MACROS FOR IDENTIFICATION OF HIGHER ORDER MODES TYPES, THEIR TRANSVERSE SHUNT IMPEDANCE AND KICK FACTOR*

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Abstract

During the design studies of accelerating cavities one of the most time-consuming tasks is classification of the higher order modes (HOM) types. To automate the process of identification of HOM types macros for CST Microwave studio was written. This article describes the programs algorithm of mode recognition and the results of the analysis on the example of single cell and 9 cell cavities. Macros for the calculation of the HOM transverse shunt impedance upon Q-factor values and kick-factor was also written.

HOM IDENTIFICATION

The higher modes damping is one of the most important tasks during design studies of accelerating cavities [1]. Their presence can lead to increasing of bunch emittance, beam deflection, beam break-up etc. To reduce the HOM impact on the beam it is necessary to reduce their shunt resistance ($R_{sh}$) and quality-factor $Q$ [2].

We can classify HOM by determining the presence of the longitudinal component of the field (TE, TM modes), the number of variations along the in the azimuthal, radial and longitudinal axes and for multicell structures determining the phase advance per cell. The procedure of determining the HOM types is one of the most time-consuming tasks during design stage of accelerating cavity development, as this classification has to be done manually.

CST Microwave studio [3] program allows the calculation of eigenmodes of the structures (Table 1).

Table 1: Modes in Cylindrical (first row) and Elliptical Cavities (second row)

<table>
<thead>
<tr>
<th>TM$_{010}$</th>
<th>TE$_{211}$</th>
<th>TE$_{412}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE$_{111}$</td>
<td>TE$_{211}$</td>
<td>TM$_{320}$</td>
</tr>
</tbody>
</table>

It also provides a possibility of creating an additional programs for the results post processing, so called VBA macros. Automation of HOM classification process will simplify the procedure for constructing the dispersion characteristics, will help to identify so called "fake modes" and save time during development of HOM damping systems for accelerating cavities. Attempts to write a macro have been taken already [4]. However it was desired to increase the precision of HOM identification and to add some additional features in later versions of the program: dispersion functions plot, universality for different types of structures etc.

The program works in several stages:

1) The first step is to find the field maximum position in the structure.

2) After it is necessary to build azimuthal, radial and longitudinal curves through this point. The field dispersion across this curves. Example of constructed curves are shown in Figure 1.

3) Further we need to identify the type of HOM (TE or TM). For this purpose we are integrating Electric and Magnetic longitudinal components along the curve lying through the maximum point. Then we are determining the HOM type at accordance from condition:

\[ \frac{\int E_z dz}{\int H_z dz} = a \sqrt{\frac{\mu_0}{\varepsilon_0}} \]

\[ a>1 \text{ – TM mode, } a<1 \text{ – TE mode} \]

Figure 1: Longitudinal, radial and azimuthal curves for field plot.

Figure 2: Typical field distribution along (a) azimuthal and (b) radial axis.

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4) After building the electric field distribution along the curve we need to count the number of extremum points which will show us the number of variations of the filed component along r, φ, z axis. By taking into account that for azimuthal and longitudinal components we have sinusoidal dispersion and Bessel function for radial (Fig. 2), we can add additional conditions, which help us to reduce errors in the identification of HOM types and to solve the problem with local maxima appearing due to numerical simulation errors.

5) For multi-cell structures we also need to determine the phase shift per cell. An example of modes calculated in CST MWS in 9 cell cavity are presented in Table 2.

The HOM identification results done by Macros are for elliptical single and 9 cell cavities are presented in the Table 3 and 4.

### Table 2: Modes in 9 Cell Cavities

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency</th>
<th>Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM010</td>
<td>8π/9</td>
<td><img src="image1.png" alt="Image" /></td>
</tr>
<tr>
<td>TE111</td>
<td>7π/9</td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>Te210</td>
<td>3π/9</td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
</tbody>
</table>

The macro was able to identify the modes in a pillbox cavity with 100% accuracy up to a 30 eigenmode (H122). The results of modes identification done manually and by macros for single cell and 9 cell cavity are presented in Table 3. Hb(M) and Hb(E) stands for hybrid mode with dominant transverse magnetic and electric field components correspondingly.

For the single cell cavity all the modes were correctly identified up to the 18th eigenvalues. For the 9 cell cavity for the first 4 passbands there were almost no errors in HOM identification. In passband 5-8 the accuracy lower due to difficulties with identification of number of radial variations for some modes. Still the accuracy is higher than 80%. Results of phase advance per cells are not included in the table because the accuracy of their identification was almost 100%.

