Abstract

One of the solutions to generate high-power, high brilliance beam of electromagnetic radiation either in THz or X-ray ranges, is to use a high charge, high average current electron beam. It is also desirable to make the light sources compact and energy efficient and one of the candidates to satisfy these demands is a light source driven by a LINAC. Adding an energy recovery stage (Energy Recovery Linac i.e. ERL) is very attractive to improve overall system energy efficiency but increasing the beam charge and repetition rate, while using conventional ERL schemas, leads to the appearance of beam break-up (BBU) instabilities [1,2]. In this paper to resolve the issue we suggest to use a dual axis asymmetric Superconducting RF (SCRF) cavity for a single turn ERL. Both accelerating and decelerating sections of this ERL consist of the same number of cells and tuned to insure overlapping of operating mode only [3]. We show via analytical and numerical studies that in this case HOMs have significantly lower Q-factors allowing high (above 1A) average currents to be driven through the system without loss.

INTRODUCTION

In this paper we discuss a single turn SCRF ERL system [3-5] i.e. the beam is transported through the whole system only once (Fig.1a). In this model, the beam is accelerated inside the acceleration section, while in the deceleration section most of the beam energy is extracted and guided through a resonantly coupled section back into the acceleration section. The schematic of the source of coherent radiation driven by a single turn SCRF ERL system is shown in Fig. 1a. Figure 1b (technical 3D drawing) illustrates both sections consisting of the same number of cells. Each individual cell is slightly detuned to insure that only the operating mode of both sections are fully overlapping each other creating a single operating mode of the cavity. The rest of the HOM frequencies are separated in the frequency domain. The two sections are linked by a resonant coupler, which insures the EM energy flows between the two sections at the well-defined set of frequencies (overlapping eigenmodes of the system’s components). There is still field leakage from one section to another at other HOMs but as it will be shown, the effect is relatively small. As a result, the possibility for the multi-pass-regenerative BBU feedback mechanism to be established is reduced (HOMs start currents are increased) allowing electron beam of high average current to be transported.

DUAL AXIS CAVITY MODEL

Let us describe the ERL model which will be considered in this work. The ERL under consideration has two axes (Fig. 2). One may assume for clarity reason that while electron bunch propagates along the first axis it is accelerated and while it propagates along the second axis it will be decelerated feeding back energy into the accelerating ERL section via resonant coupler. The cells’ tuning is achieved through small variations of their shapes and cells on each axis are tuned to insure that only the operating mode of both sections is a common for both sections while the frequency positions and Q-factors of higher order modes
are different. Due to the resonant coupling between cells located on different axes, the voltage on each axis is nearly the same for the operating mode only, while it varies strongly for all non-overlapping higher order modes (the ratio between voltages on the axis will be referred as transformer ration $N_{sp}$). Indeed, the cells act as a single cavity at a common operating mode but they behave as separate structures at all other modes. This allows decoupling of HOMs associated with accelerating and decelerating structures and thus breaking the positive feedback loop which may result in BBU development. To make some basic estimations for the cavity parameters required to transport the beam we will assume that a bunch propagates along the ‘design orbit’ is perfectly timed to ‘see’ both the maximum accelerating and decelerating potentials. Any trajectory perturbation due to interaction at IP is ignored. Assuming that at the input bunches propagate on and along the axis $z$ and $E_z(x, z) \propto \sin(k_z z)$ (pill-box like cavity), the trajectory shift at the second port (input to deceleration section) is given by:

$$
\Delta x \equiv -i \frac{c}{\omega} \frac{\partial}{\partial x} V(x) \left[ \frac{L}{W} \left( \frac{\sin(k_z L)}{k_z L} \right) R_{12} \right]
$$

where $c$ is the speed of light, $\omega$ operating frequency, $k_z L$ is the electron bunch phase shift at the cavity exit, $L$ cavity length, $R_{12}$ is the element of the transport matrix [6], $(eW)$ is the full bunch energy, $k_z$ is the longitudinal wave-number and $V(x)$ is an effective accelerating potential seen by electron. The expression gives one an opportunity to make a first estimation of an upper bound for the maximum $R_{12}$ parameter knowing some basic properties of the cavity. Indeed this expression shows the link between bunch’s trajectory deviation and machine’s parameters and therefore, to reduce the deviation on may either reduce parameter $\frac{\omega/eW}{c}$ or limit the $R_{12}$ parameter as shown:

$$
R_{12} \ll D_{sp} \frac{\omega/c}{2k_z L^2 |\partial V/\partial x|}
$$

It is clear that this condition is not sufficient for the ERL to operate as the bunch with a trajectory deviated from the ‘design orbit’ will be delayed due to a longer path and its deceleration (energy recovery) will be affected leading to interruption of ERL operation even if the bunch passed through the decelerating section. If the beam enters the decelerating cavity with some trajectory deviation the transient time $T_g$ from acceleration to deceleration sections is changed $(T_g + \Delta t)$ from its optimum value. This phenomenon is a bunch de-phasing and assuming that initial deviation $\Delta t$ from the unperturbed trajectory $S_0$ is small one can estimate the $\Delta t$:

$$
\Delta t \approx \frac{\Delta x}{2S_0} \frac{\Delta x}{c}
$$

and multiplying the expression by operating frequency $\omega$ and taking into account that dephasing $\Delta \theta$ should be much smaller than $\pi$ the following expression can be evaluated:

$$
\Delta \theta/\pi \ll \frac{D^2}{S_0 \lambda_0}
$$

where $D$ is the cavity aperture and $\lambda_0$ is the operating wavelength. Using these basic estimations and 1.3GHz TESLA cavity [6] as a starting prototype a the numerical model was created and studied using the ACE3P electromagnetic suite developed at SLAC (USA).

