Momentum Slip-Stacking Simulations for SPS Ion Beams with Collective Effects

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Acknowledgements:
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HB2018, 19/06/2018
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Motivation

• The HL-LHC requirement for ion beams is to double the peak luminosity after LIU Upgrade [1] increasing the number of bunches in the LHC

• The LIU baseline to achieve that is to decrease bunch spacing from 100 ns to 50 ns in SPS through slip-stacking (already used at Fermilab to double beam intensity [2])
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In longitudinal phase space two batches are interleaved using two independent RF systems.
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• The HL-LHC requirement for ion beams is to double the peak luminosity after LIU Upgrade [1] increasing the number of bunches in the LHC

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In longitudinal phase space two batches are interleaved using two independent RF systems.

• In SPS slip-stacking can be tested only after Upgrade
  ➢ Low Level RF of the 200 MHz RF system has to be upgraded

• Therefore simulations are the only means to check slip-stacking feasibility (see also [3])

• For the first time simulations (with the CERN BLonD code [4]) have been done using:
  ➢ Full SPS longitudinal impedance model
  ➢ Measured beam parameters
  ➢ Optimization with respect to the most significant parameters involved
Slip-stacking principle

Fundamental equations for slip-stacking dynamics (at constant magnetic field)

\[
\frac{\Delta f_{rf}}{f_{rf,0}} = -\eta_0 \frac{\Delta p}{p_0}
\]

\[
\frac{\Delta p}{p_0} = \gamma^2_{tr} \frac{\Delta R}{R_0}
\]

\[
\Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}}
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Examples above transition ($\eta_0 > 0$)
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Examples above transition \((\eta_0 > 0)\)

\( \Delta f_{rf} < 0 \)
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Examples above transition \((\eta_0 > 0)\)

\[ \Delta f_{rf} < 0 \quad \Rightarrow \quad \Delta p > 0 \]
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$$\Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}}$$

Examples above transition ($\eta_0 > 0$)

$\Delta f_{rf} < 0 \quad \Delta p > 0$
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\Delta \phi_{rf} &= \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}}
\end{align*}
\]

Examples above transition \((\eta_0 > 0)\)

\[\Delta f_{rf} < 0 \quad \rightarrow \quad \Delta p > 0 \quad \rightarrow \quad \Delta R > 0\]
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Examples above transition \((\eta_0 > 0)\)

\(\Delta f_{rf} < 0 \rightarrow \Delta p > 0 \rightarrow \Delta R > 0\)
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\Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}}
\]

Examples above transition ($\eta_0>0$)

$\Delta f_{rf} < 0 \quad \rightarrow \quad \Delta p > 0 \quad \rightarrow \quad \Delta R > 0 \quad \rightarrow \quad \Delta \varphi_{rf} < 0$
Fundamental equations for slip-stacking dynamics (at constant magnetic field)

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\frac{\Delta f_{rf}}{f_{rf,0}} = -\eta_0 \frac{\Delta p}{p_0}
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\[
\frac{\Delta p}{p_0} = \gamma^2_{tr} \frac{\Delta R}{R_0}
\]

\[
\Delta \varphi_{rf} = \frac{2 \pi \hbar \Delta f_{rf}}{f_{rf,0}}
\]

Examples above transition ($\eta_0 > 0$)

$\Delta f_{rf} < 0 \quad \Delta p > 0 \quad \Delta R > 0 \quad \Delta \varphi_{rf} < 0$

Moving one bunch
Slip-stacking principle

Fundamental equations for slip-stacking dynamics (at constant magnetic field)

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\]

Examples above transition \((\eta_0 > 0)\)

\(\Delta f_{rf} < 0 \quad \Delta p > 0 \quad \Delta R > 0 \quad \Delta \varphi_{rf} < 0\)

Moving one bunch

---

\(\Delta E [\text{GeV}]\)

# buckets
Slip-stacking principle

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\[ \Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}} \]

Examples above transition \((\eta_0 > 0)\)

\[ \Delta f_{rf} < 0 \rightarrow \Delta p > 0 \rightarrow \Delta R > 0 \rightarrow \Delta \varphi_{rf} < 0 \]

