Halo formation of the gaussian density beam in periodic solenoidal focusing field

61th ICFA Advanced Beam Dynamics Workshop
on High-intensity and High-brightness Hadron beams (HB 2018)
In Daejeon

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Intense Beam and Accelerator Laboratory (IBAL),
Ulsan National Institute of Science and Technology (UNIST)
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  - Beam physics applications
  - Nonlinear resonances and chaotic motions of envelope oscillation

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  - Halo formations
  - Uniform density charged particle motions
  - Gaussian density charged particle motions of matched beam

- Summary
High-intensity charged-particle beam physics

Applications
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High-intensity charged-particle beam physics

Applications

astrophysical nuclear reactions carrying the nucleosynthetic processes and nuclear properties
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High-intensity charged-particle beam physics

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High-intensity charged-particle beam physics

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- nuclear waste transmutation
**High-intensity charged-particle beam physics**

**Applications**

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- High energy particle physics
- Nuclear waste transmutation

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Accelerator
- (600 MeV - 4 mA proton)

Reactor
- Subcritical or Critical modes
- 65 to 100 MWth

Spallation Source

Multipurpose Flexible Irradiation Facility

Fast Neutron Source

Lead-Bismuth coolant

Nuclear waste transmutation

Available testing volume and dpa
- High >20 dpa/y in 0.5 liters
- Medium >1 dpa/y in 6 liters
- Low <1 dpa/y in 6 liters
High-intensity charged-particle beam physics

**Applications**

- astrophysical nuclear reactions carrying the nucleosynthetic processes and nuclear properties
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**Accelerator**
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**Reactor**
- Subcritical or Critical modes
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**Multipurpose Flexible Irradiation Facility**

- Spallation Source
- Lead-Bismuth coolant

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**RF Power**
- Lithium Target
- Beam footprint 200 x 50 mm²

**Input source**
- High > 20 dpa/yr in 0.5 liters
- Medium > 1 dpa/yr in 6 liters
- Low < 1 dpa/yr in 6 liters
High-intensity charged-particle beam physics

Applications

- astrophysical nuclear reactions carrying the nucleosynthetic processes and nuclear properties
- high energy particle physics
- fusion material test (IFMIF)
- nuclear waste transmutation
- HWR - Solenoidal focusing
High-intensity charged-particle beam physics

Periodic solenoidal focusing field
High-intensity charged-particle beam physics

• Periodic solenoidal focusing field
Periodic solenoidal focusing field

• Periodic solenoidal focusing field

\[ \kappa_z(s) = \kappa_z(s + S) = \left( \frac{B_{Z0}(s)}{2 [B \rho]} \right)^2 = \left( \frac{\omega_c(s)}{2 \gamma_b \beta_b c} \right)^2 \]
High-intensity charged-particle beam physics

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- The dynamics of the charged particle is easily analyzed in the Larmour frame, which rotates with the Larmour frequency around the axis of the solenoid

- Much simpler and cheaper

- Rotationally symmetric

- For a given beam emittance, the solenoid aperture required is smaller than that of the quadrupole
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Normalized envelope equation

- Introduce the dimensionless parameters and variables,

\[ \frac{s}{S} \rightarrow s, \quad \frac{r_b}{\sqrt{\varepsilon S}} \rightarrow r_b, \quad S^2 \kappa_z \rightarrow \kappa_z, \quad \frac{SK}{\varepsilon} \rightarrow K \]

- With symmetric envelope radius, \( r_x(s) = r_y(s) \equiv r_b(s) \)

- The normalized envelope equation

\[ r_b''(s) + \kappa_z(s) r_b(s) - \frac{K}{r_b(s)} - \frac{1}{r_b^3(s)} = 0 \]

- Space charge defocusing: \( K \equiv \frac{2q\lambda}{\gamma_b^2 \beta_b^2 mc^2} : \) Pervance

\[ \sigma_0 \equiv \int_0^1 \sqrt{\kappa_z(s)} \, ds = \int_0^1 \sqrt{\eta \kappa_z(0)} \, ds = \sqrt{\eta \kappa_z(0)} \]

