REVIEW OF LINEAR OPTICS MEASUREMENTS AND CORRECTIONS IN ACCELERATORS

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Abstract

The measurement and correction of optics parameters has been a major concern since the advent of strong focusing synchrotron accelerators. Traditionally, colliders have led the development of methods for optics control based on turn-by-turn centroid data, while lepton storage rings have focused on closed-orbit-response techniques. Recently considerable efforts are being invested in comparing these techniques in different light sources and colliders. An emerging class of less invasive optics control techniques based on the optimization of performance related observables is demonstrating a great potential. A review of the existing techniques is presented highlighting comparisons, merits and limitations.

THE DAWN OF A NEW DISCIPLINE

Perturbations from field imperfections and misalignments became a concern along with the conception of the strong focusing theory in 1957 [1]. However, the assumed approach was to specify design tolerances that would not impact machine performance. For example in [1] it is envisaged that with 1% rms gradient errors any particular machine would be unlikely to have more than 8% peak β-beating. In the following decade the AGS experienced tune drifts and aperture limitations due to quadrupolar errors at injection energy which were mitigated with dedicated correction circuits [2]. Actually, it seems that the modern nomenclature of tune to designate the betatron frequency [1] or betatron number [3] originates upon the intense tuning activity of this quantity. In 1972 the form tune was already widely used [4]. Tunes were, and still are, of critical importance since resonances [1] are to be avoided (an entertaining way to find resonances can be found in [5]).

In 1975 the first beam-based measurement of the average β-function over independently powered quadrupoles took place in the ISR using the tune change due to a quadrupole gradient variation [6]. This technique is referred to as k-modulation in the following.

In the same year a first beam-based measurement and correction of transverse coupling was performed in the AGS [7] while a more refined technique was being developed for the ISR [8]. These techniques are based upon turn-by-turn beam position data at a single location. The plans to build larger colliders and the use of low-β* insertions triggered the need to measure and correct chromaticity [4,9,10] in the mid ’70s.

In 1983 a major achievement took place in the ISR. The Beam Position Monitors (BPMs) around the collider were used to measure betatron phase advance and beta functions from the phase and amplitude of induced betatron oscillations [11]. This was the first realization of optics measurements from turn-by-turn BPM data with analog technology. This technique has been constantly growing in applications, scope, analytical descriptions and users.

Another major technique for optics measurements uses closed orbits excited with different orbit correctors [12–14]. Successful corrections based on these measurements were demonstrated for first time on SPEAR in 1993 [14]. An optics model of the machine is fit to reproduce the measured closed orbits. This technique is referred to as Orbit Response Measurement (ORM) in the following.

A last set of optics correction techniques may be introduced with the first sentence from [15] (1991): “For future linear colliders, [...] with demanding tolerances on final focus system alignment and magnet errors, it becomes increasingly important to use the beam as a diagnostic tool”. Extrapolating to any accelerator, a beam-based optimization of machine performance-related observables is a universal approach for the mitigation of lattice imperfections. This technique can sometimes be considered as a passive correction as the required size of the perturbations might be tolerated during machine production operation.

These first realizations of the techniques presented above (k-modulation, turn-by-turn, ORM and passive correction) appeared between 1975 and 1993, setting the ground for a new discipline: “Optics measurements and corrections in accelerators”. The materialization of this discipline came a decade later with the publication of a book [16].

MEASUREMENT AND CORRECTION TECHNIQUES

K-modulation

K-modulation has been successfully used to measure average betatron functions in almost every accelerator, for example, ISR [6,11], LEP [17,18], HERA [19], RHIC [20,21], SLS [22], Tevatron [23], ALBA [24] and LHC [25,26]. This technique is limited by the tune resolution, the knowledge of the quadrupole integrated field versus current, the quadrupole fringe fields and the unwanted tune change due to a possible orbit change during the quadrupole modulation. In SLS and ALBA the rms statistical error of this technique was in the 1-2% level [22,24] with a comparable system-
Table 1: Overview of measurement techniques. The meaning of acronyms and symbols follows. C: Calibration or tilt; FT: Fourier Transform; M: Model; SVD: Singular Value Decomposition; φ, β and $D_x$: phase advance, beta function and dispersion; $⟨β⟩$: Average beta function over a quadrupole; $ΔQ_{\text{min}}$: Closest tune approach.

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In hadron colliders this technique is mostly used to infer IP $β^*$ functions from nearby quadrupoles. However, simulations of the HL-LHC [27] show that the very pushed interaction region optics challenges the accuracy of this technique to interpolate the $β^*$.

**Turn-by-turn**

The first turn-by-turn measurements of lattice parameters in ISR [11] were followed by LEAR [28] in 1988 and LEP [29] in 1993. Measuring the $β$ functions from the amplitude of betatron oscillations requires a good absolute BPM calibration. Basically, if BPMs have an rms linear scale error of, e.g., 5% the $β$ uncertainty is 10%. This technique is referred to as “$β$ from amplitude”. Another error source of the $β$ from amplitude is the need to normalize the measured $β$’s to the model average $β$. The perturbed lattice features an average $β$ function which tends to increase with the value of the rms $β$-beating [30, 31]. The weakest point of the $β$ from amplitude method is the BPM unknown scale factors. An optics-measurement-based BPM calibration has been recently demonstrated in the LHC [32, 33] by switching off the quadrupoles in the interaction region and profiting from the parabolic behavior of the $β$ function in a drift. BPM scale calibrations around 0.5% were achieved.

