UNDERSTANDING THE TUNE SPECTRUM OF HIGH INTENSITY BEAMS

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Abstract

The tune spectra of a low intensity bunched beam consist of various head-tail modes separated from each other by the synchrotron frequency. As the beam intensity is sufficiently increased, such that its incoherent tune shift approaches a value comparable to the synchrotron tune, the low intensity tune spectrum is characterized by modified. These modifications of tune spectrum as a function of space charge and impedances were studied by means of numerical and analytical calculations based on simplified beam distributions by M. Blaskiewicz [1]. Controlled experiments were performed to study the effect of space charge on tune spectra at GSI SIS-18, and compared with these theoretical studies in [2]. This paper will present the experimental details, interpretation of the tune spectra under various beam conditions and measurement of several elusive beam parameters. Finally the relevance of these measurements for other facilities are considered.

INTRODUCTION

For low intensity beams, the particles in the beam primarily interact with the guiding external fields and any other interactions can be neglected. Therefore, the tune spectra can be approximated well by the single particle spectra, i.e.

\[ Q_k = Q_0 + \Delta Q_k, \]

usually defined as baseband tune spectrum, where \(Q_0\) is the fractional part of the machine tune, \(\Delta Q_k = \pm kQ_s\) are the synchrotron satellites and \(Q_s = Q_{s0}\) is the small amplitude synchrotron tune. For a single particle performing betatron and synchrotron oscillations, the relative amplitudes of the satellites are [3]

\[ |d_k| \sim |J_k(\chi/2)| \]

where \(\chi = 2\xi\phi_m/\eta_0\) is the chromatic phase, \(\xi\) is the chromaticity, \(\phi_m\) is the longitudinal oscillation amplitude of the particle (in rad) and \(\eta_0\) the frequency slip factor. \(J_k\) are the Bessel’s functions of first kind of order \(k\).

Two parallel tune measurement systems i.e. TOPOS [4] and BBQ [5] are installed at SIS-18. They utilize separate capacitive pick-ups and signal analysis methods to calculate the tune spectra. The systems were commissioned at various stages of acceleration, and several instances of modified asymmetric spectra were reported [6, 7] especially visible during injection. Similar “anomalous” spectra were also observed earlier at other facilities at injection energies [8]. Both systems were therefore carefully tested for different beam currents and beam excitation methods. Comparable spectra were measured using both the systems independent of the beam conditions, and any systematic noise or intermodulation were thereby discarded [2].

Several studies can be found in literature to estimate the effect of impedances and space charge on the transverse coherent beam instabilities [9, 10, 3]. The studies mainly discussed impedances and its effect on beam stability, but the treatment of space charge was not considered in adequate detail. One of the first prominent studies which accounted the effect of space charge and presented a complete analytical solution using a simplified model called “square well airbag model” was performed in [1]. The primary suggestion was that, the space charge pushes the fast head-tail instability thresholds much higher than predicted otherwise. Another significant outcome of this study was that the tune spectrum of space charge dominated beams is significantly modified. These studies were pursued and extended to realistic bunches by means of analytical and numerical studies [11, 12] and self-consistent simulations [13]. The predicted modifications of the aforementioned studies matched well with the spectra obtained in our measurements [7].

Detailed beam measurements were therefore performed to understand the effect of space charge forces on the tune spectra in comparison with the theoretical studies. All the relevant beam parameters were exclusively measured. The measurements gave two clear results. The first is that the tune spectra modification can be interpreted successfully by the analytical equations given in [1] and the follow-up studies [11, 12, 13]. The theoretical predictions can be compared with the measured modification of head-tail mode frequencies to provide a direct measurement of high intensity effects. To highlight the importance of these results for other synchrotrons which seemingly operate in quite different intensity and energy regimes, the space