: undepressed (vacuum) phase advance

- \( \sigma \equiv \int_0^1 \frac{ds}{r_b^2(s)} : \) depressed phase advance (normalized)
Nonlinear resonances and chaotic motions of envelope oscillation

Envelope oscillations
(phase plane $r_b - r_b'$)

$$r_b''(s) + \kappa_z(s) r_b(s) - \frac{K}{r_b(s)} - \frac{1}{r_b^3(s)} = 0$$

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All points are plotted in every $S$ lattice period (Poincare surface of section plots) with different envelope initial conditions for propagation over 300 lattice periods.
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High-intensity charged-particle beam physics

Nonlinear resonances and chaotic motions of envelope oscillation

Envelope oscillations
( phase plane $r_b$ - $r_b'$ )

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All points are plotted in **every S lattice period** (Poincare surface of section plots) with different envelope initial conditions for propagation **over 300 lattice periods**

**Matched beam** in solenoidal focusing (equilibrium envelope radius)

$$r_b(s) = r_b(s + S) = \text{const.}$$
Nonlinear resonances and chaotic motions of envelope oscillation

Envelope oscillations
(phase plane $r_b - r_b'$)

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**Matched beam** in solenoidal focusing
(equilibrium envelope radius)

$$r_b(s) = r_b(s + S) = \text{const.}$$

**Mismatched beam** in solenoidal focusing

$$r(s) = r_b(s; \text{matched}) + \delta r$$
High-intensity charged-particle beam physics

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(phase plane $r_b - r'_b$)

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**n-th order resonance**

\[ r(s) = r_b(s; matched) + \delta r, \quad \delta r(s) = \delta r(0) \cos(k_n s) \]

All points are plotted in every S lattice period (Poincare surface of section plots) with different envelope initial conditions for propagation over 300 lattice periods.
High-intensity charged-particle beam physics

Nonlinear resonances and chaotic motions of envelope oscillation

Envelope oscillations
(phase plane $r_b - r_b'$)

$$r_b''(s) + \kappa_z(s)r_b(s) - \frac{K}{r_b(s)} - \frac{1}{r_b^3(s)} = 0$$

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4-th

5-th

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**n-th order resonance**

$$r(s) = r_b(s; matched) + \delta r, \delta r(s) = \delta r(0) \cos(k_n s)$$

$n=5$ ; 5-th order resonance  \hspace{1cm} $k = k_5 = \frac{2\pi l}{5}$

if $s = 5, 10, 15, \ldots$,

the perturbed radius comes back its starting point
High-intensity charged-particle beam physics

Nonlinear resonances and chaotic motions of envelope oscillation

**Envelope oscillations**
(phase plane $r_b - r_b'$)

$$r_b''(s) + \kappa_z(s)r_b(s) - \frac{K}{r_b(s)} - \frac{1}{r_b^3(s)} = 0$$

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High-intensity charged-particle beam physics

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- High-intensity charged-particle beam in a periodic solenoidal focusing field
  - Beam physics applications
  - Nonlinear resonances and chaotic motions of envelope oscillation

- Halo formation of transverse particle-core model
  - Halo formations
  - Uniform density charged particle motions
  - Gaussian density charged particle motions of matched beam

- Summary
Halo formation of transverse particle-core model

Halo formations of particles along the linac
Halo formation of transverse particle-core model

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- Beam emittance growth and particle losses in accelerators
Halo formation of transverse particle-core model

Halo formations of particles along the linac

→ Beam emittance growth and particle losses in accelerators →
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- Uniform charge density
Halo formation of transverse particle-core model

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Gaussian density profile
Halo formation of transverse particle-core model

Halo formations of particles along the linac

Beam emittance growth and particle losses in accelerators → Radioactivation

- External: **periodic solenoidal magnetic focusing field**
- Uniform charge density

