Revisiting the Longitudinal $90^\circ$ Limit for Superconducting Linear Accelerators

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Ingo Hofmann
GSI Darmstadt / TU Darmstadt
Overview

- Introduction
- The longitudinal „90° structure resonance stopband“
- The sum „envelope instability“
- Irregular periodic lattices
- Discussion
- Conclusions

Acknowledgments: O. Boine-Frankenheim, J. Struckmeier, Y. Yuan
“Accepted criteria” for lattice design in high intensity accelerators

1. Keep \textbf{zero current phase advance per cell below 90}^\circ \text{ for transverse and longitudinal to avoid structure resonance/parametric instability}
2. Smooth (adiabatic) changes in transverse and longitudinal focusing
3. Avoid transverse-longitudinal emittance transfer via space charge resonance
4. Provide good matching between lattice transitions to avoid halo

1, 3 and 4 are resonant processes
Overview on discussion of $90^0$ stopband

- Was of concern as “envelope Instability” from envelope equations in 1970’s (Lambertson et al., 1977, Reiser and Struckmeier, 1984)
- Some early experimental evidence in Berkeley coasting beam channel experiment (M. Tiefenbach et al., 1985)
- Transverse $90^0$ stopband confirmed first time experimentally in a Linac – GSI-UNILAC (L. Groening et al., PRL 2009)
- No experiment on longitudinal $90^0$ mode!
- Taken for granted and $90^0$ applied to linac design transversely and longitudinally
- We found in 2017 (PRL) that longitudinally $90^0$ limitation in some cases unnecessary and re-visiting is appropriate!
Wangler:
- Longitudinal 90° stop-band can limit the accelerating gradient at low velocities
- → shorten focusing period by SR- rather than FRDR-
- → higher accelerating gradients possible

Source:

Longitudinal Beam-Dynamics
Constraint on Accelerating Gradient

T.P. Wangler, Los Alamos National Laboratory
and K.R. Crandall, TechSource

Workshop on Advanced Design of Spoke Resonators
Los Alamos, NM
October 7-8, 2002
Overview

- Introduction
- **Review longitudinal „90° structure resonance stopband“**
- The sum „envelope instability“
- Irregular periodic lattices
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1st lattice example: idealized solenoid + RF gap
“toy lattice” - for simplicity
TRACEWIN - simulations

- periodic cell: solenoid + RF gap (no acceleration)
- 3D Gaussian bunches
- strong $90^0$ effect - “as expected”
  - transversely similar to FODO
  - longitudinally?
Evidence of **longitudinal 4th order structure resonance + envelope instability**

\[ k_{0z} = 120^0 \quad k_z = 76^0 \]

- **4th order structure resonance**
  \[ 4k_z \sim 360^0 \]
  - compare with transverse UNILAC experiment

- **2nd order envelope instability**
  \[ 2k_z \sim 180^0 \]

- **Longitudinal envelope instability will occur earlier if larger mismatch**
Periodic solenoid lattice $90^0$ longitudinal stopband confirming serious effect beyond certain intensity

\[ k_{oz} = 60^0 \text{ (similar for } 80^0) \]

**Initial** 4th order followed by env-instability

no emittance growth

is it useable???

probably not as SR cells short!
2\textsuperscript{nd} lattice example: FODO + RF gap

space charge forces:
- transverse force in FR half-cell different from DR half-cell
- in an exact sense longitudinal period same as transverse due to space charge coupling
- coupling to longitudinal in practice very small
Simulation for $k_{0z} = 120^\circ$ $k_z = 76^\circ$ ($k_{0x} = 60^\circ$)

- Found no resonant effect (only initial nonlinear field energy jump)
- Contrary to same $k_{0z}$, $k_z$ in periodic solenoid case
- Apparently longitudinal space charge force in FR half-cell can be assumed nearly identical to that in DR half-cell → “identical” cells 

3D Gaussian distribution
In FODO + RF longitudinal 90° stopband absent as long as $k_{0z} < 180°$

$L_{\text{long}}$

$L_{\text{trans}} = \text{transverse period} = \text{lattice cell (long!)}$

- allows choice of $k_{0z}$ above 90°
- more design flexibility
- unless other sources of emittance degradation
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“Sum envelope instability” – a new mode of some concern

O. Boine-Frankenheim, I. Hofmann and J. Struckmeier, POP 2016
I. Hofmann and O. Boine-Frankenheim, PRL 2017
Y. Yan et al, PRAB 2018

For split tunes
\[ k_{0xy} < 90^0 \text{ and } k_{0z} > 90^0 \]
(both defined on transverse focusing period)
a “sum envelope instability” was found to exist provided that
\[ k_{0xy} + k_{0z} > 180^0 \]

smooth approximation criterion for center of stopband of
sum envelope instability:
\[ k_{0xy} + k_{0z} = 180^0 + \Delta k_{coh,\text{sum}} \]