Moving one bunch

zoom
Slip-stacking principle

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\[ \frac{\Delta f_{rf}}{f_{rf,0}} = -\eta_0 \frac{\Delta p}{p_0} \]

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\[ \Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}} \]

Examples above transition (\( \eta_0 > 0 \))

\( \Delta f_{rf} < 0 \) \quad \rightarrow \quad \Delta p > 0 \quad \rightarrow \quad \Delta R > 0 \quad \rightarrow \quad \Delta \varphi_{rf} < 0 \)

Moving one bunch

[Graph showing energy distribution over buckets]

Zoom
Slip-stacking principle

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Examples above transition ($\eta_0 > 0$)

\[ \Delta f_{rf} < 0 \quad \rightarrow \quad \Delta p > 0 \quad \rightarrow \quad \Delta R > 0 \quad \rightarrow \quad \Delta \varphi_{rf} < 0 \]

Moving one bunch

\[ \Delta E \text{ [GeV]} \]

# buckets

\[ -60 \quad 0 \quad 200 \quad 400 \quad 600 \quad 800 \quad 1000 \quad 1200 \quad 1400 \]

\[ 60 \quad 40 \quad 20 \quad 0 \quad -20 \quad -40 \quad -60 \]

zoom
Slip-stacking principle

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\]

Examples above transition \((\eta_0 > 0)\)

\(\Delta f_{rf} < 0 \rightarrow \Delta p > 0 \rightarrow \Delta R > 0 \rightarrow \Delta \varphi_{rf} < 0\)

Moving one bunch

\[
\Delta E [\text{GeV}] = f(x)
\]

\[
\text{zoom}
\]
Slip-stacking principle

Fundamental equations for slip-stacking dynamics (at constant magnetic field)

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\frac{\Delta p}{p_0} = \gamma_{tr}^2 \frac{\Delta R}{R_0}
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\Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}}
\]

Examples above transition \((\eta_0 > 0)\)

\[
\Delta f_{rf} < 0 \quad \Delta p > 0 \quad \Delta R > 0 \quad \Delta \varphi_{rf} < 0
\]

\[
\Delta f_{rf} > 0 \quad \Delta p < 0
\]

Moving one bunch

\[
\Delta E \text{[GeV]}
\]

\[
\# \text{buckets}
\]

zoom

\[
\Delta E \text{[GeV]}
\]

\[
\# \text{buckets}
\]
Slip-stacking principle

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\]

\[
\Delta \phi_{rf} = \frac{2 \pi \hbar \Delta f_{rf}}{f_{rf,0}}
\]

Examples above transition ($\eta_0 > 0$)

\[
\Delta f_{rf} < 0 \quad \Rightarrow \quad \Delta p > 0 \quad \Rightarrow \quad \Delta R > 0 \quad \Rightarrow \quad \Delta \phi_{rf} < 0
\]

\[
\Delta f_{rf} > 0 \quad \Rightarrow \quad \Delta p < 0
\]

Moving one bunch

\[
\Delta E \text{ (GeV)}
\]

\[
\text{# buckets}
\]

Zoom
Slip-stacking principle

Fundamental equations for slip-stacking dynamics (at constant magnetic field)

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\frac{\Delta f_{rf}}{f_{rf,0}} = -\eta_0 \frac{\Delta p}{p_0}
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\frac{\Delta p}{p_0} = \gamma^2 \frac{\Delta R}{R_0}
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\Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}}
\]

Examples above transition ($\eta_0 > 0$)

- $\Delta f_{rf} < 0 \rightarrow \Delta p > 0 \rightarrow \Delta R > 0 \rightarrow \Delta \varphi_{rf} < 0$
- $\Delta f_{rf} > 0 \rightarrow \Delta p < 0 \rightarrow \Delta R < 0$

Moving one bunch

![Graph showing the change in energy with the number of buckets.](image)

**zoom**

![Zoomed-in view of the graph.](image)
Slip-stacking principle

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Examples above transition ($\eta_0 > 0$)

