SIMULATION OF BEAM BEHAVIOR CAUSED BY ODD HARMONICS OF BEAM LOADING IN J-PARC RCS

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Abstract

The J-PARC RCS accelerates two bunches at a harmonic number two. The major Fourier components of the beam current are even harmonics. However, the odd harmonics grow under some conditions even though they are very small amplitude at the beginning. We describe the particle tracking simulation results for the odd harmonic beam loading effect in the RCS.

INTRODUCTION

The J-PARC Rapid Cycling Synchrotron (RCS) accelerates proton beam up to 3 GeV at a repetition rate of 25 Hz. The beam commissioning has progressed successfully and we have achieved acceleration of a 1 MW equivalent beam without significant beam loss [1].

The heavy beam loading effect is the most important issue for a longitudinal beam motion. An rf cavity of the RCS exhibits broadband characteristics to cover wide frequency range and to add the second harmonic rf to alleviate a space charge effect [2]. The beam loading compensation system using a feedforward method is utilized for the multi-harmonic beam loading. The system can successfully compensate the beam loading up to sixth harmonic [3] based on a revolution frequency.

The RCS accelerates two bunches at the harmonic number $h=2$. In this case, the major Fourier components of the beam current are even harmonics. Figure 1 shows the simulation result of the beam current harmonics up to $h=6$ during the acceleration. The horizontal axis indicates the acceleration time and the vertical axis indicates the beam current for each harmonic. The upper graph is the case without the beam loading and the lower graph is the case with the beam loading. The even harmonics of the beam loading are compensated.

APPEARANCE OF ODD HARMONICS

In order to investigate the mechanism of the growth for the odd harmonics, we focus on $h=3$ harmonic because the cavity impedance has a peak around the frequency on $h=3$ and then the wake voltage of $h=3$ becomes larger than the other odd harmonics. Furthermore, we consider stationary rf bucket at the injection energy for simplicity. The parameters used in the simulation are shown in Table 1.

Table 1: Parameters of Simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Beam energy</td>
<td>400 MeV</td>
</tr>
<tr>
<td>Number of particles</td>
<td>$8.3 \times 10^{13}$ ppb</td>
</tr>
<tr>
<td>RF voltage</td>
<td>120 kV / ring</td>
</tr>
<tr>
<td>RF frequency (h=2)</td>
<td>1.228 MHz</td>
</tr>
<tr>
<td>Cavity resonance frequency</td>
<td>2.1 MHz</td>
</tr>
<tr>
<td>Cavity Q-value</td>
<td>1.5</td>
</tr>
<tr>
<td>Cavity shunt impedance</td>
<td>28.8 kΩ / ring</td>
</tr>
</tbody>
</table>

The odd harmonics appear under the following conditions if a charge amount is same on each bunch: (a) by the difference of the bunch shape on each bunch and/or (b) by

We describe that a mechanism for the growth of the odd harmonics by using the particle tracking simulation.

![Figure 1: Simulation result for each harmonic of the beam current during the acceleration. The upper graph is the case without the beam loading and the lower graph is the case with the beam loading. The even harmonics of the beam loading are compensated.](image-url)
the displacement of the bunch position with respect to the zero cross of the fundamental rf voltage.

Figure 2 is an example of the case (a) where the first graph shows the bunch shape, the second graph shows the beam harmonic of $h = 3$, the third graph shows the wake voltage of $h = 3$ and the bottom graph shows the potential well formed by the fundamental rf voltage and the wake voltage of $h = 3$ where the vertical axis is normalized by the potential of the fundamental rf. The horizontal axis indicates the phase of the fundamental rf.

Figure 3 is also an example of the case (a), however, the bunch deformation is the opposite in contrast with Fig. 2. The first bunch becomes $10\%$ narrower and the second one becomes $10\%$ narrower. The phase of the beam harmonic is antisymmetric of Fig. 2. This bunch deformation makes the bunches move apart from each other as seen from the potential well.