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Uniform density charged particle motions
Halo formation of transverse particle-core model

Uniform density charged particle motions

*Equation of motion* (Larmor frame)
Halo formation of transverse particle-core model

**Uniform density charged particle motions**

**Equation of motion** (Larmor frame)

\[ x''(s) + \kappa_z(s)x(s) - KF(x, r_b) = 0 \]

\[ F(x, r_b) = \begin{cases} \frac{x(s)}{r_b^2(s)} & \text{for } x(s) < r_b(s), \\ \frac{1}{x(s)} & \text{for } x(s) > r_b(s) \end{cases} \]
Halo formation of transverse particle-core model

**Uniform density charged particle motions**

**Equation of motion** (Larmor frame)

(Phase plane \(x/r_b - x\))

\[
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Halo formation of transverse particle-core model

**Uniform density charged particle motions**

**Equation of motion** (Larmor frame) (phase plane $x/r_b - x'$)

$$x''(s) + \kappa_z(s)x(s) - KF(x,r_b) = 0$$

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Halo formation of transverse particle-core model

Uniform density charged particle motions

Equation of motion (Larmor frame)

(phase plane $x/r_b \cdot x'$)

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Matched core – test particles

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**Matched core – test particles**

- $K = 0$
  - $\sigma_0 = 45.5^\circ$

- $K = 3$
  - $\sigma_0 = 45.5^\circ$

**Mismatched core – test particles**

- $K = 3$
  - $\sigma_0 = 45.5^\circ$

All points are plotted in every 5 lattice period (Poincare surface of section plots) with different particle initial conditions for propagation over 300 lattice periods.
Halo formation of transverse particle-core model

**Uniform density charged particle motions**

**Equation of motion** (Larmor frame)

(Phase plane $x/r_b - x'$)

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**Matched core – test particles**

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**Mismatched core – test particles**

- $K = 3$
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All points are plotted in every 5 lattice period (Poincare surface of section plots) with different particle initial conditions for propagation over 300 lattice periods.

5th resonance core – test particles (plot in every 5 period)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Space charge field of gaussian density particles
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Space charge field of gaussian density particles

For Gaussian charge density,

$$\rho(x) = \frac{\lambda}{2\pi\sigma_x\sigma_y} \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right)$$

$$E_{sc,x}(x, y) = 2\lambda \frac{1 - e^{-r^2/\sigma^2}}{r^2} x$$
$$E_{sc,y}(x, y) = 2\lambda \frac{1 - e^{-r^2/\sigma^2}}{r^2} y$$

$$r^2 = x^2 + y^2$$
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Space charge field of gaussian density particles**

For Gaussian charge density,

\[
\rho(x) = \frac{\lambda}{2\pi\sigma_x\sigma_y} \exp\left(\frac{-x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)
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\[
r^2 = x^2 + y^2
\]

For symmetric case, \( \sigma_r = \sqrt{2}\sigma_x = \sqrt{2}\sigma_y \)

\[
\rho(r) = \frac{\lambda}{\pi\sigma_r^2} \exp\left(-\frac{r^2}{\sigma_r^2}\right)
\]

\[
E_{sc,r}(r) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r}
\]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Space charge field of gaussian density particles

For Gaussian charge density,

\[
\rho(x) = \frac{\lambda}{2\pi\sigma_x\sigma_y} \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right)
\]

\[
E_{sc,x}(x,y) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r^2} x,
\]

\[
E_{sc,y}(x,y) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r^2} y
\]

\[
r^2 = x^2 + y^2
\]

For symmetric case, \( \sigma_r = \sqrt{2}\sigma_x = \sqrt{2}\sigma_y \)

\[
\rho(r) = \frac{\lambda}{\pi\sigma_r^2} \exp\left(-\frac{r^2}{\sigma_r^2}\right)
\]

\[
E_{sc,r}(r) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r}
\]

\[
\sigma_r(s) = n_b/\sqrt{2}
\]

