Longitudinal Diagnostics Methods and Limits for Hadron LINACs

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Outline

• SNS Accelerator

• Longitudinal Diagnostics in SNS

• BPM for Bunch Length Measurements
  – How to
  – Limitations

• Longitudinal Twiss Parameters Measurements
  – Method = RF + Drift + BPM
  – Conditions of Applicability and Errors

• Results for SNS Superconducting Linac

• Conclusions
Spallation Neutron Source Accelerator

(SNS)

Front-End:
Produce a 1-msec long, chopped, H- beam

1 GeV LINAC

Accumulator Ring:
Compress 1 msec long pulse to 700 nsec

<table>
<thead>
<tr>
<th>Ion Source RFQ</th>
<th>2.5 MeV</th>
<th>87 MeV</th>
<th>186 MeV</th>
<th>387 MeV</th>
<th>1000 MeV</th>
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<tbody>
<tr>
<td>MEBT</td>
<td>DTL</td>
<td>CCL</td>
<td>SRF, β=0.61</td>
<td>SRF, β=0.81</td>
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Design parameters

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>Kinetic Energy [GeV]</td>
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<td>Beam Power [MW]</td>
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<td>Repetition Rate [Hz]</td>
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<td>Peak Linac Current [mA]</td>
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<tr>
<td>Linac pulse length [msec]</td>
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<tr>
<td>SRF Cavities</td>
<td>81</td>
</tr>
</tbody>
</table>

150 kW injection dump

7.5 kW beam dump

7.5 kW beam dump

Liquid Hg Target
Longitudinal Diagnostics in SNS Linac

• Acceptance phase scan
  – was used at the DTL entrance to measure the bunch length
  – was used in SCL to estimate the longitudinal emittance

• Bunch Shape Monitors (BSMs) in CCL
  – Were used to measure bunch length in CCL
  – Were used to confirm the new method

• SCL RF + Drift + Beam Position Monitors with the amplitude signals (non-intercepting method)
There is no direct way to interpret the stripline BPM signal as the bunch density.

\[ I \propto \frac{A_\omega}{I_0 \left( 2\pi \frac{R}{\gamma \lambda} \right)} \]

\( A_\omega \) - Fourier harmonics of the longitudinal density distribution.

Hadron accelerators: \( v < c \)

Case (b)

Picture from Alex Chao’s book “Collective Instabilities in Accelerators”
• The frequency of the BPM response is limited even by the simple geometry

• There is a dependency on the energy of the beam. It should be taken into account during the calibration

$$I_0 \left( \frac{1}{2\pi \frac{R}{\gamma \lambda_\omega}} \right)$$

SNS SCL Case:
- $R = 40 \text{ mm}$
- $\beta = 0.55$
- $\gamma = 1.2$
- $f_0 = 402.5 \text{ MHz}$
Harmonic of Gaussian Distribution

\[ \lambda(z) = q \cdot N \cdot \frac{1}{\sqrt{2\pi\sigma_z^2}} \cdot \exp\left(-\frac{z^2}{2\sigma_z^2}\right) \]

Gaussian Longitudinal Distribution

BPM harmonic after Fourier transformation

\[ A_\omega(\sigma_\varphi) = A_{\text{max}} \cdot \exp\left(-2\pi^2 \left(\frac{\sigma_\varphi}{360^0}\right)^2\right) \]

\[ \sigma_\varphi \] - Longitudinal RMS bunch size in degrees

\[ \sigma_\varphi = \frac{360^0}{\sqrt{2 \cdot \pi}} \sqrt{\ln\left(\frac{A_{\text{max}}}{A_\omega}\right)} \]

\[ A_{\text{max}} \] - should be found during the calibration
Acceptable Bunch Length Range

Reasonable bunch length for this method is between 30° and 120°

Bunches in linacs are much shorter!

In a normal situation this method is useless!

\[
\frac{A}{A_{\text{max}}} = \exp \left( -2 \cdot \pi^2 \cdot \left( \frac{\sigma_\varphi}{360^\circ} \right)^2 \right)
\]

Errors for Bunch Length assuming \( \delta A/A_{\text{max}} = 1\% \)

A between 10% and 90% \( A_{\text{max}} \) will give us the acceptable errors
The New Method: RF + Drift + BPM

- We want to know the bunch length at places where it is not possible to measure with BPMs.
- We want to know not only the bunch length but also Twiss parameters which can be used in the tracking models.