### RESULTS OF NUMERICAL ANALYSIS

To study property of asymmetric cavity for ERL both frequency (Omega3P [7]) and time (T3P [8]) domain analyses were carried out. For all cases, curvilinear tetrahedral mesh elements were used. The eigenmode spectrum was analysed and start currents of HOMs which can lead to BBU development were estimated. In figure 2 the schematic of the system studied is shown with bunches 1 and 2 indicating incoming bunches into accelerating and decelerating sections.

![Figure 2: Schematic of dual axis cavity with accelerating (axis 1) and decelerating (axis 2) sections linked by resonant coupling cell.](image)

In figures 3-5 the contour plots showing the result of numerical studies are presented. In figure 3 operating mode which has the highes Q factor and field structure appropriate for ERL operation is demonstrated. Let us note that the split of the sections allows independent (from IP) tuning of the cavity i.e. an individual cell and overall cavity. For instance by introducing either introducing a phase shifter in the resonant cell or tuning cell itself one has additional control on energy recovery and footprint of the system. In figure 4 an example of

![Figure 3: Contour plot of cavity operating eigenmode.](image)

excitation of HOM inside cells located on the second axis is shown. It is clear that due to the detuning of the sections and presence of the resonant couple this HOM can-
not be excited inside the cells on the axis 1 (accelerating section). The accelerating section and coupler are seen by this mode as an effective loads decreasing its Q-factor and limiting the feedback (responsible for one of the BBU development scenarios) between the sections. Let us note that in such a system one can distinguish following families of eigenmodes: overlapping modes i.e. common for all sections; non-overlapping modes i.e. specific for accelerating, decelerating sections and resonant coupler. Using the data from numerical studies the transverse and longitudinal R/Q’s were evaluated. It has been found that the R/Q’s are not equal for each axis as desired. A cluster of common modes between 1.2GHz and 1.4 GHz were found. The majority of these modes lie inside the fundamental pass-band (around 1.3GHz) and include the operating mode. Either side of this pass-band (at 1.1GHz and 1.5GHz) specific modes having equal R/Q were identified to be those of resonant coupler (Fig. 5). These modes are confined/localised to the coupling cell. The finale clusters of the specific modes are dipole modes (Fig.4) and they would be of most concern for preventing the development of BBU if the system is symmetric. In Fig. 5 another example of the specific HOM (located at frequency 1.4855 GHz) of resonant coupler is shown with maximum of the electric field in the middle. These are not eigenmodes of the whole cavity and localised inside the coupler. This mode can be potentially dangerous and correct design of the RF coupler is important. Indeed, it is clear that locating a HOM coupler or any other alternative absorber in the middle of the cell will not affect the beam transportation or operating mode and will load the HOMs thus solving the problem.

Taking into account the results observed, it is possible to compare the start currents of symmetric and asymmetric cavities. It is clear that the start currents of asymmetric cavity’s HOMs should increase as compared with the symmetric cavity. Indeed each HOM in symmetric cavity has equal field in each structure (i.e. transformer ratio $N_{sp}=1$) and using telegraphist approach one can derive the following relation between the start currents:

$$I_{assym} = I_{sym} \left[1 + \left(\frac{1}{N_{sp}}\right)\right] \frac{1}{2N_{sp}}$$

Analysing the expression, one notes, that the ratio between the currents has the minimum value 1 if $N_{sp}=1$ (a symmetric cavity). One also note that for $N_{sp}=1$ the cavity has twice the power losses of a standard cavity and thus the HOM start current is doubled compared to a conventional single axis recirculating machine. In figure 6 the comparison of the HOMs start currents for symmetric and asymmetric machines is shown. The diamond points indicate the start currents values of the asymmetric system’s HOMs, while triangular points shows the start currents for the most dangerous modes of the symmetric cavity. The start currents in asymmetric cavity are at least 5 times above the currents observed in symmetric ERL which allows assuming that it will be possible to increase average beam current without beam loss due to BBU instabilities. Also the start current values can be further increased by adding HOMs’ couplers and absorbers.

**CONCLUSION**

We introduced a new concept of asymmetric system of accelerating and decelerating cells coupled by resonant coupler for ERL. The operational principles of new cavity were discussed using both numerical and analytical approaches. We compared it with conventional symmetric design and discussed the advantages of the asymmetric layout. Indeed it was demonstrated that asymmetry allows transporting through the ERL an electron bunch having significantly increased (at least by factor of 5) current without limiting BBU. This potentially is very attractive for applications in which it is important to increase THz and x-ray photon yield as well as bunch energy recovery.

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**REFERENCES**