IBS near transition crossing in NICA collider

S. Kostromin (JINR), V. Lebedev (FNAL), I. Gorelyshev (JINR), A. Sidorin (JINR)
IBS - Coulomb scattering of charged particles in a beam results in an exchange of energy between different degrees of freedom

⇒ Causes the beam size to grow up ⇒ limits luminosity lifetime

\[
\begin{align*}
\frac{1}{\tau_x} &= \frac{1}{\varepsilon_x} \frac{d\varepsilon_x}{dt} \\
\frac{1}{\tau_y} &= \frac{1}{\varepsilon_y} \frac{d\varepsilon_y}{dt} \\
\frac{1}{\tau_p} &= \frac{1}{6r_p^2} \frac{d(6r_p^2)}{dt}
\end{align*}
\]

1974 --- A. Piwinski derived original theory of IBS applicable for weak focusing only [1]


\( \gamma < \gamma_{tr} \) - quasi-equilibrium between three “temperatures” (of each degree of freedom) may exists

IBS leads to

- relaxation (equation) between 3 “temperatures” in the beam faster than 6D-emittance growth rate

\[
NICA \\
\gamma = 5.8 \\
\gamma_{tr} = 7.1
\]

\( \gamma \approx \gamma_{tr} \)

\( \gamma > \gamma_{tr} \) - quasi-equilibrium between local temperatures in the beam does not exists

IBS leads to

- infinite beam 6D-phase space volume growth in circular machines

\[
distr. \ function, \\
f(x,x') = \frac{1}{2^{\frac{5}{2}} \sqrt{5\pi} \delta_x \delta_{x'}}, \exp\{-\frac{x^2}{2\delta_x^2} - \frac{x'^2}{2\delta_{x'}^2}\}
\]

Beam temp.

\[
T = T_{def} = \frac{m \delta_x^2}{2k_B} = \frac{m \delta_{\theta_x}^2}{2k_B}
\]

\[
\delta_{\theta_x} = \sqrt{\delta_{\theta_x}^2}
\]
Evolution of the velocity distribution function is described by Landau collision integral

\[
\frac{df}{dt} = -2\pi n r_0^2 c^4 L_c \frac{\partial}{\partial v_i} \int \left( f \frac{\partial f'}{\partial v_j} - f' \frac{\partial f}{\partial v_j} \right) \omega_{ij} d^3v'
\]

\[
\omega_{ij} = \frac{(v - v')^2 \delta_{ij} - (v_i - v_i')(v_j - v_j')}{|v - v'|^3} \int f(v) d^3v = 1,
\]

Plasma perturbation theory
Works only when \(L_c >> 1\)!
(logarithmic approximation)

When the particles kinetic energy much higher than their interaction potential

\[
L_c = \ln(\rho_{\text{max}}/\rho_{\text{min}}) \text{ is the Coulomb logarithm.}
\]

\[
\rho_{\text{min}} = r_0 c^2 / \bar{v}^2, \quad \rho_{\text{max}} = \sqrt{\bar{v}^2 / 4\pi n r_0 c^2}, \quad \bar{v}^2 = \sigma_{\text{vx}}^2 + \sigma_{\text{vy}}^2 + \sigma_{\text{vz}}^2.
\]

\[
\sigma_{vi} = \sqrt{v_i^2}, \quad i = (x,y,z) \text{ are the rms velocity spreads}
\]

General time-dependent solution does not exist
Non-relativistic one component plasma

Under the assumption that the initial particles' distribution is Gaussian, it can be reduced to a 3-temperature distribution function:

\[ f(x, y, z) = \frac{1}{(2\pi)^{3/2} \sigma_x \sigma_y \sigma_z} \exp\left(-\frac{1}{2} \left( \frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2} + \frac{z^2}{\sigma_z^2} \right) \right) \]

The growth rate for the distribution function is given by:

\[ \Sigma_{ij} = \int f v_i v_j d^3v. \]

Rate of change of these second order moments due to Coulomb scattering in plasma:

\[ \frac{d}{dt} \Sigma_{ij} = \int \frac{\partial f}{\partial t} v_i v_j d^3v. \]