(equivalent beams)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Space charge field of gaussian density particles**

For Gaussian charge density,

\[ \rho(x) = \frac{\lambda}{2\pi \sigma_x \sigma_y} \exp \left( -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) \]

\[ E_{sc,x}(x,y) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r} x, \quad E_{sc,y}(x,y) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r} y \]

\[ r^2 = x^2 + y^2 \]

For symmetric case, \( \sigma_r = \sqrt{2}\sigma_x = \sqrt{2}\sigma_y \)

\[ \rho(r) = \frac{\lambda}{\pi \sigma_r^2} \exp(-\frac{r^2}{\sigma_r^2}) \]

\[ E_{sc,r}(r) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r} \]

**Equation of motion** (real frame)

Coupled equation of motion

\[ \begin{cases} x''(s) - 2\sqrt{\kappa_x(s)}y'(s) - \frac{K}{2} F_{sc,x}(x,y) = 0 \\ y''(s) + 2\sqrt{\kappa_x(s)}x'(s) - \frac{K}{2} F_{sc,y}(x,y) = 0 \end{cases} \]

\[ F_{sc,x}(x,y) = 2 \frac{1 - e^{-r^2/\sigma_r^2}}{r^2} x \]

\[ F_{sc,y}(x,y) = 2 \frac{1 - e^{-r^2/\sigma_r^2}}{r^2} y \]

When \( p_\theta \neq 0 \), \( \gamma' = \gamma'' = 0 \)

\[ \sigma_r(s) = \frac{r_b}{\sqrt{2}} \]

(equivalent beams)
**Halo formation of transverse particle-core model**

**Gaussian density charged particle motions of matched beam**

### Space charge field of gaussian density particles

For Gaussian charge density,

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\rho(x) = \frac{\lambda}{2\pi\sigma_x\sigma_y} \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)
\]

\[
E_{sc,x}(x,y) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r} x,
E_{sc,y}(x,y) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r} y
\]

\[r^2 = x^2 + y^2\]

For symmetric case, \(\sigma_r = \sqrt{2}\sigma_x = \sqrt{2}\sigma_y\)

\[
\rho(r) = \frac{\lambda}{\pi\sigma_r^2} \exp\left(-\frac{r^2}{\sigma_r^2}\right)
\]

\[
E_{sc,r}(r) = 2\lambda \frac{1 - e^{-r^2/\sigma_r^2}}{r}
\]

(\text{equivalent beams})

### Equation of motion (real frame)

**Coupled equation of motion**

\[
\begin{aligned}
x''(s) - 2\sqrt{\kappa_2(s)}y'(s) - \frac{K}{2} F_{sc,x}(x,y) &= 0 \\
y''(s) + 2\sqrt{\kappa_2(s)}x'(s) - \frac{K}{2} F_{sc,y}(x,y) &= 0
\end{aligned}
\]

When \(p_\theta \neq 0\), \(\gamma' = \gamma'' = 0\)

**Radial equation of motion** (real frame)

\[
\begin{aligned}
r''(s) + \kappa_2(s)r(s) - \frac{K}{2} F_{sc}(r) &= 0 \\
F_{sc}(r) &= 2 \frac{1 - e^{-r^2/\sigma_r^2}}{r}
\end{aligned}
\]

When \(p_\theta = 0\), \(y = y' = 0\), \(\gamma' = \gamma'' = 0\)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions** (real frame)

Radial equation of motion (phase plane $r/r_b$-$r'$)

$$r''(s) + \kappa_z(s)r(s) - \frac{K}{2} F_{sc}(r) = 0$$

$$F_{sc}(r) = 2 \frac{1 - e^{-r^2/\sigma_r^2}}{r}$$

All points are plotted in every S lattice period (Poincare surface of section plots) with different particle initial conditions for propagation over 300 lattice periods.
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions** (real frame)