- $\Delta f_{rf} < 0 \Rightarrow \Delta p > 0 \Rightarrow \Delta R > 0 \Rightarrow \Delta \varphi_{rf} < 0$
- $\Delta f_{rf} > 0 \Rightarrow \Delta p < 0 \Rightarrow \Delta R < 0 \Rightarrow \Delta \varphi_{rf} < 0$

Moving one bunch

![Graph showing energy vs. number of buckets with zoomed-in view](image-url)
Slip-stacking principle

Fundamental equations for slip-stacking dynamics (at constant magnetic field)

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Examples above transition ($\eta_0 > 0$)

\[
\Delta f_{rf} < 0 \Rightarrow \Delta p > 0 \Rightarrow \Delta R > 0 \Rightarrow \Delta \varphi_{rf} < 0 \\
\Delta f_{rf} > 0 \Rightarrow \Delta p < 0 \Rightarrow \Delta R < 0 \Rightarrow \Delta \varphi_{rf} > 0
\]

Moving one bunch

![Graph showing energy distribution over buckets](image)

zoom
Slip-stacking principle

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Examples above transition ($\eta_0 > 0$)

- $\Delta f_{rf} < 0$  $\Rightarrow$  $\Delta p > 0$  $\Rightarrow$  $\Delta R > 0$  $\Rightarrow$  $\Delta \varphi_{rf} < 0$
- $\Delta f_{rf} > 0$  $\Rightarrow$  $\Delta p < 0$  $\Rightarrow$  $\Delta R < 0$  $\Rightarrow$  $\Delta \varphi_{rf} > 0$

Moving one bunch  Bringing closer two bunches

[Graphs and diagrams showing energy distribution and bucket counts]
Slip-stacking principle

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\begin{align*}
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\end{align*}
\]

Examples above transition ($\eta_0 > 0$)

- Moving one bunch
  - $\Delta f_{rf} < 0$ → $\Delta p < 0$ → $\Delta R < 0$ → $\Delta \varphi_{rf} > 0$
- Bringing closer two bunches
  - $\Delta f_{rf} > 0$ → $\Delta p > 0$ → $\Delta R > 0$ → $\Delta \varphi_{rf} < 0$
Slip-stacking principle

Fundamental equations for slip-stacking dynamics (at constant magnetic field)

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\]

Examples above transition \( (\eta_0 > 0) \)

\( \Delta f_{rf} < 0 \) \quad \Delta p > 0 \quad \Delta R > 0 \quad \Delta \varphi_{rf} < 0
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\( \Delta f_{rf} > 0 \) \quad \Delta p < 0 \quad \Delta R < 0 \quad \Delta \varphi_{rf} > 0

Moving one bunch

Bringing closer two bunches
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Examples above transition \((\eta_0 > 0)\)

- \(\Delta f_{rf} < 0\)  \(\Rightarrow\)  \(\Delta p > 0\)  \(\Rightarrow\)  \(\Delta R > 0\)  \(\Rightarrow\)  \(\Delta \varphi_{rf} < 0\)
- \(\Delta f_{rf} > 0\)  \(\Rightarrow\)  \(\Delta p < 0\)  \(\Rightarrow\)  \(\Delta R < 0\)  \(\Rightarrow\)  \(\Delta \varphi_{rf} > 0\)

Moving one bunch  Bringing closer two bunches

![Zoom](image1.png)  ![Zoom](image2.png)
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Examples above transition (\(\eta_0 > 0\))

- \(\Delta f_{rf} < 0\) \(\Rightarrow\) \(\Delta p > 0\) \(\Rightarrow\) \(\Delta R > 0\) \(\Rightarrow\) \(\Delta \varphi_{rf} < 0\)
- \(\Delta f_{rf} > 0\) \(\Rightarrow\) \(\Delta p < 0\) \(\Rightarrow\) \(\Delta R < 0\) \(\Rightarrow\) \(\Delta \varphi_{rf} > 0\)

Moving one bunch

Bringing closer two bunches

Interleaving two batches
Slip-stacking principle

Fundamental equations for slip-stacking dynamics (at constant magnetic field)

$$\Delta f_{rf} = -\eta_0 \frac{\Delta p}{p_0}$$
$$\Delta p = \gamma_{tr}^2 \frac{\Delta R}{R_0}$$
$$\Delta \varphi_{rf} = \frac{2 \pi h \Delta f_{rf}}{f_{rf,0}}$$