Solution is a combination of three components:
- RF cavity to manipulate the longitudinal phase space. It is a point of Twiss parameters measurements.
- Drift – long enough for de-bunching of the beam to $30^0$-$120^0$
- BPM for the bunch length measurements after the drift.

$$E_{\text{out}} = E_{\text{in}} + qV_0 \cdot \cos(\phi_{RF})$$
Definitions of Variables and Parameters

Longitudinal phase space

- \((\varphi, dE)\) Longitudinal coordinates – phase and energy deviation from the synchronous particle

Statistics:

- \(\langle \varphi^2 \rangle = \sigma_{\varphi}^2\) square of RMS bunch length
- \(\langle \varphi \cdot dE \rangle = K_{\text{corr}} \cdot \sigma_{\varphi} \cdot \Delta E\) phase-energy correlation
- \(\langle dE^2 \rangle = \Delta E^2\) square of RMS energy spread

Initial Twiss Parameters:

- \(\varepsilon_{\text{rms}} = \sqrt{\langle \varphi^2 \rangle \cdot \langle dE^2 \rangle - \langle \varphi \cdot dE \rangle^2}\)
- \(\alpha_{\text{Twiss}} = -\frac{\langle \varphi \cdot dE \rangle}{\varepsilon_{\text{rms}}}\)
- \(\beta_{\text{Twiss}} = \frac{\langle \varphi^2 \rangle}{\varepsilon_{\text{rms}}}\)
Transformation of Parameters

--- Coordinate transformation by transport matrices ---

\[
M_{Drift} = \begin{pmatrix}
1 & 2\pi \frac{L}{\lambda_{RF}} \cdot \frac{1}{m\gamma^3 \beta^3} \\
0 & 1
\end{pmatrix}
\]

\[
M_{RF} = \begin{pmatrix}
1 & 0 \\
-qV_0 \cdot \sin(\phi_{RF}) & 1
\end{pmatrix}
\]

\[
\lambda_{RF} = 2\pi \frac{c\beta}{\omega_{RF}}
\]

\[
M = M_{Drift} \times M_{RF} = \begin{pmatrix}
m_{1,1} & m_{1,2} \\
m_{2,1} & m_{2,2}
\end{pmatrix}
\]

Total transport matrix

\[
\begin{pmatrix}
\phi_{BPM} \\
dE_{BPM}
\end{pmatrix} = M \times \begin{pmatrix}
\phi_0 \\
dE_0
\end{pmatrix}
\]

No space charge effects included!

\[
\phi_{BPM} = m_{1,1} \cdot \phi_0 + m_{1,2} \cdot dE_0
\]

\[
\langle \phi_{BPM}^2 \rangle = m_{1,1}^2 \langle \phi_0^2 \rangle + 2m_{1,1}m_{1,2} \langle \phi_0 dE_0 \rangle + m_{1,2}^2 \langle dE_0^2 \rangle
\]

\[
\sigma_{BPM}^2 (\phi_{RF}) = \left( 1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3 \beta^3} \right)^2 \sigma_0^2 + \{Corr\} + \left( 2\pi \frac{L}{\lambda_{RF}} \frac{\Delta E}{m\gamma^3 \beta^3} \right)^2
\]

\[
\{Corr\} = 2 \left( 1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3 \beta^3} \right) \left( 2\pi \frac{L}{\lambda_{RF}} \frac{1}{m\gamma^3 \beta^3} \right) \cdot K_{corr} \cdot \sigma_0 \Delta E_0
\]
RF Cavity Effect

$$\sigma_{BPM}(\phi_{RF}) = 1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3} \sigma_0$$

No correlation, no energy spread.
Bunch length should grow from few to 30⁰-60⁰.
Maximal size at \( \sin(\phi_{RF}) = \pm 1 \)

For SNS:

At SCL entrance \( E_{kin} = 186 \text{ MeV} \)

\( qV_0 \approx 10 \text{ MeV} \)
\( m\gamma^3\beta^3 \approx 939 \cdot (1.2)^3 \cdot (0.55)^3 \approx 270 \text{ MeV} \)

\( \lambda_{RF} = 0.37 \text{ m} \)
\( \sigma_{BPM} = 60^0 \)
\( \sigma_0 = 3^0 \)

\( L \geq \frac{\lambda_{RF} \cdot m\gamma^3\beta^3 \cdot \sigma_{BPM}}{2\pi \cdot qV_0 \cdot \sigma_0} \)

\( L \geq \frac{0.37 \cdot 270 \cdot 60}{2\pi \cdot 10 \cdot 3} = 32 \text{ m} \)

- We have to transport the beam on significant distance.
- Weaker the cavity – more drift space we need.
- Higher the energy – more drift space we need.
- At the end of SCL_Med we need more than 200 m drift.
Energy Spread Effect

\[ \sigma_{BPM}^2 = \left( 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0}{m\gamma^3 \beta^3} \right)^2 \sigma_0^2 + \left( 2\pi \frac{L}{\lambda_{RF}} \frac{\Delta E}{m\gamma^3 \beta^3} \right)^2 \]

- **Cavity**
- **Energy spread**

Effects are equal when (bunch length in radians)

\[ qV_0 \cdot \sigma_0 \approx \Delta E \]

No correlation.

