinal damping time</td>
<td>6.07 ms</td>
</tr>
<tr>
<td>$Q_0$ of main SRF cavity</td>
<td>1.8E9</td>
</tr>
<tr>
<td>$Q_l$ of main SRF cavity</td>
<td>6.6E4</td>
</tr>
<tr>
<td>R/Q of main SRF cavity</td>
<td>47.5</td>
</tr>
</tbody>
</table>

At such low emittance, the bunch length is about 3 mm at designed beam current of 500 mA and operating gap voltage of 3.2 MV. The short bunch length results in short Touschek lifetime and high parasitic loss of insertion device (ID). With 70% filled pattern, the total lifetime at 500 mA with 1 nTorr vacuum pressure is about 8 hours. The parasitic loss of an in-vacuum undulator (IU) with 2.2 m length at 500 mA for electron with 3 mm bunch length is about 64 W. To reduce the head load is an important issue for IUs. To install higher harmonic cavity is a solution for improving the Touschek lifetime and the heat load by lengthening the bunch length [2]. This approach is effectively employed in many other light sources, such as BESSY-II, SLS, ELETTRA, NSLS-II and others [3]-[5].

In this paper, the effect of installing 3rd harmonic cavity (3HC) for TPS is investigated. The expected maximum elongation factor for bunch lengthening, and the effect on the Touschek lifetime and heat load of IUs are presented as well.

ISSUES FOR THE 3RD HARMONIC CAVITY AT TPS

Lifetime

The Total lifetime can be written as:

$$\tau_{tot} = \tau_q + \tau_T + \tau_{Br} + \tau_{Co}$$

where $\tau_q$ is the quantum lifetime; $\tau_T$ is the Touschek lifetime due to intra-beam scattering loss of electrons in the bunch; $\tau_{Br}$ is the bremsstrahlung scattering lifetime, and $\tau_{Co}$ is the Coulomb scattering lifetime. The quantum lifetime for TPS with gap voltage larger than 1000 kV is larger than 100 hours. The dominant lifetime are from gas scattering lifetime and Touschek lifetime. The Coulomb and bremsstrahlung scattering lifetime are 46.5 hours and 76.4 hours respectively for TPS with gas pressure 1 nTorr of N₂ equivalent residual gas [1]. The Touschek lifetime can be written as [6]:

$$\tau_T = \frac{r_0^2 c q}{8 e \gamma^2 \sigma_z C} \int \frac{F\left(\delta_{ac}(s)\right)}{\delta_{ac}(s)} ds$$

where $r_0$ is the classical electron radius; $c$ is the speed of light, $q$ is the bunch charge; $\gamma$ is the relativistic gamma factor; $\sigma_{x,y,z}$ are the RMS horizontal, vertical, and bunch sizes, respectively. The function $F(x)$ is defined as:

$$F(x) = \int_{-\infty}^{\infty} \left(1 - \frac{1}{u^2} \ln \left(1 - \frac{1}{u^2}\right) - e^{-ux} du$$

For TPS with gap voltage of 3.5 MV and bunch current of 0.5 mA, the Touschek lifetime is 19.8 hours. The total lifetime at 500 mA with 1 nTorr...
vacuum pressure and 70% filled pattern is about 8 hours. One can improve the Touschek lifetime by lengthening the bunch length. Figure 1 shows the total lifetime as function of beam current with gap voltage of 3.2 MV, 70% filled pattern and different elongation factors of bunch length.

![Figure 1. Total lifetime as function of beam current.](image)

**Heat Load of IUs**

The heating power of IDs due to parasitic loss can be written as:

$$ P_{heat} = k_1 \cdot q^2 \cdot N_b \cdot f_{rev} $$

(4)

where $f_{rev}$ is the revolution frequency; $N_b$ is the filled bunch number; $q$ is the bunch charge, and the $k_1$ is the loss factor. The loss factor can be written as [7]:

$$ k_1(\sigma_z) = \frac{1}{2\pi} \int daZ_\perp(\omega)h(\omega, \sigma_z) $$

(5)

where $h(\omega, \sigma_z)$ is the spectral power density of the bunch of rms length $\sigma_z$, and the $Z_\perp$ is the longitudinal impedance. For a Gaussian bunch with rms $\sigma_z$, the $h(\omega, \sigma_z)$ can be written as:

$$ h(\omega, \sigma_z) = e^{-\frac{\omega^2}{\alpha_c^2}} $$

(6)

The conventional thick-wall formula for the impedance can be written as [8]:

$$ Z(\omega) = \frac{1 - \text{sgn}(\omega) \cdot i}{2 \cdot \pi \cdot b \cdot \delta_{\text{skin}} \cdot \sigma} \cdot L $$

(7)

where $b$ is the pipe radius; $\sigma$ is the conductivity of the material, and the $\delta_{\text{skin}}$ is the skin depth which defines as:

$$ \delta_{\text{skin}} = \left(\frac{2}{\mu \cdot \omega \cdot \sigma}\right)^\frac{1}{2} $$

(8)

The parameters of IUs used in the calculation are listed in the Table 2.

**Table 2: IU parameters**

<table>
<thead>
<tr>
<th>IU Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>IU length</td>
<td>2.2 m</td>
</tr>
<tr>
<td>Minimum gap</td>
<td>5.0 mm</td>
</tr>
<tr>
<td>Maximum beam current</td>
<td>500 mA</td>
</tr>
<tr>
<td>Conductivity (copper)</td>
<td>5.9E7 Ω m⁻¹</td>
</tr>
<tr>
<td>RF Voltage</td>
<td>3.2 MV</td>
</tr>
<tr>
<td>Bunch length</td>
<td>3.0 mm</td>
</tr>
</tbody>
</table>

The loss factor calculated from above equations and parameters is 0.089 V/pC. The average heating power with 70% filled pattern is about 64W. The heating power can be improved by lengthening the bunch length. Figure 2 shows the loss factor reducing ratio as function of the bunch lengthening ratio, and the average heating power as function of bunch filled pattern with different bunch lengthening ratio.