Examples above transition ($\eta_0 > 0$)

- $\Delta f_{rf} < 0 \quad \Delta p > 0 \quad \Delta R > 0 \quad \Delta \varphi_{rf} < 0$
- $\Delta f_{rf} > 0 \quad \Delta p < 0 \quad \Delta R < 0 \quad \Delta \varphi_{rf} > 0$

Moving one bunch

Bringing closer two bunches

Interleaving two batches

Zoom
Slip-stacking principle

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Examples above transition (\(\eta_0 > 0\))

\(\Delta f_{rf} < 0\) \(\Rightarrow\) \(\Delta p > 0\) \(\Rightarrow\) \(\Delta R > 0\) \(\Rightarrow\) \(\Delta \varphi_{rf} < 0\)
\(\Delta f_{rf} > 0\) \(\Rightarrow\) \(\Delta p < 0\) \(\Rightarrow\) \(\Delta R < 0\) \(\Rightarrow\) \(\Delta \varphi_{rf} > 0\)

Moving one bunch

Bringing closer two bunches

Interleaving two batches
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Examples above transition ($\eta_0 > 0$)

- $\Delta f_{rf} < 0$ $\rightarrow$ $\Delta p > 0$ $\rightarrow$ $\Delta R > 0$ $\rightarrow$ $\Delta \varphi_{rf} < 0$
- $\Delta f_{rf} > 0$ $\rightarrow$ $\Delta p < 0$ $\rightarrow$ $\Delta R < 0$ $\rightarrow$ $\Delta \varphi_{rf} > 0$

Moving one bunch

Bringing closer two bunches

Interleaving two batches

Images showing changes in $\Delta E$ (GeV) with respect to the number of buckets, with zoom-in sections highlighting specific regions.
RF perturbation

- Each RF system, while accelerates one batch, perturbs the other batch.

- The perturbation is zero when the batch distance $T_b$ is greater than the time $T_{th}^b$ needed to switch alternatively the two RF systems on and off depending on which batch passes by.

- The perturbation has less damaging effects when $\Delta f_{rf}^b$ is large relative to $f_{s0}$
  - the perturbation averages within a synchrotron oscillation period

- Perturbation severity

$$\alpha = \frac{\Delta f_{rf}^b}{f_{s0}} = 2 \frac{\Delta E^b}{H_b}$$

Difference in RF frequency and absolute energy between the two batches

Core synchrotron frequency and bucket half height

- It has been proven [5] that a lower limit for stable motion is defined by $\alpha = 4$
RF perturbation

• Each RF system, while accelerates one batch, perturbs the other batch.

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$$\alpha = \frac{\Delta f_{rf}^b}{f_{s0}} = 2 \frac{\Delta E^b}{H_b}$$

  Difference in RF frequency and absolute energy between the two batches
  Core synchrotron frequency and bucket half height

• It has been proven [5] that a lower limit for stable motion is defined by $\alpha = 4$

$\alpha = 8$ -> small perturbation effect but large needed recapture voltage and large emittance after filamentation
RF perturbation

- Each RF system, while accelerates one batch, perturbs the other batch.

- The perturbation is zero when the batch distance $T_b$ is greater than the time $T^{th}_b$ needed to switch alternatively the two RF systems on and off depending on which batch passes by.

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  - the perturbation averages within a synchrotron oscillation period

- Perturbation severity

\[
\alpha = \frac{\Delta f_{rf}^b}{f_{s0}} = 2 \frac{\Delta E_b^b}{H_b}
\]

- Difference in RF frequency and absolute energy between the two batches
- Core synchrotron frequency and bucket half height

- It has been proven [5] that a lower limit for stable motion is defined by $\alpha = 4$

$\alpha = 8$ -> small perturbation effect but large needed recapture voltage and large emittance after filamentation

$\alpha = 4$ -> smaller recapture voltage needed (and small emittance growth after recapture) but the perturbation has more effect
RF perturbation

- Each RF system, while accelerates one batch, perturbs the other batch.