Cavity at max focusing/defocusing

\[ \sin(\phi_{RF}) = \pm 1 \]

Border value for the bunch length

\[ \sigma_0 \approx \frac{\Delta E}{qV_0} \approx \frac{0.1 - 0.3 \text{ MeV}}{10 \text{ MeV}} \approx 0.6^0 - 2^0 \]

- **Energy spread should be small**
  \[ \Delta E \ll qV_0 \]

- **We will see only energy spread if**
  \[ \sigma_0 \ll \frac{\Delta E}{qV_0} \]

- **Space charge may significantly increase the energy spread, so the cavity should be strong enough**
Phase-Energy Correlation Effect - No

\[ \sigma_{BPM}^2(\phi_{RF}) = \left( m_{1,1}(\phi_{RF}) \right)^2 \sigma_0^2 + 2m_{1,1}(\phi_{RF}) \cdot m_{1,2} \cdot K_{corr} \cdot \sigma_0 \cdot \Delta E + m_{1,2}^2 \cdot \Delta E^2 \]

\[ K_{corr} = 0 \quad qV_0 = 10 \text{ MeV} \quad \sigma_0 = 4^\circ \quad \Delta E = 235 \text{ keV} \]

- Bunch length in degrees
- Almost \( \sin^2 \) from cavity
- Const. from energy spread
- BPM’s amplitudes
- Almost the same level

SNS SCL:
\[ f_{RF}=805 \text{ MHz} \quad f_{BPM}=402.5 \text{ MHz} \]
Phase-Energy Correlation Effect - Yes

The shape of the BPM’s amplitude vs. RF phase curve is sensitive to Phase – Energy Correlation

- $K_{corr} = 0$ (No correlation)
- $K_{corr} = 0.9 > 0$
- $K_{corr} = -0.9 < 0$
How to Get Parameters from Curves

We scan RF phase

These values are from the model (like Trace3D)

That we measure with BPM amplitude

These 3 we want to know!

\[
\sigma_{BPM}^2(\phi_{RF}) = \left( m_{1,1}(\phi_{RF}) \right)^2 \sigma_0^2 + 2 m_{1,1}(\phi_{RF}) \cdot m_{1,2} \cdot K_{corr} \cdot \sigma_0 \cdot \Delta E + m_{1,2}^2 \cdot \Delta E^2
\]

\[
\sigma_{BPM}^2(\phi^{(1)}_{RF}) = \left( m_{1,1}(\phi^{(1)}_{RF}) \right)^2 \sigma_0^2 + 2 m_{1,1}(\phi^{(1)}_{RF}) \cdot m_{1,2} \cdot K_{corr} \cdot \sigma_0 \cdot \Delta E + m_{1,2}^2 \cdot \Delta E^2
\]

\[
\sigma_{BPM}^2(\phi^{(2)}_{RF}) = \left( m_{1,1}(\phi^{(2)}_{RF}) \right)^2 \sigma_0^2 + 2 m_{1,1}(\phi^{(2)}_{RF}) \cdot m_{1,2} \cdot K_{corr} \cdot \sigma_0 \cdot \Delta E + m_{1,2}^2 \cdot \Delta E^2
\]

\[
\sigma_{BPM}^2(\phi^{(3)}_{RF}) = \left( m_{1,1}(\phi^{(3)}_{RF}) \right)^2 \sigma_0^2 + 2 m_{1,1}(\phi^{(3)}_{RF}) \cdot m_{1,2} \cdot K_{corr} \cdot \sigma_0 \cdot \Delta E + m_{1,2}^2 \cdot \Delta E^2
\]

- We have 3 unknown variables

- 3 equations - it is enough (but more is better!)

- If there are more than 3 eq. we have a linear system for the least square method
Errors of Parameters

\[
\begin{pmatrix}
\sigma_{2BPM}(\phi_{RF}^{(1)}) \\
\vdots \\
\sigma_{2BPM}(\phi_{RF}^{(N)})
\end{pmatrix} = M_{N \times 3} \times \begin{pmatrix}
\sigma_0^2 \\
K_{corr} \cdot \sigma_0 \cdot \Delta E \\
\Delta E^2
\end{pmatrix}
\]

\[
\begin{pmatrix}
\sigma_0^2 \\
K_{corr} \cdot \sigma_0 \cdot \Delta E \\
\Delta E^2
\end{pmatrix} = (M^T \cdot W \cdot M)^{-1} M^T W
\begin{pmatrix}
\sigma_{2BPM}(\phi_{RF}^{(1)}) \\
\vdots \\
\sigma_{2BPM}(\phi_{RF}^{(N)})
\end{pmatrix}
\]

\[
W_{N \times N} = \begin{pmatrix}
1 & 0 & 0 \\
\delta^2(\sigma_{BPM}^{(1)}) & 0 & 0 \\
0 & \ldots & 0 \\
0 & 0 & \delta^2(\sigma_{BPM}^{(N)})
\end{pmatrix}
\]

\[
\begin{pmatrix}
\delta^2(\sigma_0^2) \\
\delta^2(K_{corr} \cdot \sigma_0 \cdot \Delta E) \\
\delta^2(\Delta E^2)
\end{pmatrix} = \left[(M^T \cdot W \cdot M)^{-1}\right]_{Diagonal}
\]