Radial equation of motion

(phase plane $r/r_b-r'$)

\[
r''(s) + \kappa z(s)r(s) - \frac{K}{2} F_{sc}(r) = 0
\]

\[
F_{sc}(r) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r}
\]

\[K=0, \ \sigma_0 = 45.5^\circ, \sigma = 46^\circ \]

\[\sigma/\sigma_0 = 1\]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Radial equation of motion

\[ r''(s) + \kappa_2(s)r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[ F_{sc}(r) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r} \]

\[ K=3, \sigma = 45.5^\circ, \sigma = 12^\circ \]

\[ \frac{\sigma}{\sigma_0} = 0.26 \]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Radial equation of motion

\[ r''(s) + \kappa_z(s)r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[ F_{sc}(r) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r} \]

\[ K=0, \quad \sigma_0 = 45.5^\circ, \sigma = 46^\circ \]
\[ \frac{\sigma}{\sigma_0} = 1 \]

\[ K=2.3, \quad \sigma_0 = 115^\circ, \sigma = 90^\circ \]
\[ \frac{\sigma}{\sigma_0} = 0.78 \]

\[ K=3, \quad \sigma_0 = 45.5^\circ, \sigma = 12^\circ \]
\[ \frac{\sigma}{\sigma_0} = 0.26 \]

All points are plotted in every S lattice period (Poincare surface of section plots) with different particle initial conditions for propagation over 300 lattice periods.
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Radial equation of motion

\[ r''(s) + \kappa(s) r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[ F_{sc}(r) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r} \]

\[ K=2.3, \quad \sigma_0 = 115^\circ, \quad \sigma = 90^\circ \]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions (real frame)**

Radial equation of motion

\[ r''(s) + \kappa_z(s)r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[ F_{sc}(r) = 2 \left( 1 - \frac{r^2/\sigma_r^2}{r} \right) \]

\[ e^{-r^2/\sigma_r^2} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( -\frac{r^2}{\sigma_r^2} \right)^n \]

K = 2.3, \( \sigma_0 = 115^\circ \), \( \sigma = 90^\circ \)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions** (real frame)

**Radial equation of motion**

\[
r''(s) + \kappa_z(s)r(s) - \frac{K}{2} F_{sc}(r) = 0
\]

\[
F_{sc}(r) = 2 \left(1 - e^{-r^2/\sigma_r^2}\right) = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{r^2}{\sigma_r^2}\right)^n
\]

With \(n \leq 2\); \(n=1\) (linear), \(n=2\) (2nd order)

\(- r'''(s) + \sigma_{\perp}^2 r(s) \sim r^3 \cdot e^{i\sigma_{\text{env}} s} ; r \sim e^{\pm i\sigma_s}
\]

< Resonance condition >

\(- > \sigma_{\text{env}} = 4\sigma_{\perp} ; \sigma_{\text{env}} = 360^\circ\) (matched beam)

\(- > \sigma_{\perp} = 90^\circ : 4^{\text{th}}\text{order resonance}
\]

\(K=2.3 , \sigma_0 = 115^\circ , \sigma = 90^\circ\)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Radial equation of motion

\[ r''(s) + \kappa_z(s) r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[ F_{sc}(r) = 2 \frac{1 - e^{-r^2/\sigma_r^2}}{r} \]

\[ e^{-r^2/\sigma_r^2} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( -\frac{r^2}{\sigma_r^2} \right)^n \]

With \( n \leq 2 \); \( n = 1 \) (linear), \( n = 2 \) (2nd order)

\[ r''(s) + \sigma_r^2 r(s) \sim r^3 \cdot e^{i\sigma_{env} s} ; r \sim e^{\pm i\sigma_s} \]

< Resonance condition >

\[ \sigma_{env} = 4\sigma_r ; \sigma_{env} = 360^\circ \] (matched beam)

\[ \sigma_r = 90^\circ : 4^{th} order resonance \]