- The perturbation is zero when the batch distance $T_b$ is greater than the time $T_{b}^{th}$ needed to switch alternatively the two RF systems on and off depending on which batch passes by.

- The perturbation has less damaging effects when $\Delta f_{rf}^b$ is large relative to $f_{s0}$
  - the perturbation averages within a synchrotron oscillation period

- Perturbation severity
  \[
  \alpha = \frac{\Delta f_{rf}^b}{f_{s0}} = 2 \frac{\Delta E^b}{H_b}
  \]
  - Difference in RF frequency and absolute energy between the two batches
  - Core synchrotron frequency and bucket half height

- It has been proven [5] that a lower limit for stable motion is defined by $\alpha = 4$

  $\alpha = 8 \rightarrow$ small perturbation effect but large needed recapture voltage and large emittance after filamentation

  $\alpha = 4 \rightarrow$ smaller recapture voltage needed (and small emittance growth after recapture) but the perturbation has more effect

  $\alpha = 0 \rightarrow$ bunches are lost, large quantity of satellite particles after recapture due to chaotic motion in $\Delta E = 0$
RF perturbation

- Each RF system, while accelerates one batch, perturbs the other batch.

- The perturbation is zero when the batch distance $T_b$ is greater than the time $T_b^{th}$ needed to switch alternatively the two RF systems on and off depending on which batch passes by.

- The perturbation has less damaging effects when $\Delta f_{rf}^b$ is large relative to $f_{s0}$
  - the perturbation averages within a synchrotron oscillation period

- Perturbation severity
  \[ \alpha = \frac{\Delta f_{rf}^b}{f_{s0}} = 2 \frac{\Delta E^b}{H_b} \]
  - Difference in RF frequency and absolute energy between the two batches
  - Core synchrotron frequency and bucket half height

- It has been proven [5] that a lower limit for stable motion is defined by $\alpha = 4$

  \( \alpha = 8 \) -> small perturbation effect but large needed recapture voltage and large emittance after filamentation

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  - the perturbation averages within a synchrotron oscillation period

- Perturbation severity

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- It has been proven [5] that a lower limit for stable motion is defined by $\alpha = 4$

  $\alpha = 8$ -> small perturbation effect but large needed recapture voltage and large emittance after filamentation
  $\alpha = 4$ -> smaller recapture voltage needed (and small emittance growth after recapture) but the perturbation has more effect
  $\alpha = 0$ -> bunches are lost, large quantity of satellite particles after recapture due to chaotic motion in $\Delta E=0$
RF perturbation

• Each RF system, while accelerates one batch, perturbs the other batch.

• The perturbation is zero when the batch distance $T_b$ is greater than the time $T_b^{th}$ needed to switch alternatively the two RF systems on and off depending on which batch passes by.

• The perturbation has less damaging effects when $\Delta f_{rf}^b$ is large relative to $f_{s0}$
  ➢ the perturbation averages within a synchrotron oscillation period

• Perturbation severity
  
  $$\alpha \doteq \frac{\Delta f_{rf}^b}{f_{s0}} = 2 \frac{\Delta E^b}{H_b}$$

  
  Difference in RF frequency and absolute energy between the two batches
  Core synchrotron frequency and bucket half height

• It has been proven [5] that a lower limit for stable motion is defined by $\alpha = 4$

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
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Slip-stacking energy in SPS

- At which energy should slip-stacking be performed?
  - Flat bottom: strong space charge effects, IBS and RF noise (observed during operation)
  - Flat top: uncaptured beam is transferred to the LHC and lowest instability threshold
  - Intermediate energy plateau: good compromise (the energy 300 GeV/qc is chosen)

- Only multiples of 1.2 s (PSB cycle length) can be added to the SPS cycle for slip-stacking
  - 1.2 s are added at 300 GeV/c (0.8 s for slip-stacking and 0.4 s for filamentation after recapture)

Operational momentum program and derivative

Momentum program with slip-stacking and derivative

- Simulations started at TX₁ (beginning of slip-stacking)
• In measurements [6] significant bunch by bunch variation in intensity and emittance at 300 GeV/qc due to losses at SPS flat bottom.
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- Emittance increases by a factor of 2
Initial parameters