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SIS-18</th>
<th>SPS</th>
<th>RHIC</th>
<th>ANKA</th>
</tr>
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<tbody>
<tr>
<td>(I_p) (mA)</td>
<td>~ 12</td>
<td>~ 1600</td>
<td>~ 500</td>
<td>~ 10</td>
</tr>
<tr>
<td>(\gamma_0)</td>
<td>~ 27</td>
<td>~ 4.5</td>
<td>~ 100</td>
<td></td>
</tr>
<tr>
<td>(\xi s(2\sigma))</td>
<td>~ 12</td>
<td>~ 3</td>
<td>~ 10</td>
<td>~ 0.153</td>
</tr>
<tr>
<td>(\Delta Q_{sc})</td>
<td>~ 0.05</td>
<td>~ 0.05</td>
<td>~ 0.02</td>
<td>~ 10^{-4}</td>
</tr>
<tr>
<td>(Q_{s0})</td>
<td>~ 0.007</td>
<td>~ 0.005</td>
<td>~ 0.0015</td>
<td>~ 0.008</td>
</tr>
<tr>
<td>(q_{sc})</td>
<td>~ 7</td>
<td>~ 10</td>
<td>~ 12</td>
<td>~ 0.2</td>
</tr>
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</table>
charge parameters \( q_{sc} \) at injection were estimated (based on published data) for other synchrotrons as shown in Table 1. The space charge effects were found to be comparable to SIS-18. The definition of the parameters described in the table are discussed in the next sections. The SPS at CERN operates at much higher energies, but a strong effect of space charge at its injection was hinted in [14] where the “slow” modes were found to be strongly damped. Similarly RHIC saw a series of anomalous spectra in its low energy operations during run 7 [8, 15]. The only electron machine considered is the ANKA booster which looks to be unaffected by space charge. Next section mentions the theoretical results concerning the effect of high intensity beams on tune spectra. It is followed by a section describing the experimental conditions, and the various measurement results. Following section discusses the applications scenarios of these results and comments on examples from other facilities.

**HIGH INTENSITY EFFECTS**

As the beam intensity is increased, the interaction with the environment i.e. beam pipe, diagnostic devices and other insertions become significant. These interactions are often combined under a single framework of “machine impedances” [16, 17]. In addition to impedances, the beam particles also interact with each other, and this is referred as the space charge effect. These high intensity effects are normalized to the synchrotron tune to account for the coupling between transverse and longitudinal phase space.

**Transverse Impedances**

Machine impedances encapsulate the coherent interaction of beam with its environment. In general, impedances are described by a frequency dependent complex parameter. In this contribution, we will only consider the purely imaginary transverse dipolar impedances for perfectly conducting beam pipe of radius \( b_x \) since we are dealing with transverse motion of space charge dominated beams. The relation of transverse dipolar impedance with pipe radius is given by [17, 18],

\[
Z_x = -i \frac{Z_0}{2\pi(\beta_0\gamma_0 b_x)^2} \tag{3}
\]

It is a broadband impedance which is independent of frequency. The real coherent tune shift is then given by,

\[
\Delta Q_{c,x} = -\frac{qI_pF_f R^2 Z_x}{2Q_s^2\beta_0 W_0} \tag{4}
\]

where \( I_p \) is the peak bunch current, \( q \) is the particle charge and \( W_0 = \gamma_0\rho_0 c^2 \) is the rest energy. The relativistic parameters are \( \gamma_0 \) and \( \beta_0 \), \( R \) is the ring radius, \( F_f \) is the Laslett form factor given by the shape of the boundary, and \( Z_0 \) is the characteristic impedance of vacuum. The index \( x \) denotes the parameters for the horizontal plane, \( x \) can be simply replaced by \( y \) for vertical plane and in case of no indices, the equations represent either of the planes.