### Table 3: Results of HOM Identification done with VBA Macro and Manually for Single Cell Cavity

<table>
<thead>
<tr>
<th>#</th>
<th>Mode</th>
<th>Macro</th>
<th>Manually</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td>TM010</td>
<td>0 1 0</td>
<td>0 1 0</td>
</tr>
<tr>
<td>10-13</td>
<td>TE111</td>
<td>1 1 1</td>
<td>1 1 1</td>
</tr>
<tr>
<td>14-18</td>
<td>Hb(M)</td>
<td>1 1 1</td>
<td>1 1 1</td>
</tr>
<tr>
<td>19-23</td>
<td>TM210</td>
<td>1 0 0</td>
<td>1 0 0</td>
</tr>
<tr>
<td>24-27</td>
<td>Hb(E)</td>
<td>1 1 0</td>
<td>1 1 0</td>
</tr>
<tr>
<td>28</td>
<td>TE020</td>
<td>2 0 0</td>
<td>2 0 0</td>
</tr>
<tr>
<td>29</td>
<td>Hb(E)</td>
<td>2 0 0</td>
<td>2 0 0</td>
</tr>
<tr>
<td>30</td>
<td>Hb(E)</td>
<td>2 0 0</td>
<td>2 0 0</td>
</tr>
</tbody>
</table>

### TRANSVERSE R/Q

CST Microwave Studio has rich functionality that can significantly facilitate the calculation of electrodynamic characteristics of accelerating structures, such as the longitudinal R/Q, the quality factor, and so on. However in order to calculate \( \frac{R_{sh}}{Q} \) a number of operations should be done.

Transverse shunt resistance to the Q-factor can be calculated through Panofsky-Wenzel theorem, where W-stored energy, \( k_z \)-wave number:

\[
\frac{R_{sh}}{Q} = \frac{\int_0^1 \frac{1}{K_z} \frac{\partial E_z}{\partial r} e^{ik_x z} dz|^2}{\omega W} \tag{1}
\]

and through direct integration of magnetic and electric fields:

\[
\frac{R_{sh}}{Q} = \frac{\int_0^1 (i \cdot c \cdot \mu_0 \cdot H_x(z) + E_y(z)) e^{ik_x z} dz|^2}{\omega W} \tag{2}
\]
For axially symmetric structures gradient longitudinal field in the formula 1 can be replaced by the difference, and given that the longitudinal field dipole waves on the axis of the structure is zero, the resulting expression will look like this:

\[
\frac{R_{sh \perp}}{Q} = \frac{1}{k_\varepsilon} \int_0^L 1 \left( \frac{E_z(r=1)(z)}{\Delta r} \right) e^{ik z^2 dz} dz^2 \quad (3)
\]

However, for axially symmetrical structures \( E_z(x=0, y=0) \neq 0 \) and correspondingly equality in Equation 3 are not true anymore. To account for the field on the axis of the structure in the CST is necessary to perform a series of time-consuming procedures. Therefore, the macro was written to automate the process of calculating a cross shunt in VBA. The graphical user interface is shown in Fig. 3. This macro allows you to calculate the cross-shunt resistance at different positions of the axes of integration according to the formulas (1) and (2).

![Figure 3: Macros for transverse R/Q calculation interface.](image)

Work macro has been tested on several structures [5]. Comparing R/Q values obtained in two different ways in an axially asymmetrical structure shown and exclude field on the axis in Table 3.

![Figure 4: Structures for macros testing (a) cylindrically symmetrical, (b) with HOM couplers, (c) with HOM couplers and power input.](image)

Another negative factor affecting the dynamics of the beam in an accelerator is a kick-factor [6,7]. When an object that perturbs the cylindrical symmetry of the accelerating cavity is introduced to the structure, such as power input or HOM couplers it gave a rise to the transverse component of electrical fields appear which can lead to emmitans growth of the beam. To calculate the kick-factor Formula 4 is used:

\[
kick = \frac{v_x}{v_z} \quad (4)
\]

To calculate this characteristic is usually also necessary use the third-party programs. To automate the process of calculating the kick factor we changed a part of our macros responsible for transverse \( \frac{R_{sh \perp}}{Q} \) of the structure. The structures from Fig. 4b and 4c were used to test the macro. The results of the comparison calculation of Kick-factors are presented in the Table 4.

![Table 4: Kick Factor Calculation Results](image)

From the Table 4 we can see that results calculated be direct integration and Panofsky-Wenzel theorem are in a good agreement.

**CONCLUSIONS**

Thus it was written 3 macros to automate the process of HOM identification and process of the calculation of the \( \frac{R_{sh \perp}}{Q} \) and kick-factor. These macros will be available on website of NRNU-MEPHI RF-LAB after beta-testing or they can be requested by correspondent author e-mail.

**REFERENCES**


[3] [https://www.cst.com](https://www.cst.com)


[7] A. Lunin et al., Coupler RF Kick in the Input 1.3 GHz Accelerating Cavity of the LCLS-II Linac, in Proc. IPAC’15, Richmond, VA, USA, pp. 571-574.