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<table>
<thead>
<tr>
<th>Bunch number</th>
<th>Intensity [arbitrary units]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
</tr>
<tr>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>25</td>
<td>22</td>
</tr>
</tbody>
</table>

- Emittance increases by a factor of 2

---

<table>
<thead>
<tr>
<th>Bunch number</th>
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</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.04</td>
</tr>
<tr>
<td>5</td>
<td>0.06</td>
</tr>
<tr>
<td>10</td>
<td>0.08</td>
</tr>
<tr>
<td>15</td>
<td>0.10</td>
</tr>
<tr>
<td>20</td>
<td>0.12</td>
</tr>
<tr>
<td>25</td>
<td>0.14</td>
</tr>
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  - Measured strong tails reproduced by binomial distributions with $\mu = 5$

![Graph showing intensity increases by a factor of 1.5 and emittance increases by a factor of 2](image)
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**EXAMPLE**

- ‘Low’ RF voltage needed (1.4 MV) relative to current operational one (7 MV)
- Induced voltage cumulates along each batch of 24 bunches
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• The two batches are separated by large $T_b = 2.7 \mu s$ ($T_b^{th} = 1 \mu s$)
  ➢ In this way $\alpha \gg 4$ when $T_b=T_b^{th}$ and adiabaticity is preserved

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Measurements of cavity voltage rise during batch passage (courtesy T. Bohl)
Design of RF programs

- The RF voltage program during slip-stacking is calculated from the momentum program for constant filling factor of bucket in energy $q_e^{mss}$.
- The batches move in opposite directions relative to $\Delta E=0$ ($f_{r1} + f_{r2} = 2f_{rf,0}$, $V_{rf1} = V_{rf2}$).

**EXAMPLE** with $q_e^{mss} = 0.65$ and $\alpha_{TX2} = 4.5$.

- The capture voltage $V_{rf}^{rc}$ is used in $[TX_2, TX_3]$, then the filling factor in energy at $TX_3$ of the highest emittance bunch is computed and used to calculate the voltage program in $[TX_3, TX_4]$.
- At flat top two options are considered:
  - $V_{rf}$ is increased adiabatically to 15 MV (*bunch compression*).
  - $V_{rf}$ is increased in few turns to 15 MV (*bunch rotation*).
Constraints and optimizations

• Two constraints to take into account
  - $L_{tot} < 5\%$ (total losses due to momentum slip-stacking, LIU constraint)
  - $\tau_{max} < 1.65$ ns (maximum bunch length at SPS extraction, 400 MHz LHC buckets cannot accept more)

• We divide the total losses in $L_{tot} = L_{SPS} + S_{LHC}$

With all the assumptions given before, simulations are completely defined fixing $\alpha_{TX2}$, $q_{e}^{mss}$, $V_{rf}^{rc}$ and the RF manipulation at flat top (compression or rotation)
  - It is important to find the combinations which give the lowest $L_{tot}$ and $\tau_{max}$ (optimal solutions)
  - The dynamics are too complex to rely only on qualitative studies
  - Parameter scan: $\alpha_{TX2}$ (3.5->8), $q_{e}^{mss}$ (0.45->9), $V_{rf}^{rc}$ (1 MV->9 MV)

• The Q20 optics ($\gamma_{tr} = 18$) is currently used in operation (detailed analysis)
  - The other two optics (Q22, $\gamma_{tr} = 20$), (Q26, $\gamma_{tr} = 23$) are briefly mentioned
Q20 optics: bunch compression

Scan results

Optimal solutions

- Constraints are practically not satisfied
- Strong limitation on $\tau_{\text{max}}$ rather than on $L_{\text{tot}}$
- No significant difference if intensity effects are off
- Intensity effects don't cause additional losses or blow-up
Q20 optics: bunch rotation

Scan results

Optimal solutions

- Bunch rotation allows to significantly reduce $\tau_{\text{max}}$ without increasing $L_{\text{tot}}$
- Constraints satisfied by half of the combinations
- However ‘S’ shape at extraction could complicate transmission into LHC

Again no significant difference if intensity effects are off
Q20 optics: proposed solution

- Only bunch rotation can fulfil the requirements.