Result - rate of energy exchange between degrees of freedom in plasma:

\[ \frac{d\Sigma}{dt} = \frac{d}{dt} \begin{pmatrix} \sigma_{xx}^2 \\ \sigma_{yy}^2 \\ \sigma_{zz}^2 \end{pmatrix} = \frac{(2\pi)^{3/2} nr_0^2 c^4 L_c}{\sqrt{\sigma_{xx}^2 + \sigma_{yy}^2 + \sigma_{zz}^2}} \begin{pmatrix} \Psi(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}) \\ \Psi(\sigma_{yy}, \sigma_{zz}, \sigma_{xx}) \\ \Psi(\sigma_{zz}, \sigma_{xx}, \sigma_{yy}) \end{pmatrix} \]

Assumptions:

- Initial particles' distribution – Gaussian \( \Rightarrow \) does not stay Gaussian-like in evolution process (but stay similar)
- Integral does not take into account single collisions (responsible for non-Gaussian tails)
Non-relativistic one component plasma

expressed through the symmetric elliptic integral of the second kind

\[
\Psi(x, y, z) = \frac{\sqrt{2r}}{3\pi} \left( y^2 R_D(x^2, y^2, z^2) + z^2 R_D(x^2, z^2, y^2) - 2x^2 R_D(y^2, z^2, x^2) \right)
\]

\[
R_D(u, v, w) = \frac{3}{2} \int_0^{\infty} \frac{dt}{\sqrt{(t+u)(t+v)(t+w)^3}}
\]

\[
r = \sqrt{x^2 + y^2 + z^2} \quad x, y, z \geq 0
\]

- \(\Psi(1,1,1)\) - depends on ratios of its variables (not on r)
- normalized that \(\Psi(0,1,1)=1\)
- \(\Psi(1,1,1)=0\) – no energy transfer between degrees of freedom
- \(\Psi(x,y,z) + \Psi(y,z,x) + \Psi(z,x,y) = 0\) energy conservation

Function for two equal temperatures
In the ring accelerator (collider)

In difference to plasma where the energy is conserved, in a storage ring the binary collisions do not conserve energy in the beam frame (BF).
It results in unlimited 3D-emittance growth supported by energy transfer from the longitudinal beam motion to the internal particle motion in BF.

How to calculate??

- Be sure that particle collision time $\rho_{\text{max}}/v$ in BF is much smaller than period of betatron oscillations
- Assume that in each location of the accelerator the distribution function in the BF is Gaussian in 6D phase space
- Use results for plasma in each location of the ring => calculate the growth rate in BF
- Convert these rates into the Laboratory frame (LF) emittance growth rates
- Average the this results over whole accelerator length to obtain overall IBS rates:

$$\text{Rate} := \sum_{i} \frac{\text{rate}_i \, ds}{C_{\text{ring}}}$$

- local rate of the emittance growth at the lattice element of small length $ds$ with fixed Twiss parameters
Smooth focusing, unbunched beam (variation of beta- and dispersion- functions \(\sim 0\))

\[
\Sigma_V = \gamma \cdot \beta \cdot c \cdot 
\begin{pmatrix}
\theta_x^2 & 0 & 0 \\
0 & \theta_y^2 & 0 \\
0 & 0 & \theta_p^2
\end{pmatrix}
\]

- matrix of second moments of local velocity distribution in BF

\[
\begin{align*}
\theta_x^2 &= \frac{\varepsilon_x}{\beta_x} \\
\theta_y^2 &= \frac{\varepsilon_y}{\beta_y} \\
\theta_p^2 &= \sigma_p^2 \cdot \frac{\beta_x \cdot \varepsilon_x}{\gamma^2 \cdot \sigma_x^2}
\end{align*}
\]

\[
\sigma_x = \sqrt{\varepsilon_x \cdot \beta_x + \sigma_p^2 \cdot D^2} \\
\sigma_y = \sqrt{\varepsilon_y \cdot \beta_y}
\]

\[
\frac{d}{dt} \varepsilon_k = \frac{\sqrt{\pi}}{2 \cdot \sqrt{2}} \cdot \frac{e^4 \cdot N \cdot L_c}{M^2 \cdot c^3 \cdot \sigma_x \cdot \sigma_y \cdot L \cdot \beta^3 \cdot \gamma} \cdot \sqrt{\theta_x^2 + \theta_y^2 + \theta_p^2} 
\]