K=2.3 , \( \sigma_0 = 115^\circ \) , \( \sigma = 90^\circ \)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Radial equation of motion

\[ r''(s) + \kappa_z(s)r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[ F_{sc}(r) = 2 \left(1 - \frac{e^{-r^2/\sigma_r^2}}{r} \right) \]

\[ e^{-r^2/\sigma_r^2} = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{r^2}{\sigma_r^2}\right)^n \]

With \( n \leq 2 \); \( n = 1 \) (linear), \( n = 2 \) (2nd order)

- \( r''(s) + \sigma_r^2 r(s) \sim r^3 \cdot e^{i\sigma_{env} s} ; r \sim e^{i\sigma_{ls} s} \)

< Resonance condition >

- \( \sigma_{env} = 4\sigma_{\perp} \); \( \sigma_{env} = 360^\circ \) (matched beam)

- \( \sigma_{\perp} = 90^\circ : 4^{th}\text{order resonance} \)

K=2.3 , \( \sigma_0 = 115^\circ \), \( \sigma = 90^\circ \)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions** (real frame)

Radial equation of motion

\[ r''(s) + \kappa_z(s)r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[ F_{sc}(r) = 2 \left( 1 - e^{-r^2/\sigma_r^2} \right) \]

\[ e^{-r^2/\sigma_r^2} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( -\frac{r^2}{\sigma_r^2} \right)^n \]

With \( n \leq 2 \); \( n=1 \) (linear), \( n=2 \) (2nd order)

\[ \rightarrow r''(s) + \sigma_\perp^2 r(s) \sim r^3 \cdot e^{i\sigma_{env}s} ; r \sim e^{\pm i\sigma_{\perp}s} \]

< Resonance condition >

\[ \rightarrow \sigma_{env} = 4\sigma_\perp ; \sigma_{env} = 360^\circ \text{ (matched beam)} \]

\[ \rightarrow \sigma_{\perp} = 90^\circ : 4^{th} \text{order resonance} \]

\[ K=2.3 , \sigma_0 = 115^\circ , \sigma = 90^\circ \]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions** (real frame)

Radial equation of motion

\[ r''(s) + \kappa_z(s) r(s) - \frac{K}{2} F_{sc}(r) = 0 \]

\[
F_{sc}(r) = 2 \frac{1 - e^{-r^2/\sigma_r^2}}{r} \quad e^{-r^2/\sigma_r^2} = \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{-r^2}{\sigma_r^2} \right)^n
\]

With \( n \leq 2 \); \( n=1 \) (linear), \( n=2 \) (2nd order)

\[ r''(s) + \sigma_r^{-2} r(s) \sim r^{-3} \cdot e^{i\sigma_{env}s} ; r \sim e^{\pm i\sigma z s} \]

< Resonance condition >

\[ \sigma_{env} = 4\sigma_z ; \sigma_{env} = 360^\circ \text{ (matched beam)} \]

\[ \sigma_z = 90^\circ : 4^{th} \text{ order resonance} \]

\[ K=2.3 , \sigma_0 = 115^\circ , \sigma = 90^\circ \]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions** (real frame)

**Coupled equation of motion**

\[
\begin{align*}
    x''(s) &= 2\sqrt{\kappa_x(s)}y'(s) - \frac{K}{2}F_{sc.x}(x, y) = 0 \\
    y''(s) &= 2\sqrt{\kappa_y(s)}x'(s) - \frac{K}{2}F_{sc.y}(x, y) = 0
\end{align*}
\]

\[
F_{sc.x}(x, y) = 2\frac{1 - e^{-r^2/\sigma^2}}{r^2} x
\]
\[
F_{sc.y}(x, y) = 2\frac{1 - e^{-r^2/\sigma^2}}{r^2} y
\]

