Classification of Space-Charge Resonances and Instabilities

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Drowned in a swamp of terms...?
Space-charge mechanisms

• There are two families of space-charge mechanisms, and yet they need to be differentiated: instabilities and resonances.

• Instabilities: a.k.a. parametric resonances, coherent resonances, coherent instabilities, parametric instabilities …
• Resonances: a.k.a. (single) particle resonances, incoherent resonances …

• Both families are loosely called “resonances”.
• Many names for the same thing … → confusing even to experts.
• It is beneficial to differentiate the two families of mechanisms.
Instabilities

- Instabilities of a KV distribution were reported in the early literatures, and the 2\textsuperscript{nd} order instability is widely known as “the envelope instability”.

- These instabilities of the beam envelope are also called parametric resonances.
- They are parametric resonances of the envelope equation:
  \[ x'' + k(s)x - \frac{\varepsilon^2}{x^3} - \frac{K(s)}{x} = 0 \]
  where \( x \) is the beam envelope not the particle coordinate.

- They are parametric resonances of the beam envelope.
- Are they resonances of the beam particle? No.
Resonances

- **Resonances** are well known in circular accelerators. In fact, they are resonances of the beam particle.
- Particle resonances were discovered in high intensity linear accelerators in 2009.
  - described by a particle Hamiltonian.
- Space-charge resonances and instabilities may look alike in the phase space!
Resonances

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- Space-charge resonances and instabilities may look alike in the phase space!

- However, there is a fundamental difference between resonances and instabilities!!
What is the difference?

**Instabilities** (or parametric resonances) of **beam envelope**

- No resonance frequency component

**Resonances** (or particle resonances) of **beam particle**

- Yes resonance frequency component

Instabilities of the beam envelope → no fixed points in phase space

- Instability of KV distribution was first found by Haber (1979).
- Instabilities of envelope equation were studied analytically by Hofmann et al (1983).
  - 2nd, 3rd, 4th order envelope instabilities have been observed.

Resonances of the beam particle → fixed points in phase space

- $4\sigma = 360^\circ$ 4th order resonance was found by Jeon et al (2009) and verified experimentally by Groening et al (2009).
- $6\sigma = 720^\circ$ 6th order resonance was found (2015).
- 8th, 10th order resonances were found by Hofmann (2016).
Instabilities of the beam envelope
a.k.a. parametric resonances or envelope instabilities
2nd order envelope instability for high intensity linear accelerators

- $2\sigma_o - \Delta\sigma_{2,coh} = 180^\circ$ second order instability for a constant-$\sigma_o$ lattice with $\sigma_o = 100^\circ$ and $\sigma = 70^\circ$ with Gaussian distribution.
- Observed for KV, Gaussian, waterbag distributions.
- The envelope instability is excited following the 4th order resonance for a constant-$\sigma_o$ lattice.
3rd order envelope instability for high intensity linear accelerators

- $3\sigma_o - \Delta \sigma_{3,\text{coh}} = 180^\circ$ third order instability for a constant-$\sigma_o$ lattice $\sigma_o = 92^\circ$ and $\sigma = 40^\circ$ (90 mA beam).
- Observed for KV and waterbag distributions, but no for Gaussian distribution.
- Not a resonance: no resonance peaks around 1/3 or 1/6 in the FFT spectrum.

Jeon et al., NIM A 832 (2016) 43
4th order envelope instability for high intensity linear accelerators

- $4\sigma_0 - \Delta \sigma_{4,\text{coh}} = 2 \cdot 180^\circ$ fourth order instability for a lattice with $\sigma_0 = 112^\circ$ and $\sigma = 85^\circ$
- Observed only for a KV distribution.
- Not a resonance: no resonance peak around $1/4 = 90^\circ/360$ in the FFT spectrum.
4th order envelope instability for high intensity linear accelerators

\[ 4\sigma_o - \Delta\sigma_{4,coh} = 180^\circ \text{ fourth order instability} \]

- Observed for KV and waterbag distributions.

Courtesy of Haber and Maschke, PRL 42, 1479 (1979)
KV distribution
\( \sigma_o = 90^\circ \) and \( \sigma = 30^\circ \)

Courtesy of Hofmann (HB2016)
Waterbag distribution
\( \sigma_o = 70^\circ \) and \( \sigma = 35^\circ \)
Applying KV instabilities to non-KV beams

- Beam envelope equation was derived for a KV distribution by Kapchinskij and Vladmirskij.
- The envelope equation was extended to any charge distribution with elliptical symmetry by Sacherer, noting that second moments of any particle distribution linear part of the force.
- Vlasov-Poisson-equation approach relying on a KV distribution is also subject to similar limitations.