- Giving priority to loss reduction and keeping some safety margin for the bunch length we propose the following combination (see also the Appendix for more detailed analysis):

  - $L_{tot} = 0.43\%$
  - $\tau_{max} = 1.55\text{ ns}$
  - $S_{LHC} = 0.13\%$
  - $L_{SPS} = 0.30\%$
  - $\alpha_{TX2} = 4.5$
  - $q_{e^{mss}} = 0.65$
  - $V_{rf}^{rc} = 8\text{ MV}$

  Low $q_{e^{mss}}$ helps limiting the losses in SPS during slip-stacking.

  Low $q_{e^{mss}}$ and $\alpha_{TX2} > 4$ helps limiting influence of chaotic motion in $\Delta E = 0$ making $S_{LHC}$ low.

The solution largely fulfills LIU requirements

The solution is feasible

- $\Delta R_{mss}^{max} = 8\text{ mm} < 20\text{ mm}$ (one-side aperture limitation)
- $\Delta f_{rf,max}^{mss} = 1\text{ kHz} << 1\text{ MHz}$ (200 MHz TWC bandwidth)
- $V_{rf}^{max} = 14.6\text{ MV} < 15\text{ MV}$ (available peak voltage)
Proposed solution: slip-stacking

- As expected only few particle lost during slip-stacking ($q_e^{mss} = 0.65$)
- Low impact of chaotic motion in $\Delta E = 0$ ($\alpha_{TX2} = 4.5$)
Proposed solution: slip-stacking

As expected only few particle lost during slip-stacking \( q_{emss} = 0.65 \)

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**General frame** bunches are yellow at $TX_3$

- Few losses due to recapture and acceleration of higher emittance bunches
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- Few losses due to recapture and acceleration of higher emittance bunches
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Middle bunch
Proposed solution: capture and ramp

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![Graph showing phase space distribution for first and middle bunches]
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**First bunch**

$\epsilon_{l,1} = 0.16, \epsilon_{l,2} = 0.12, A_b = 0.83 \text{ eVs/A}$

**Middle bunch**
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First bunch
- $\epsilon_{l,1} = 0.16, \epsilon_{l,2} = 0.12, A_b = 0.83 \text{ eVs/A}$
- $\tau_1 = 1.61 \text{ ns}, \tau_2 = 1.39 \text{ ns}, N_{bu} = 1.384e+10 \text{ p}$

Middle bunch
- $\epsilon_{l,1} = 0.29, \epsilon_{l,2} = 0.25, A_b = 0.83 \text{ eVs/A}$
- $\tau_1 = 2.24 \text{ ns}, \tau_2 = 2.04 \text{ ns}, N_{bu} = 1.942e+10 \text{ p}$

Last bunch
- $\epsilon_{l,1} = 0.32, \epsilon_{l,2} = 0.29, A_b = 0.83 \text{ eVs/A}$
- $\tau_1 = 2.38 \text{ ns}, \tau_2 = 2.25 \text{ ns}, N_{bu} = 2.063e+10 \text{ p}$
Proposed solution: flat top

General frame

- The dense area remains up to bunch rotation (shortest bunch)
- Structure also for middle and last bunches but less strong

First bunch:
\[ \varepsilon_{i,1} = 0.11, \varepsilon_{i,2} = 0.28, A_b = 0.83 \text{ eVs/A} \]
\[ \tau_1 = 0.71 \text{ ns}, \tau_2 = 2.18 \text{ ns}, N_{bu} = 1.384 \times 10^9 \]

Middle bunch:
\[ \varepsilon_{i,1} = 0.58, \varepsilon_{i,2} = 0.37, A_b = 0.83 \text{ eVs/A} \]
\[ \tau_1 = 3.44 \text{ ns}, \tau_2 = 2.58 \text{ ns}, N_{bu} = 1.935 \times 10^9 \]

Last bunch:
\[ \varepsilon_{i,1} = 0.61, \varepsilon_{i,2} = 0.42, A_b = 0.83 \text{ eVs/A} \]
\[ \tau_1 = 3.54 \text{ ns}, \tau_2 = 2.79 \text{ ns}, N_{bu} = 2.051 \times 10^9 \]
Loss of Landau damping