**Space Charge**

Repulsion of beam particles with each other due to Coulomb forces counteracts the external focussing, and results in depression of tune or incoherent tune shift for each particle distinctly based on its exact position in the beam. This effect along with indirect incoherent interaction of particles via the environment are collectively referred as space charge effects and is well described in [18]. Here we just refer to the analytical results obtained for the incoherent tune shifts,

\[
\Delta Q_{sc,x} = \frac{qI_p R}{4\pi\epsilon_0 c W_0\gamma_0^2\beta_0^4 b_x^2} \tag{5}
\]

with \( \varepsilon_x \) as the rms emittance of the equivalent K-V distribution. In the case of an elliptic transverse cross-section, the emittance \( \varepsilon_x \) in Eq. 5 should be replaced by

\[
\frac{1}{2} \left( \varepsilon_x + \sqrt{\varepsilon_x^2 - \frac{Q_{xy}}{Q_{yy0}}} \right) \tag{6}
\]

It should be noted that the incoherent tune shift has a strong dependence on energy since the beam interaction due to electric and magnetic forces tend to cancel each other as the beam gets relativistic. Considering the scaling of beam emittance with beam energy, \( \Delta Q_{sc} \) is inversely proportional to \( \gamma_0^2 \).

At this point, we define an important quantity, the space charge parameter \( q_{sc} \) which is given as a ratio of the space charge tune shift (Eq. 5) to the synchrotron tune \( (Q_s) \),

\[
q_{sc} = \frac{\Delta Q_{sc}}{Q_s} \tag{7}
\]

The physical relevance of the space charge parameter can be interpreted as the mitigation of transverse space charge forces due to longitudinal motion of the beam particles along the bunch.

**Head-tail Mode Frequency Shifts**

Based on the solution of Vlasov equation for a “square well airbag model” with arbitrary impedances, the following equation emerges for head-tail mode frequency shifts [1],

\[
\Delta Q_k = -\frac{\Delta Q_{sc} + \Delta Q_{cc}}{2} \pm \sqrt{(\Delta Q_{sc} - \Delta Q_{cc})^2/4 + (kQ_s)^2} \tag{8}
\]

where the + is used for \( k > 0 \). For \( k = 0 \) one obtains \( \Delta Q_{k=0} = -\Delta Q_{cc} \). It should be noted that the analytic solution for the eigenvalues (Eq. 8) is obtained from a simplified approach, where the transverse space charge force is assumed to be constant for all particles. This assumption is correct if there are only dipolar oscillations. Moreover, the coherent tune shift due to transverse impedances is not exactly represented in comparison on the original derivation (Page 10 of Ref. [1]). It should be pointed out that the head-tail mode frequency shifts (\( \Delta Q_k \)) is dependent on intensity, unlike the low intensity description given in Eq. 1.

\[ ISBN 978-3-95450-127-4\]
Table 2: Measured Beam and Machine Parameters During the $U^{73+}$ and $N^{7+}$ Experiments

<table>
<thead>
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<th>Parameters</th>
<th>$U^{73+}$</th>
<th>$N^{7+}$</th>
</tr>
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<tbody>
<tr>
<td>$W_{kin}$ (MeV/u)</td>
<td>11.4</td>
<td>11.56</td>
</tr>
<tr>
<td>$I_p$ (mA)</td>
<td>0.9 – 7.5</td>
<td>0.8 – 11</td>
</tr>
<tr>
<td>$\varepsilon_x, \varepsilon_y(2\sigma)$ (mm-mrad)</td>
<td>45, 22</td>
<td>33, 12</td>
</tr>
<tr>
<td>$Q_s0, Q_s1$</td>
<td>0.007, 0.0065</td>
<td>0.006, 0.0057</td>
</tr>
<tr>
<td>$q_{sc}$</td>
<td>0.14 – 1.7</td>
<td>2.2 – 10.8</td>
</tr>
</tbody>
</table>

The eigenfunctions of the head-tail eigenmodes mentioned in Eq. 8 are given as, [1]

$$\bar{x}_k(\phi) = \cos(k\pi\phi/\phi_0)\exp(-i\chi\phi/\phi_0)$$ (9)

where $\bar{x}_k$ is the local transverse bunch offset for the $k^{th}$ mode, $\chi = \xi\phi_0/\eta_0$ is the chromatic phase, $\phi_0$ is the full bunch length (in rad) and $\eta_0$ is slip factor. Bunch length in radians is directly related to the bunch length in time i.e. $\tau = \phi/\omega_0$.