All points are plotted in every 5 lattice period (Poincare surface of section plots) with different particle initial conditions for propagation over 300 lattice periods.
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Coupled equation of motion

\[
\begin{align*}
    x''(s) - 2\sqrt{\kappa_z(s)}y'(s) - \frac{K}{2} F_{sc,x}(x, y) &= 0 \\
y''(s) + 2\sqrt{\kappa_z(s)}x'(s) - \frac{K}{2} F_{sc,y}(x, y) &= 0
\end{align*}
\]

\[
F_{sc,x}(x, y) = 2 \frac{1 - e^{-r^2/\sigma_r^2}}{r^2} x
\]

\[
F_{sc,y}(x, y) = 2 \frac{1 - e^{-r^2/\sigma_{ry}^2}}{r^2} y
\]

Many test particles with different initial conditions

All points are plotted in every S lattice period (Poincare surface of section plots) with different particle initial conditions for propagation over 300 lattice periods.
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Coupled equation of motion

\[
\begin{align*}
x''(s) - 2\sqrt{\kappa_z(s)}y'(s) - \frac{K}{2}F_{sc,x}(x, y) &= 0 \\
y''(s) + 2\sqrt{\kappa_z(s)}x'(s) - \frac{K}{2}F_{sc,y}(x, y) &= 0
\end{align*}
\]

\[
F_{sc,x}(x, y) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r^2} x
\]

\[
F_{sc,y}(x, y) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r^2} y
\]

Many test particles with different initial conditions

(\text{phase plane } x/r_b - x', y/r_b - y', x/r_b - y/r_b)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions (real frame)

Coupled equation of motion

\[
\begin{align*}
    x''(s) - 2\sqrt{\kappa_z(s)}y'(s) - \frac{K}{2} F_{sx}(x, y) &= 0, \\
    y''(s) + 2\sqrt{\kappa_z(s)}x'(s) - \frac{K}{2} F_{sy}(x, y) &= 0
\end{align*}
\]

where

\[
F_{sx}(x, y) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r^2} x
\]

\[
F_{sy}(x, y) = 2 \frac{1 - e^{-r^2/\sigma^2}}{r^2} y
\]

Many test particles with different initial conditions

(Phase plane \(x/r_b - x', y/r_b - y', x/r_b - y/r_b\))

All points are plotted in every 5 lattice period (Poincare surface of section plots) with different particle initial conditions for propagation over 300 lattice periods

- \(K = 2.3, \sigma_0 = 115^\circ, \sigma = 90^\circ\)
  \[
  \frac{\sigma}{\sigma_0} = 0.78
  \]

- \(K = 3, \sigma_0 = 45.5^\circ, \sigma = 12^\circ\)
  \[
  \frac{\sigma}{\sigma_0} = 0.26
  \]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions
(real frame)

All points are plotted in every 5 lattice period (Poincare surface of section plots) of a single particle for propagation over 300 lattice periods
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions

(real frame)

Single test particle motion

(phase plane $x/\tau_b - y/\tau_b$)

$K=2.3, \sigma_0 = 115^\circ, \sigma = 90^\circ$

$\frac{\sigma}{\sigma_0} = 0.78$

All points are plotted in every 5 lattice period (Poincare surface of section plots) of a single particle for propagation over 300 lattice periods.
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions
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Single test particle motion
(phase plane $x/r_b - y/r_b$)

$K=2.3$, $\sigma_0 = 115^\circ$, $\sigma = 90^\circ$

$\frac{\sigma}{\sigma_0} = 0.78$
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions
(real frame)

Initial condition
\[
\left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.1)^2
\]
\[
\left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.7)^2
\]

Single test particle motion
(phase plane \( x/r_b \) - \( y/r_b \))

\( K=2.3 \), \( \sigma_0 = 115^\circ \), \( \sigma = 90^\circ \)

\( \frac{\sigma}{\sigma_0} = 0.78 \)

All points are plotted in every 5 lattice period (Poincare surface of section plots) of a single particle for propagation over 300 lattice periods.
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions
(real frame)