In summary, with lengthening factor of 3, the total lifetime at 500 mA with 1 nTorr vacuum pressure is about 16 hours (about 8 hours without bunch lengthening) and the heat load of the ID is reduced to 12W (about 64 W without bunch lengthening).

![Figure 2. The loss factor reducing ratio as function of the bunch lengthening ratio [LEFT] and the heating power as function of bunch filled pattern [RIGHT].](image)

**BUNCH LENGTHENING AND INSTABILITY**

The longitudinal density distribution of the bunch can be written as:

$$ \rho(\phi) = \rho_0 \exp\left(-\frac{U(\phi)}{\alpha_c^2 \sigma_z^2}\right) $$

(9)

where $\alpha_c$ is the momentum compaction factor; $\sigma_z$ is the energy spread; $\rho_0$ is the normalization constant, and the $U(\phi)$ is the RF potential formed by the total voltages, which can be written as:

$$ U(\phi) = -\frac{\alpha_c e}{E r_0} \int_0^\infty \left[V(\phi') - \frac{U}{e}\right] d\phi $$

(10)

For the double RF system with passive 3HC, the total RF voltage seen by beam can be written as:

$$ V(t) = V_{max} \cos(\omega_0 t + \phi) - 2I_0 F_2 R_{3HC} \cos(3\omega_0 t + \phi_{3HC}) $$

(11)

where $I_0$ is the beam current; $F_2$ is the form factor, which can be written as $F_2 = \exp(-n^2 \sigma_{\phi}/2)$ ($n$=3 for 3HC), and the $\phi_{3HC}$ is the tuning angle of the 3HC, which is defined by:

$$ \tan\phi_{3HC} = -Q_{3HC} \left[\frac{3\omega_{ij}}{\omega_{3HC}} - \frac{\omega_{ij}}{3\omega_{uj}}\right] $$

(12)

Then the bunch length can be written as:

$$ \sigma_{\phi}^2 = \frac{\int \phi^2 \rho(\phi) d\phi}{\int \rho(\phi) d\phi} $$

(13)
One can determine the longitudinal density distribution and bunch length by using numerical method to get the self-consistent form factor. Detail discussion can be found in the reference [9, 10]. Figure 3 shows the calculated bunch length as function of induced voltage from the 3HC at beam current of 500 mA and gap voltage of 3.2 MV. For the high induced voltage, it may cause the double-hump bunch shape, as shown in the Fig. 4. The maximum bunch lengthening ratio without double-hump bunch shape is about 4 for TPS with 3.2 MV gap voltages. However, in this study the transient effect is not considered. The maximum bunch lengthening ratio may reduce if the transient effect was included [11].

Figure 3. The bunch lengthening ratio as function of induced voltage.

Figure 4: The bunch density distribution for bunch lengthening ratio 4 [LEFT] and 5.5 [RIGHT].

The instability issues are also studied. We follow the Bosch’s [10] method. The dipole mode zero frequency instability, which means that RF generator voltage do not provide restoring force, is predicted when

\[ \omega_d^2 = \frac{\alpha_e e_0 a_\rho}{E T_0} I_0 \left( R_{11} F_1^2 \sin(-2\phi_2) + nR_{12} F_2^2 \sin(-2\phi_{2H}) \right) < 0 \]

where \( \omega_d \) is the collective dipole oscillations frequency. Figure 5 shows the zero frequency instabilities are all larger than zero with different fixed detuning of the 3HC. The Robinson damping rate for dipole mode can be written as:

\[ \alpha_d = \frac{\alpha_e e_0 a_\rho I_0}{2\Omega E T_0} Z_R \]

(14)

where

\[ Z_R = F_1^2 (\text{Re} Z_1^- - \text{Re} Z_1^+) + n F_2^2 (\text{Re} Z_2^- - \text{Re} Z_2^+) \]

(15)

\[ \text{Re} Z_{1,2}^2 = (1 + \frac{m_1 a_{\rho_1} a_{\varphi} - a_{\varphi}}{m_2 a_{\rho_2} a_{\varphi}})^{-1} \]

(16)

and \( m_1 = 1 \) and \( m_2 = 3 \) for main cavity and 3HC respectively. Figure 6 shows the \( Z_R \) as function of beam current with different fixed detuning of the 3HC. The values are all larger than zero, which means that the Robinson damping rates for dipole mode are all for damping. For low momentum compaction machine, the instability from coupling between dipole and quadrupole Robinson modes need to be taken into account [10], which is not calculated here. More studies for effect of low momentum compaction are needed for TPS case.

Figure 5: The dipole mode zero frequency instability as function of beam current for different detuning of 3HC.

Figure 6: The \( Z_R \) as function of beam current for different detuning of 3HC.

CONCLUSION

To install the harmonic cavity can improve the Touschek lifetime and the heat load by lengthening the bunch length. The maximum bunch lengthening ratio without double-hump bunch shape is about 4 for TPS with 3.2 MV gap voltages. The dipole mode zero frequency instabilities and Robinson instabilities are all stable. However, more studied for the low momentum compaction and the transient effect are needed. Those effects would limit the performance of harmonic cavity.
REFERENCES