Selective Damping of Head-tail Modes

In Ref. [13], Eq. 8 has been compared to Schottky spectra obtained from 3D self-consistent simulations for realistic bunch distributions in rf buckets. It has been pointed out, that the negative $-k$ eigenmodes (or slow modes) are suppressed due to intrinsic Landau damping. Analytic and numerical solutions for Gaussian and other bunch distributions valid for $q_{sc} \gg 1$ were presented in [11, 12] which validated the simulation results.

EXPERIMENTAL OBSERVATIONS

The beam experiments were performed with the primary motive of interpretation of the “anomalous” tune spectra for space charge dominated beams. Therefore, the measurement conditions with a high space charge parameters were prepared. In this section, we will present the interpretation of the measured spectra in comparison with the analytical predictions, and further extract the high intensity parameters based on head-tail mode frequency shifts. Chromaticity measurements based on fitting with head-tail eigenmodes will also be presented.

Experimental Details

The measurements were mainly performed at injection energy of 11.4 MeV/u on a long injection plateau of SIS-18. Some tune measurements were also performed during the acceleration ramp to understand its evolution with the beam energy. Both tune measurement systems i.e. BBQ and TOPOS were used to record the tune spectra during all measurement times, while TOPOS was also used to record the “raw signal” separately for each plate from all the 12 BPMs in the “raw data” mode. The sampling rate in 125 MSa/s. Further details of SIS-18 installations of both these systems can be found in [2]. Beam excitation types such as band-limited noise, frequency sweep and a single kick excitation were used. The measurements concerning the effect of excitation are reported elsewhere [7].

The relevant beam parameters for the described experiments are summarized in Table 2. A Schottky probe was used to measure the beam energy. Ionization profile monitor [19] (IPM) is used for recording the transverse beam profiles, DC current transformer [20] for current measurement and TOPOS for the longitudinal beam profile and longitudinal synchrotron tune $Q_{s1}$. It should be noted that measured longitudinal synchrotron tune $Q_{s0}$ is different from the small amplitude synchrotron tune $Q_{s0}$ (calculated from rf voltage and machine parameters) for longer bunches [21]. The space charge parameter $q_{sc} = \Delta Q_{sc}/Q_{s1}$ is then calculated using Eq. 5 and Eq. 4 at each beam current setting and the head-tail mode shifts are then predicted by Eq. 8.

Interpretation of Tune Spectra

Figure 1: Horizontal tune spectra for $U^{73+}$ ions and beam parameters given in Table 2 (see text). The dashed lines indicate the head-tail tune shifts from Eq. 8.

Figure 1 shows the horizontal tune spectra obtained with the BBQ system using bandwidth limited noise at three different intensities. Fig. 2 shows the predicted head-tail mode shifts (red lines) from Eq. 8 for the $U^{73+}$ beam with parameters given in Table 2 as a function of space charge parameter. Space charge parameter is a function of peak beam current since all other parameters were unmodified (see Table 2). The dashed lines in Fig. 1 correspond to the predicted values according to Fig. 2. The lower part of the Fig. 1 (1(a)) shows the horizontal tune spectrum at low intensity. Here the $k = 1, 0, -1$ peaks are almost equidistant, which is expected for low intensity bunches. The space charge parameter obtained using the beam parameters and Eq. 5 is $q_{sc} \approx 0.14$. The vertical lines indicate the positions of the synchrotron satellites obtained from Eq. 8 (with
beam given in Table 2) as a function of space charge parameter. Figure 4 shows the vertical tune spectrum obtained by the BBQ system with band limited noise excitation for $N^{7+}$ beams with $q_{sc}$ values larger than 2. Here the negative modes ($k < 0$) could not be seen anymore. In the vertical plane, the coherent tune shift is larger due to the smaller SIS-18 beam pipe diameter ($\Delta Q_c \approx \Delta Q_{sc}/10$). The shift of the $k = 0$ peak due to the effect of the pipe impedance is clearly visible in Fig. 4. The measured mode peaks from Fig. 4 are plotted back in Fig. 3.