Figure 4 is an example of the case (b) where the bunch shape is fixed, whereas the bunch position is moved away from the original zero cross phase. The deviation of the first bunch is $+10$ degrees and it is $-10$ degrees on the second bunch, that is, the bunches get closer to each other. The beam harmonic of $h = 3$ appears as shown in the second graph and the phase is the quadrature of the case (a). The resultant potential well distortion suggests that the bunches get further closer, that the first bunch becomes wider, and that the second bunch becomes narrower next step.

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**Figure 2:** Bunch shape, beam current, wake voltage of $h = 3$ and potential well of the fundamental rf and the wake voltage of $h = 3$ arisen from the bunch deformation where the first bunch becomes wider and the second bunch becomes narrower.

**Figure 3:** Bunch shape, beam current, wake voltage of $h = 3$ and potential well of the fundamental rf and the wake voltage of $h = 3$ arisen from the bunch deformation where the first bunch becomes wider and the second bunch becomes narrower.

**Figure 4:** Bunch shape, beam current, wake voltage of $h = 3$ and potential well of the fundamental rf and the wake voltage of $h = 3$ arisen from the bunch displacement where the bunches get closer to each other.
Figure 5 is also an example of the case (b), however, the bunches move apart from each other in contrast with Fig. 4. The deviation of the first bunch is $-10$ degrees and it is $+10$ degrees on the second bunch. The phase of the beam harmonic of $h = 3$ is antisymmetric of Fig. 4. The resultant potential well distortion suggests that the bunches move further apart from each other, that the first bunch becomes narrower, and that the second bunch becomes wider next step.

The way for the appearance of $h = 3$ harmonic is the individual or the combination among Figs. 2 – 5.

**GROWTH OF ODD HARMONICS**

The particle tracking simulation helps to understand how $h = 3$ harmonic grows up. In the simulation, the beam is injected at once for simplicity, whereas the RCS uses a multi-turn injection scheme in reality.

Figure 6 shows the simulation result using the parameters in Table 1. The simulation condition is that only the beam loading of $h = 3$ affects on the bunch and the other harmonics of the beam loading are compensated. The upper graph shows the bunch displacement from the zero cross phase and the lower graph shows the bunch width. The red line indicates the first bunch and the blue one indicates the second bunch.

The bunch position and the width are almost same for the both bunches until 80 turns. After that, each bunch moves individually. The bunches start to get closer around 100 turns and then the first bunch becomes wider and the second bunch becomes narrower around 125 turns suggested by the bottom graph of Fig. 4.

Next, the bunches move apart from each other around 160 turns suggested by the bottom graph of Fig. 3. This displacement promotes the bunch deformation as shown in the bottom graph of Fig. 5, that is, the first bunch becomes narrower and the second bunch becomes wider around 200 turns. Furthermore, the bunches get closer around 230 turns by the effect as shown in the bottom graph of Fig. 2.

After that, the bunches trace the same way from Fig. 4. The difference of the bunch displacement and the width become larger and larger during the successive appearance of $h = 3$ among Figs. 2 – 5 and the beam loss occurs around 290 turns at the end.

The reason why the beam loading effect of $h = 3$ rapidly grow up is that the behavior of the bunch shape deformation and the bunch displacement are synchronized with the behavior of the potential well distortion caused by $h = 3$ harmonic. This phenomenon depends on the phase of the wake voltage and the cavity resonance frequency of 2.1 MHz is just the phase for the growth. The behavior of the bunch is different in the other resonance frequency and it should be investigated.

**SUMMARY**

We have investigated the beam loading effect by the odd harmonics focused on $h = 3$ harmonic in the J-PARC RCS. Although the odd harmonics are very small at the beginning, they rapidly grow up and cause the beam loss. The simulation results suggest that the synchronization of the bunch behavior with the potential well distortion promotes the growth of $h = 3$ harmonic. In principle, this phenomenon can be suppressed by compensating the odd harmonics of the beam loading completely. We will consider a sophisticated beam loading compensation system dedicated to the odd harmonics under the two bunch operation.

**REFERENCES**