To avoid the aforementioned limitations of the “$β$ from amplitude” methods, $β$ functions were computed at LEP from the phase advance between 3 BPMs [29, 34] assuming the exact knowledge of the focusing elements in between the 3 BPMs. This method, usually named “$β$ from phase”, was also used in CESR [35] in 2000 giving a $Δβ/β$ in the 100% level before corrections. The CESR optics correction was based on fitting a model to the measured phases, reaching an rms $Δβ/β$ of 2% thanks to the independently powered quadrupoles. The 3 BPM method developed at LEP has been recently extended to consider any number of BPMs [36], N-BPM method, considerably boosting the resolution of the measurement in the LHC. For this the knowledge of the optics model uncertainties is fundamental. This method has been also applied in ALBA [37].

Large scale BPM systems inevitably feature a set of malfunctioning BPMs. This can happen in very subtle ways with only one faulty reading out of 1000 turns (known as ghost data) [19, 38]. Efficient ways to detect these bad BPMs were developed for SLC [39] and SPS [38], based on SVD and FT decompositions, respectively. A comparison of these techniques was performed later at RHIC [40]. The SVD technique was also applied in many circular machines [41–43], to measure lattice parameters. In order to get the best out of the two techniques, it is possible first to condition, or clean, the BPM data matrix with the SVD using the singular values to the largest ones and then to apply the FT to the cleaned data. Concerning the FT algorithms, NAFF [44] and Sussix [45] feature better accuracy in the measurement of main frequencies than the regular FFT, however for the phase advance between 2 BPMs the FFT is preferred [46].

A second key point for optics measurements is the excitation required to induce a beam oscillation around the closed orbit. In lepton rings this is traditionally done with a fast kicker and measurements are performed while the beam naturally damps back towards its closed orbit. On the other hand, in hadron machines any applied excitation leads to irreversible transverse emittance blow-up and beam degradation. An important progress occurred in 1998 when AC dipoles...
were proposed to excite forced, coherent and non-destructive betatron oscillations [47] with a first application to optics measurements in RHIC [48]. It is noted that an adiabatic excitation of the AC dipole minimizes the emittance growth after the measurement [49]. In [50] it is shown analytically how linear and non-linear resonance driving terms are modified by the AC dipole with experimental measurements in RHIC [51]. In Tevatron [52] it is demonstrated that the AC dipole perturbation to the linear optics is equivalent to a quadrupole at the same location with a gradient depending only on the machine and AC dipole tunes.

For coupling measurements the situation is less intuitive. The corresponding analytical equations are derived in [53]. The coupling measurement is easily extended to measure chromatic coupling with or without AC dipole [54, 55]. The AC dipole has been fundamental in the commissioning of the LHC [56, 57] since about 20 optics during the β∗ squeeze are to be measured within tolerance for machine protection. Recently two new applications of AC dipole have been proposed to identify impedance sources [46] and to measure a short term dynamic aperture [58].

A thorough study of the systematic errors involved in turn-by-turn optics measurements techniques is presented in [31]. A new analytical formula for the 3 BPM method is derived taking into account quadrupolar errors in between the BPMs. This should speed-up the implementation of the N-BPM method, which is currently based on Monte Carlo simulations. Analytical formulas are also derived for the perturbations to the phase advance and coupling measurements from nonlinear dynamics. These limit the accuracy of the turn-by-turn optics measurements at ESRF in ultra-low coupling mode.

In hadron colliders, it is fundamental to perform local corrections in the interaction regions. Two techniques have successfully demonstrated these local corrections: action and phase jump [59] and segment-by-segment [43, 56]. After local corrections, optics errors can be further reduced by applying a global correction using a response matrix of phase advances on the available quadrupoles [30, 57]. In RHIC successful global corrections were achieved using β from amplitude [60]. In light sources it is more customary to compute corrections by fitting a model to the measurements [61, 62].

**ORM**

Optics correction based on ORM [12–14] is widely used in electron storage rings. ORM consists of the changes in BPM readings in response to corrector excitations, which is a matrix containing a large number of elements. The simplest approach to extract machine parameters from ORM is a direct fit of φ and β functions at every BPM using the analytical equation describing the orbit response [12, 13]. This was successfully used in KEKB [63] with a reduced set of orbit correctors. The β functions obtained this way are directly affected by BPM gain errors. This limitation is mitigated by using all available orbit correctors and fitting the optics model, as proposed in [14]. The model parameters usually include: quadrupole gradients error, BPM and corrector calibration error and roll errors of these components. More parameters may be included if the measured ORM is not reproduced within the measurement noise level. Finally, the beta-beating is inferred from the fitted optics model. The fitted quadrupole gradient errors can be reversely applied to the machine to correct the beta-beating. Two modern implementations of the complete ORM algorithm were developed in the NSLS VUV Ring, known as LOCO [64], and in the ESRF [65].