**Measurement of Beam Pipe Impedances**

The coherent tune shift $\Delta Q_c$ can be obtained by measuring shift of the $k = 0$ mode as a function of the peak bunch current as shown in Fig. 5. The transverse impedance of the beam pipe is obtained by a linear least square error fit of the measured shifts in both planes to Eq. 3 and Eq. 4. The impedance values obtained at injection energy are $Z_x = -i(0.23 \pm 0.04) \text{ M}\Omega/m^2$ and $Z_y = -i(1.78 \pm 0.04) \text{ M}\Omega/m^2$. Using Eq. 3 correspond to the beam pipe radii of $b_x = 105 \pm 7 \text{ mm}$ and $b_y = 38 \pm 0.6 \text{ mm}$ which roughly agree to pipe width in the straight sections of SIS-18.

**Measurement of Space Charge Parameter**

The space charge parameter ($q_{sc}$) or the incoherent space charge tune shift ($\Delta Q_{sc}$) can be determined directly from the tune spectra by measuring the separation between the $k = 0$ and $k = 1$ peaks. For $k = 0$ mode, $\Delta Q_{k=0} = \Delta Q_c$ from Eq. 8, therefore the separation between head-tail modes $k = 0$ and $k = 1$ ($\Delta Q_{k=1} = \Delta Q_{k=1} - \Delta Q_{k=0}$) is an exclusive function of $\Delta Q_{sc}$ and can be estimated numerically using Eq. 8. Equivalently, the solu-
Figure 5: Cumulative measurements over several beam-times (including very high intensity $Ar^{18+}$ beam not mentioned in Table 2) showing coherent tune shift in the horizontal and the vertical planes as a function of the peak beam current. The dashed lines correspond to a linear least square error fit.

Figure 6: Combining the results from the Figures 2 and 3, a plot of predicted $q_{sc}$ using Eq. 5 against measured $q_{sc}$ using the distance between modes $k = 0$ and $k = 1$ is obtained.

Figure 7: Frequency sweep excites individual head-tail modes, the mode number can be identified by the number of “valleys” between the peaks. Here $k = 1, 2$ and 3 modes are excited one after another as the excitation frequency sweeps through them.

**Measurement of Chromaticity**

Chromaticity measurement based on head-tail phase shifts was first proposed in [23]. The main idea is that the betatron phase difference $\Delta \phi$ between any two longitudinal bunch "slices" spaced by $\Delta \tau$ oscillates with synchrotron frequency and the amplitude of these oscillations is a function of chromaticity as depicted in Eq. 10.

$$\Delta \phi = \frac{\xi \omega_0}{\eta_0} \Delta \tau (1 - \cos(2\pi n Q_{sc}))$$

where $n$ is the turn number. The technique considers the measurement error tolerances due to transverse impedances and cable response, but ignores the effect of space charge forces [24].

Figure 8: The change in the head-tail mode structure for $k = 0$ mode with change in chromaticity. It is visually evident that the head-tail phase shift increases with chromaticity. The beam current and thus the amplifier noise levels vary with the chromaticity setting.