\[
\begin{pmatrix}
\Psi_{IBS}(\theta_x, \theta_y, \theta_p) & 0 & 0 \\
0 & \Psi_{IBS}(\theta_y, \theta_p, \theta_x) & 0 \\
0 & 0 & \gamma^2 \cdot \Psi_{IBS}(\theta_p, \theta_x, \theta_y)
\end{pmatrix}
\]

where

\[
E_x = \begin{pmatrix}
\beta_x & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & \frac{D^2}{\beta_x}
\end{pmatrix} \quad E_y = \begin{pmatrix}
0 & 0 & 0 \\
0 & \beta_y & 0 \\
0 & 0 & 0
\end{pmatrix} \quad E_s = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 2
\end{pmatrix}
\]

\[
\frac{d}{dt} \begin{pmatrix}
\varepsilon_x \\
\varepsilon_y \\
\sigma_p^2
\end{pmatrix} = \frac{\sqrt{\pi}}{2 \cdot \sqrt{2}} \cdot \frac{e^4 \cdot N \cdot L_c}{M^2 \cdot c^3 \cdot \sigma_x \cdot \sigma_y \cdot L \cdot \beta^3 \cdot \gamma} \cdot \sqrt{\theta_x^2 + \theta_y^2 + \theta_p^2} 
\]

\[
\begin{pmatrix}
\beta_x \cdot \Psi_{IBS}(\theta_x, \theta_y, \theta_p) + \gamma^2 \cdot \frac{D^2}{\beta_x} \cdot \Psi_{IBS}(\theta_p, \theta_x, \theta_y) \\
\beta_y \cdot \Psi_{IBS}(\theta_y, \theta_p, \theta_x) \\
2 \cdot \gamma^2 \cdot \Psi_{IBS}(\theta_p, \theta_x, \theta_y)
\end{pmatrix}
\]

R. Carrigan, V. Lebedev, N. Mokhov, S. Nagaitsev, V. Shiltsev, G. Stancari, D. Still, and A. Valishev, chapt. 6. Accelerator Physics at the Tevatron Collider
Smooth focusing, unbunched beam at quasi-equilibrium state of the coasting beam:

\[ \theta_x = \theta_y = \theta_p \]

\[ \Psi_{\text{IBS}}(\theta_x, \theta_y, \theta_p) = \Psi_{\text{IBS}}(\theta_y, \theta_p, \theta_x) = \Psi_{\text{IBS}}(\theta_p, \theta_x, \theta_y) = 0 \]

This equivalent to:

\[ \frac{\varepsilon_x}{\beta_x} = \frac{\varepsilon_y}{\beta_y} = \sigma_p \cdot \frac{\beta_x \cdot \varepsilon_x}{\gamma^2 \cdot (\varepsilon_x \cdot \beta_x + \sigma_p \cdot D^2)} \]

\[ \frac{\sigma_p^2}{\gamma^2} = \frac{\varepsilon_x}{\beta_x} \cdot \frac{1}{1 - \frac{\gamma^2}{\gamma_{tr}^2}} \]

can be fulfilled only below critical energy

\[ 1 - \frac{\gamma^2}{\gamma_{tr}^2} > 0 \]

For fixed transverse emittance the equilibrium momentum spread grows to infinity when the beam energy approaches transition.

Equilibrium does not exist above transition in smooth approximation.

For FODO equilibrium does not exist. 6D emittance grows before and after, and there is no jump for emittance growth at transition.
The NICA accelerator facility will consist of:
- cryogenic heavy ion source KRION of ESIS type,
- heavy ion linear accelerator (HILac)
- a superconducting Booster synchrotron
- the superconducting heavy ion synchrotron Nuclotron
- collider: two new superconducting storage rings with two interaction points

Collider basic parameters:
\[ \sqrt{s_{NN}} = 4 - 11 \text{ GeV}; \text{ beams: from } p \text{ to } Au; \, L \sim 10^{27} \text{ cm}^{-2} \text{ c}^{-1} (Au), \]
IBS beam emittance growth rate minimization

\[
\begin{align*}
\frac{1}{\tau_x} &= \frac{1}{\varepsilon_x} \frac{d\varepsilon_x}{dt} \\
\frac{1}{\tau_y} &= \frac{1}{\varepsilon_y} \frac{d\varepsilon_y}{dt} \\
\frac{1}{\tau_p} &= \frac{1}{6p^2} \frac{d(6p^2)}{dt}
\end{align*}
\]