All points are plotted in every S lattice period (Poincare surface of section plots) of a single particle for propagation over 300 lattice periods

**Transverse particle motions**

(initial condition \( \left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.1)^2 \))

\( \left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.7)^2 \)

\( \left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.9)^2 \)

**Single test particle motion**

(Phase plane \( x/r_b - y/r_b \))

\( K = 2.3, \quad \sigma_0 = 115^\circ, \quad \sigma = 90^\circ \)

\( \frac{\sigma}{\sigma_0} = 0.78 \)
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

**Transverse particle motions**
(real frame)

All points are plotted *in every S lattice period* (Poincare surface of section plots) of a single particle for propagation *over 300 lattice periods*

**Initial condition**

\[
\left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.1)^2
\]

\[
\left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.7)^2
\]

\[
\left( \frac{x}{r_b} \right)^2 + \left( \frac{y}{r_b} \right)^2 = (0.9)^2
\]

**Single test particle motion**
(phase plane \( x/r_b - y/r_b \))

\[\frac{x}{r_b}, \frac{y}{r_b} = (0.1)^2\]

\[\frac{x}{r_b}, \frac{y}{r_b} = (0.7)^2\]

\[\frac{x}{r_b}, \frac{y}{r_b} = (0.9)^2\]

\[\frac{x}{r_b}, \frac{y}{r_b} = (1.2)^2\]

\[K = 2.3, \sigma_0 = 115^\circ, \sigma = 90^\circ\]

\[\sigma = 0.78\]
Halo formation of transverse particle-core model

Gaussian density charged particle motions of matched beam

Transverse particle motions
(real frame)

Single test particle motion
(phase plane $x/r_b - y/r_b$

$K = 2.3$, $\sigma_0 = 115^\circ$, $\sigma = 90^\circ$

$\frac{\sigma}{\sigma_0} = 0.78$

All points are plotted in every $S$ lattice period (Poincare surface of section plots) of a single particle for propagation over 300 lattice periods
Contents

- High-intensity charged-particle beam in a periodic solenoidal focusing field
  - Beam physics applications
  - Nonlinear resonances and chaotic motions of envelope oscillation
- Halo formation of transverse particle-core model
  - Halo formations
  - Uniform density charged particle motions
  - Gaussian density charged particle motions of matched beam
- Summary
Summary

• The periodic solenoidal focusing field is important for several reasons.
The periodic solenoidal focusing field is important for several reasons.

- Halo formations
Summary

• The periodic solenoidal focusing field is important for several reasons.

• Halo formations
  ✓ Uniform charge density
Summary

- The periodic solenoidal focusing field is important for several reasons.
- Halo formations
  - Uniform charge density

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## Summary

- The periodic solenoidal focusing field is important for several reasons.
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Summary

• The periodic solenoidal focusing field is important for several reasons.

• Halo formations

  ✓ Uniform charge density

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✓ Non-uniform charge density (Gaussian)
• The periodic solenoidal focusing field is important for several reasons.

• Halo formations

✓ Uniform charge density

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Envelope oscillation

Resonance

Particle frequency

✓ Non-uniform charge density (Gaussian)

| Envelope | Matched | Gaussian density profile | Non-linear space charge force |
Summary

• The periodic solenoidal focusing field is important for several reasons.

• Halo formations

  ✓ Uniform charge density

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  ✓ Non-uniform charge density (**Gaussian**)

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- Symmetric gaussian -> radial motion
- Non symmetric gaussian -> coupled motions of x, y -> many test particles / single particle motions
Future plan

Reference

Future plan

- Transverse particle beam dynamics
  - particle-core model compare with PIC simulation of self-consistence

- Longitudinal beam dynamics

- Apply to the beam halo and beam loss measurement design input

Reference

Thank you for your attention!

61th ICFA Advanced Beam Dynamics Workshop
on High-intensity and High-brightness Hadron beams (HB 2018)
In Daejeon

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