As we saw in the previous section, the tune spectra is no longer symmetric under moderate space charge effects
(q_{sc} \gtrsim 1), and the mentioned technique might not give expected results. An alternative approach in these conditions would be to excite each head-tail mode individually as shown in Fig. 7 where transverse center-of-mass for \( k = 1, 2 \) and 3 modes can be identified. A fit of the mode-structure as a function of chromaticity with the predicted analytical eigenmodes (Eq. 9) gives a direct measurement of chromaticity. In Fig. 8, three consecutive turns showing transverse center-of-mass for \( k = 0 \) mode in vertical plane are plotted for different values of chromaticity. The chromaticity values can be extracted by a least squares fit in comparison with the analytical eigenmodes. Figure 9 shows the measured chromaticity values using this method against the set chromaticity values in the machine. The results are consistent with the measurements with conventional methods at SIS-18. More details can be found in [2].

![Figure 9](image1)

**Figure 9**: Vertical chromaticity values are measured in the setting range of 0 to \(-4\) at GSI SIS-18. The measured values deviate from the set values, however they are consistent with other measurement methods. The tune value is also modified for different values of chromaticity.

The relative amplitudes of the head-tail modes are found to vary with chromaticity as shown in [2]. For a high intensity bunched beam, they are defined by an overlap between the mode spectra (symmetric around the chromatic frequency) and the transverse machine impedances [9]. Therefore, methods estimating chromaticity based on their relative heights are prone to errors.

**APPLICATIONS**

The modification of tune spectra has direct implications on the various feedback systems which use it as an input. An interesting example was demonstrated in [8], where it was reported that the phase locked loop (PLL) for tune measurement failed to lock at the correct tune peak during the “low energy operations” with 3.5 GeV/u Gold ions at RHIC. Figure 10 shows one of the tune spectra measured in RHIC during run 7 [8]. It is clear from the theoretical predictions that the spectra is modified due to space charge effects, and the left peak is the \( k = 0 \) mode.

Though the incoherent tune shift sharply decreases with increase in beam energy, the space charge parameter experiences a weaker dependence due to normalization by synchrotron tune. Measurement of tune spectra over acceleration confirms this assertion, and space charges are found to play an important role during the initial stages of acceleration. As exemplary tune spectra during acceleration at SIS-18 is shown in Fig. 11 where \( k = -1 \) mode is farther from \( k = 0 \) mode when compared with the \( k = 1 \) mode.

Secondly, the \( k = -1 \) mode is strongly damped in comparison to \( k = 1 \) in line with predictions for space charge dominated beam.

![Figure 10](image2)

**Figure 10**: The tune spectra measured at RHIC [15] (Courtesy: P. Cameron). The irregularly spaced head-tail modes can be well described by head-tail mode frequency shifts predicted by Eq. 8.

![Figure 11](image3)

**Figure 11**: Tune measurement during acceleration at GSI SIS-18. The spectra is asymmetric around \( k = 0 \) mode until the head-tail modes are not resolvable anymore.

**SUMMARY**

Careful and directed beam measurements at GSI SIS-18 showed systematic modification of tune spectra for high intensity beams. The measurements compared very well with the theoretical studies which predicted the head-tail mode frequency shifts. The measurements were further used to estimate the incoherent tune shifts and transverse impedances. It was also shown that the tune and chromaticity...
maticity measurement systems should evaluate the impact of high intensity effects on them. Relevance of these studies for other synchrotons were also briefly discussed.

ACKNOWLEDGEMENTS

This work is partially supported by DITANET (novel Diagnostic Techniques for future particle Accelerators: A Marie Curie Initial Training NETwork), Project Number ITN-2008-215080. Experimental support from the operating crew and GSI beam diagnostics group is sincerely acknowledged. We are indebted to O. Boine-Frankenheim for extremely helpful theoretical discussions on the subject. We thank Klaus-Peter Ningel from GSI RF group who supported us with the adiabatic rf amplitude ramp set-up. Finally, Marek Gasior from CERN beam instrumentation is acknowledged for helpful discussions of BBQ system.

REFERENCES