Lattices with FODO- and triplet- focusing were tested
Results of IBS Tests

Ideal storage ring - no IPs

- perimeter of the ring \(\rightarrow\) keep constant
- change focusing strength \(\rightarrow\) adjust total ring tune (number periods per ring is varied)
- fix \(\delta v_{SC} = 0.05\) - limited \(\rightarrow\) number of ions in the beam
- adjust \(\varepsilon_x, \varepsilon_y\) and \(\sigma_p\) to keep equal all 3 growth times

Triplet focusing preferable. It results in doubling IBS growth time

NICA: Conceptual Proposal for Collider, Valeri Lebedev, Fermilab, January 11, 2010
Results of IBS Tests

**Ideal storage ring - no IPs**

- Minimum heating at $\gamma \approx \gamma_{tr}$

☐ below transition (large $\gamma_{tr}$)
  - large $\Delta \nu$ per cell $\Rightarrow$ strong heating

☐ above transition (small $\gamma_{tr}$)
  - heating due to $\Delta T$ between $H$ & $L$ planes

Resulting $\varepsilon_x \varepsilon_y$ has weak dependence on other parameters

$\varepsilon_x = \varepsilon_y \pm 30\%$ at thermal equilibrium point

NICA: Conceptual Proposal for Collider, Valeri Lebedev, Fermilab, January 11, 2010
Results of IBS Tests

Add IPs → Ideal storage ring

- Straight lines and IPs increase IBS heating by about 4.5 times
- Operation in vicinity of thermal equilibrium still significantly reduces IBS heating

<table>
<thead>
<tr>
<th>Energy</th>
<th>$\varepsilon_x$ [mm mrad]</th>
<th>$\varepsilon_y$ [mm mrad]</th>
<th>$\sigma_p$ %</th>
<th>$\tau_x = \tau_y = \tau_s$ [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 GeV/n</td>
<td>1.117</td>
<td>0.692</td>
<td>0.156</td>
<td>1025</td>
</tr>
<tr>
<td>4.5 GeV/n</td>
<td>1.291</td>
<td>0.684</td>
<td>0.192</td>
<td>1350</td>
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</table>

ODFDO- and FODO- give not more than 30% difference in the IBS growth times in “real” rings

- FODO- was chosen for NICA
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Circumference, m</td>
<td>503.04</td>
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<tr>
<td>Number of bunches</td>
<td>22</td>
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<tr>
<td>Rms bunch length, m</td>
<td>0.6</td>
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<tr>
<td>Beta-function at IP, m</td>
<td>0.35</td>
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<tr>
<td>Betatron tunes, Qx / Qy</td>
<td>9.44 / 9.44</td>
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<tr>
<td>Chromaticity, Q'x / Q'y</td>
<td>-33 / -28</td>
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<tr>
<td>Ring acceptance, π·mm·mrad</td>
<td>40</td>
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<tr>
<td>Long. acceptance, Δp/p</td>
<td>±0.010</td>
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<td>Gamma-transition, $\delta_{tr}$</td>
<td>7.088</td>
</tr>
<tr>
<td>Ion energy, GeV/u</td>
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<tr>
<td>Ion number per bunch</td>
<td>2.0·10^8</td>
</tr>
<tr>
<td></td>
<td>2.4·10^9</td>
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<tr>
<td></td>
<td>2.3·10^9</td>
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<tr>
<td>Rms $\Delta p/p$, 10^{-3}</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>1.15</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Rms beam emittance, hor/vert, (unnormalized), π·mm·mrad</td>
<td>1.1/</td>
</tr>
<tr>
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<td>1.1/</td>
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<tr>
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<td>1.1/</td>
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<tr>
<td></td>
<td>0.95</td>
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<td></td>
<td>0.85</td>
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<td>0.75</td>
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<td>Luminosity, cm^{-2}s^{-1}</td>
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<td>1.0·10^{27}</td>
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<td>1.0·10^{27}</td>
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<td>IBS growth time, sec</td>
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<tr>
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<td>1800</td>
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