LOCO code is re-implemented into Matlab-LOCO [66] with graphical user interface to ease the optics correction. Several fitting algorithms are available in this code, namely, Gauss-Newton, (Scaled) Levenberg-Marquardt, and constrained fitting. The last one is used to solve degeneracy problems, e.g., two or more quadrupoles are situated between two BPMs. Third generation light sources often face this problem, and the quadrupole gradient errors found from LOCO fitting tend to be too vigorous [66] when a single fitting algorithm is employed. A fitting based on SVD with proper eigenvalue cut also avoids this problem [22].

Due to the large number of data points, the LOCO fit is quite robust against statistical measurement error, and the corresponding statistical beta function error in the LOCO-fitted optics model can be 0.1%. This does not include systematic uncertainties. Inferred beta-beating below 1% and coupling corrections to 0.01% level have been achieved at light sources [67, 68], where the quadrupole magnets are individually powered and enough skew quadrupoles are installed.

Even when the LOCO fitting is successful, the measured ORM after correction may not converge towards the model ORM. This was observed at the SLS [22] and illustrates the limitation of the parametrized model to represent the real machine. Therefore, the uncertainty of the inferred beta-beating must be well above the inferred beta-beating when the convergence is not satisfactory. Measuring ORM only in a section of the ring proved successful to detect possible locations of the differences between the model and the machine [69].

ORM measurement is a lengthy procedure, varying the corrector excitation current one-by-one and recording BPM readings. A fast measurement in about one minute using a fast orbit feedback network is under development at Diamond [70]. In small and medium-size hadron machines ORM techniques have demonstrated successful only in estimating the β-beating [71–74] while in large colliders first attempts to use LOCO resulted impractical [75] or in unrealistic corrections [76].

Techniques based on closed orbit bumps were successful in identifying gradient errors in Tristan and RHIC [77, 78]. Another technique that has been applied to excited closed orbits is the already mentioned action and phase analysis [79]. A single quadrupole error could be identified with excellent accuracy [80].
**Turn-by-turn and ORM Comparisons**

Large efforts are being done to compare the different measurement techniques. Turn-by-turn measurements are conceptually faster than ORM, however first attempts in light sources faced important BPM limitations [22, 81]. Once these limitations were overcome the \( \beta \) functions showed an agreement slightly above the 1% rms level between turn-by-turn and ORM [37, 82–84]. There is no experimental evidence of an accuracy below the 1% rms level for any of the techniques.

Concerning coupling measurements, no direct comparison of coupling terms from the two techniques has been presented. In [83] coupling corrections based on both techniques yield similar 1% emittance ratio. Analytical considerations in [31] challenge the coupling measurement from turn-by-turn at emittance ratios of 0.1% and below.

**Passive Corrections**

Since the very first accelerators, performance is optimized by scanning available parameters. In the framework of correcting linear optics aberrations first realizations can be found in the linear collider SLC [85]. The strategy at the SLC was to develop a set of orthogonal knobs connected with the different phase space degrees of freedom at the IP. These knobs were individually scanned until a minimum beam size was found with the help of a parabolic fit to mitigate measurement errors. This technique is still applied in Final Focus Systems (FFS) such as ATF2 [86] and foreseen for future linear colliders. The Simplex algorithm was used to tune the KEKB injector lincac in 1998 [87]. Simulations show that the Simplex is also needed in the CLIC [88] FFS to achieve acceptable performance.

In lepton and hadron circular colliders the luminosity has been maximized using multivariate optimization algorithms over many physics fills [89, 90].

Recently renewed versions of this concept have been also successfully applied in light sources. In SLS an optimization based on the random walk successfully corrected coupling to unprecedented levels achieving a record low vertical emittance [91]. Other applications of optimization techniques in light sources can be found in [92, 93]

**SUMMARY AND OUTLOOK**

Beam linear optics, understood as the arrangement of bending and focusing elements, is one of the fundamental pillars of modern accelerators. Machine performance and protection aspects rest upon linear optics parameters. The high demands of modern accelerators has boosted the “optics measurement and correction” to grow into a discipline of its own. Table 1 summarizes the various techniques following the classification used above. The main challenge faced by all accelerators and measurement techniques is the required machine time. Conceptually optics correction could be as fast as orbit correction. First steps in this direction have been done for ORM in Diamond [70] and for turn-by-turn in LHC [94, 95] and NSLS-II [83]. In particular, turn-by-turn techniques require more flexible and accurate BPM systems and possibly the generalized use of AC dipoles to excite long-lasting and small betatron oscillations.

Large experimental programs have demonstrated a 1% accuracy in the \( \beta \) function measurement from the various techniques. Equivalent comparative studies are still required for coupling. Future projects, like HL-LHC, SuperKEKB, FCC, ESRF upgrade, MAX IV, SLS-II, etc, will continue challenging optics control techniques in terms of accuracy, resolution, speed and instrumentation. Developments in other disciplines, like collective effects leading to particle loss as impedance, space charge and Touschek, also require improving the measurement and control of linear optics [96–100].

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