(single envelope instability \( k_{0xyz} = 90^0 + \Delta k_{coh} \))

- not to be confused with “sum resonance” \( k_{0x} + k_{0y} = 360^0 \) by skew quads,
  which is a single particle resonance!
Sum envelope instability criterion:

\[ k_{0z} + k_{0xy} = 180^\circ + \Delta k_{\text{coherent}} \]

\[ k_{0z} = 120^\circ \quad (k_z = 92^\circ) \quad \text{and} \quad k_{0xy} = 90^\circ \]
Combined stopbands in xy and z

\[ k_{0z} + k_{0xy} = 180^\circ + \Delta k_{coh} \text{ for stopband center} \]

here: \[ \Delta k_{coh} \sim \Delta k_{incoh,z} \sim 30^\circ \]
Summary
for avoiding the 90° and sum mode

Periodic solenoid channel SR-SR-...
- $k_{0xyz} < 90°$ safe
- no sum mode ($k_{0z} + k_{0xy} < 180°$)

Periodic quadrupole channel FRDR-FRDR-...
- $k_{0xy} < 90°$ safe
- $k_{0z} > 90°$ also ok, provided that:
  - sum mode condition $k_{0xy} + k_{0z} = 180° + \Delta k_{coh}$ is avoided
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Linacs often not strictly periodic

test case: breaking RF gap strength symmetry

asymmetric gap voltage: 2→1 cell

width of stopband ~ asymmetry
→ small asymmetry ignorable
Periodic interruptions – from tank to tank

“toy” example: missing every 6th gap (between tanks)

The missing 6th gap generates a space charge harmonic with a period over 2 cells equivalent to a phase advance $2k_{0z} = 120^0$ - but no evidence for envelope instability
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Interference with “stability chart”
enable operation far to the right of “main resonance”

\[ \frac{\varepsilon_{xy}}{\varepsilon_{z}} = 0.1/0.1 \]

FODO - \( k_{oz} = 120^0 \)

- \( k_{oxy} < 85^0 \) stable against sum mode
- \( k_z + k_{oxy} < 180^0 \)

\( k_{oz} = 120^0 \)
\( k_z = 100^0 \)
\( k_{oxy} = 85^0 \)
\( k_{xy} = 54^0 \)
Scan of envelope instabilities in $x$ - $y$
(similar in $xy$ – $z$)

linearized envelope equation
**growth rates** in transverse FODO channel with **symmetric** focusing in $x$-$y$

source: Yao-Shuo Yan et al., PRAB 2107
3D chart of structure resonances

- k_0,x,y
- k_0,z
- 900
- 600
- 1200
- 1800
- shorter longitudinal cell

- x,y - envelope instability
- x,y,z - sum envelope instability
- conventional design region
- extended design region
- 2:2 emittance exchange resonance
- space charge shift
- 3D chart of structure resonances
- shorter longitudinal cell
- z - envelope instability
- k_0,z 180°
Application example
Discussion of CW s.c. linac C-Neutrino Driver (J-Y. Tang)

1 longitudinal period, where "effective" $k_{0z} < 45^0$
- could be chosen larger!

R3FD

R5FR5D (> 1GeV)

R5FD

R5FR5D (> 1GeV)
Conclusions

- Retrieved interplay of fourth order and envelope instability in longitudinal plane for Solenoid + RF – similar to transverse $90^0$
- Demonstrated that in FODO+RF longitudinal period effectively halved and absence of $90^0$ stopband
- Allows longitudinal phase advance (per focusing cell) above $90^0$
- Watch additional constraint for $k_{0z}>90^0$: “sum envelope instability”
- No need to replace FODO by shorter solenoid cells
- Added design flexibility in high gradient superconducting linacs
- Expect that full linac studies including high acceleration gradients also shows mitigated $90^0$ effects in transverse plane
- ➔ conventional “under-90-degrees” (longitudinally) is an over-emphasized criterion – needs to be relaxed in number of cases!
Thank you for your attention!
Resonance – Instability
- 2 distinct sources of emittance growth -

resonant excitation
- single particle resonances
- coherent resonances
- in linacs only structure resonances
- here: fourth order, driven by space charge

instability
- parametric resonance
- here: envelope instabilities

Beam potential from lattice and self-consistent electric field
Sum mode activated by increased $\varepsilon_z$ but only “delayed” growth, if beam well-matched

$k_{oz}=120^0$

$\varepsilon_{xy}/\varepsilon_z=0.1/0.15$