- Errors of the initial parameters are defined by the BPMs’ amplitudes errors, cavity’s strength, and RF phases
- More the RF phase points is better for accuracy
- Usually the Twiss parameters errors will be small if the our second order correlations have small errors, but is not always true
Summary of Method

• We need:
  – several lattice elements: RF+Drift+BPM
  – Data
  – transport matrices from the RF entrance to BPM

• The method will give us beam longitudinal Twiss parameters

• Is it model based?
  – If there is no space charge – all formulas are here!
  – If there is space charge effect – we need only something very simple – envelope model like Trace3D
  – We can use more complicated models (IMPACT, TraceWin etc.), but the error estimation should be done with linear lattice model
Some Special Cases for Twiss

- Our results bunch length, energy spread, and phase-energy correlation. We have errors for these values.
- If the correlation coefficient between phase and energy is close to 1, even the small errors will give us a big relative error for the emittance.
- RMS emittance is important integral of motion in linear lattices, so it could be measured in convenient places.

\[
\varepsilon_{\text{rms}} \ll \sigma_{\varphi} \cdot \Delta E
\]

\[
\varepsilon_{\text{rms}} = \sqrt{\left(\sigma_{\varphi} \cdot \Delta E\right)^2 - \left(K_{\text{corr}} \cdot \sigma_{\varphi} \cdot \Delta E\right)^2} = \sigma_{\varphi} \cdot \Delta E \cdot \sqrt{1 - K_{\text{corr}}^2}
\]

\[
K_{\text{corr}} \approx 1
\]

\[
\frac{\delta \varepsilon_{\text{rms}}}{\varepsilon_{\text{rms}}} \gg 1
\]
BPM’s Amplitude Calibration

\[ u_{\omega}(\sigma_{\omega}) \propto \frac{A_{\omega}(\sigma_{\omega})}{I_0 \left( \frac{2\pi R}{\gamma \lambda} \right)} \]

\[ A_{\omega}(\sigma_{\phi}) = A_{\text{max}} \cdot \exp \left( -2 \cdot \pi^2 \cdot \left( \frac{\sigma_{\phi}}{360^0} \right)^2 \right) \]

- We have to know \( A_{\text{max}} \)
- We can use a very short bunch (few degrees) for calibration
- We have to take into account the beam energy
- For the case of SNS Superconducting linac we used the production setup when we knew the energy at each BPM and the fact that the bunches are short
We included all BPMs up to 80 m downstream (BPM1-BPM14). The phase scan was with 5° step. The system of equations had \((72 \times 14) = 1008\) equations.

Results (XAL units):
- \(\alpha = 0.56 \pm 0.02\)
- \(\beta = 5.33 \pm 0.13\)
- \(\epsilon = (0.928 \pm 0.012) \times 10^{-6}\)


- The data is a byproduct of tuning procedure
- We included all BPMs’ data because we have them all anyway
Each Cavity is A Measuring Device

- We can use each cavity in SCL as the measuring point for the longitudinal Twiss
- We assumed a constant normalized emittance. It means we fitted $\alpha$, $\beta$ Twiss parameters only.
- This is a benchmark of the method and the model
Improvements—Some Unchecked Ideas

- Two-Three cavities simultaneous phase scan
- We can reduce the distance

\[ L \geq \frac{\lambda_{RF} \cdot m\gamma^3\beta^3 \cdot \sigma_{BPM}}{2\pi qV_0 \sigma_0} \]

- RF1 + (Drift #1) + RF2 + (Drift #2) + BPM
- Use the initial energy spread for the beam size increase at the entrance of the 2\textsuperscript{nd} cavity

\[ \sigma_{BPM}^2(\phi_{RF}) = \left(1 - 2\pi \frac{L}{\lambda_{RF}} \frac{qV_0 \sin(\phi_{RF})}{m\gamma^3\beta^3} \right)^2 \sigma_0^2 + \{Corr\} + \left(2\pi \frac{L}{\lambda_{RF}} \frac{\Delta E}{m\gamma^3\beta^3} \right)^2 \]
Summary

• The method of the longitudinal Twiss parameters measurements based on the BPMs’ signals was developed for the hadron linacs

• The limitations and errors of the method have been analyzed

• The applicability of the new method was demonstrated at the SNS Superconducting Linac
Thanks!