Dipole oscillations with intensity effects

- Large dipole oscillations never damp
- Shortest bunch: amplitude 10% relative to bucket length, LLD at $TX_2$
- Longest bunch: amplitude 2%, LLD not at $TX_2$ but at $TX_3$

Dipole oscillations without intensity effects

- Very small dipole oscillations for all bunches
- Amplitude only 0.2% relative to bucket length
**LLD analytical estimations**

**LLD threshold formula**

\[
N_b < \frac{|\eta|E}{q^2 \beta^2} \frac{\tau}{\text{Im}Z/n} \left( \frac{\Delta E}{E} \right)^2 \frac{\Delta \omega_s}{\omega_s}
\]

\[\text{Im}Z/n = 3\Omega \ [6]\]

- \(V_{rf} = 1.5\ MV\) (during slip-stacking)
- \(V_{rf} = 8\ MV\) (at recapture)

- LLD for shortest bunches even using ‘low’ RF voltage during slip-stacking
- Analytical estimations confirm what seen in simulations
  - LLD for **shortest bunch** at recapture
  - Not LLD for **longest bunch** at recapture
Q22 and Q26: bunch compression

- Lower $\eta_0 \rightarrow$ lower $q_e^{mss} \propto \eta_0^{1/4} \rightarrow$ significant lower $L_{SPS}$ and $S_{LHC}$

- On average lower $L_{SPS}$ and $\tau_{max}$ than Q20 optics

- Constraints even better satisfied

Scan results

Optimal solutions

Q22
- On average lower $L_{tot}$ and $\tau_{max}$ than Q20 optics
- Acceptable margin for constraint fulfillment

Q26
- On average lower $L_{tot}$ and $\tau_{max}$ than Q22 optics
- Constraints even better satisfied

• However Q22 and Q26 are more sensitive to IBS and transverse space charge
Conclusions
Conclusions

- Momentum slip-stacking for LHC ion beams in SPS after LIU upgrade is fundamental to fulfil the HL-LHC requirements.

- The optimum parameters involved in this complicated beam manipulation have been suggested through simulations.

- An accurate impedance model and realistic beam parameters have been used.

- Simulations show that momentum slip-stacking can be applied under certain conditions, providing at extraction the beam parameters required by the LIU project.

- Intensity effects do not increase losses or bunch length at SPS extraction.

- However loss of Landau damping was observed
  - Analytical estimations confirm what found in simulations
  - Further studies are needed to find possible cures
References


LHC Injectors Upgrade

THANK YOU FOR YOUR ATTENTION!
APPENDIX 1. Parameter behaviours for the optimal solutions (Q20 optics, rotation)

- All optimal solutions have roughly the same $\alpha_{TX2}$ as expected
  - Different configurations with same $\alpha_{TX2}$ are similar from a slip-stacking perspective
- $\alpha_{TX2} \approx 4$, because 4 is the lower limit for stability and the solutions are optimal
  - $\alpha_{TX2} = 4.5$ is preferable (margin due to highest emittance bunches filling the bucket)
- Qualitatively, a lower $\tau_{\text{max}}$ implies
  - Lower $\varepsilon_{\text{max}}$ (lower emittance for lower bunch length at extraction)
  - Lower $V_{rf}^{rc}$ (bucket area after recapture has to decrease)
  - Lower $\Delta E_b$ (otherwise particles outside the recapture bucket will be lost)
  - Higher $S_{LHC}$ (stronger perturbations due to lower $\Delta E_b$)
  - Lower $H_b$ and $V_{rf,1}$ ($\alpha_{TX2}$ is roughly constant)
  - Higher $q_e^{mss}$ (it scales as $V_{rf,1}^{-1/4}$)
  - Higher $L_{SPS}$ (due to higher $q_e^{mss}$) and $L_{tot}$
APPENDIX 2. Proposed solution: capture and ramp, no intensity effects

- Without intensity effects the hollow bunches are uniform
- The two peaks of the line density have the same height