- One-to-one correlation between instabilities of KV and non-KV distributions may be limited.
- The 3rd and 4th instabilities have been observed only for waterbag distributions (non-KV).
- No high order instabilities have been observed for Gaussian distributions.
- This suggests the possibility that high order instabilities may not be observable for real beams.
Instabilities

- Beam envelope becomes identical to itself when the particle makes 180° phase advance.
- Instability condition is $m\sigma_o - \Delta\sigma_{m,coh} = n180°$.
- Mathieu-type instabilities.
- Called “half integer resonance” by some.

- But half integer resonances known in circular accelerators are $2\sigma = n360°$.
- Particle resonance condition $m\sigma = n360°$ comes from the Fourier expansion of the Hamiltonian.

- Terminologies of two different worlds got mixed.
Resonances of the beam particle
a.k.a. (single) particle resonances, incoherent resonances
• The 4th order resonance of the beam particle was discovered in high-intensity linear accelerators in 2009.

• Stable fixed points do exist and their properties are observed.
  - The resonant frequency component is observed at the tune $1/4 = 90°/360°$.
  - Behavior difference depending on whether to cross the resonance “from above” or “from below” due to stable fixed points.
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Jeon et al, PRST-AB 12, 054204 (2009)
Appearance may be deceiving!

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- No resonance frequency component is observed for the 4\textsuperscript{th} order instability of a KV distribution.

Resonance frequency peak

• Clear resonance frequency peak at $1/4 = 90^\circ/360^\circ$ is observed for non-KV beam distributions.
• The 4th order resonance was verified in the two experiments.
Resonance frequency peak

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Experiment 1 of the 4th order resonance using GSI UNILAC

Groening et al., PRL 102, 234801 (2009)

Graphs showing measurement and simulation data for different phase advances. The horizontal and vertical emittance growth is measured and simulated for zero current phase advances of 80°, 100°, and 120°. The emittance growth is presented as a percentage and is compared with the "Parmila" simulation. The simulation data is overlaid on the measurement data to show the agreement or discrepancy between the two.
Experiment 2 of the 4th order resonance
SNS linac, Simulations

Simulations

$\sigma = 65^\circ$

$\sigma = 80^\circ$

$\sigma = 88.5^\circ$

$\sigma = 93^\circ$
Experiment 2 of the 4th order resonance
SNS linac, Experiment

simulations

experiments

Jeon, PRAB 19, 010101 (2016)
6th order resonance
for high intensity linear accelerators

• $6\sigma = 720^\circ$ sixth order resonance for $\sigma < 120^\circ$.
• No resonance effects for $\sigma > 120^\circ$ (Hamiltonian property).
• Frequency analysis shows a peak at $1/3 = 120^\circ/360^\circ$.
• Result of the perturbation of $2\sigma = 360^\circ$ and $4\sigma = 360^\circ$ resonances.
6th order resonance for high intensity linear accelerators

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Jeon et al., PRL 114, 184802 (2015)
6th order resonance for high intensity linear accelerators

- Resonance frequency peak at 1/3 for lattice < 120° for non-KV beams.
- No resonance frequency peak for > 120°.

Jeon, J Korean Phys Soc, in press
6th order resonance for high intensity linear accelerators

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6th order resonance
10-emA Gaussian distribution

6th order resonance
20-emA waterbag distribution

Jeon, J Korean Phys Soc, in press
Particle Resonances

• The $4\sigma = 360^\circ$ resonance in high intensity linacs was discovered in 2009. [Jeon et al., PRSTAB 12, 054204 (2009)]

• The $6\sigma = 720^\circ$ resonance was discovered, which was a perturbation of two strong resonances: $2\sigma = 360^\circ$ resonance and $4\sigma = 360^\circ$ resonance. [Jeon et al., PRL 114, 184802 (2015)]

• The $6\sigma = 360^\circ$ resonance was too weak to observe for Gaussian distribution. [Jeon et al., PRSTAB 12, 054204 (2009)]

• Weak sign was observed for waterbag distribution. [Hofmann et al., PRL 115, 204802 (2015)]

• Higher order resonances were discovered:
  - $8\sigma = 1080^\circ$ resonance $(8:3) = (6:2) \oplus (2:1)$
  - $10\sigma = 1440^\circ$ resonance $(10:4) = (8:3) \oplus (2:1)$
    [Hofmann, Proc. of HB2016]
Resonances: a particle Hamiltonian property
More on 4th order resonance
emittance growth vs $\sigma$

Emittance growth factor ($\varepsilon_f/\varepsilon_i$) plot as a function of $\sigma$ and initial tune depression ($\sigma_o - \sigma$).

$\sigma$ is the relevant parameter of the 4th order resonance.

Constant-$\sigma$ lattices are used for all the data points.

No evidence of the envelope instability is found.

well-matched 3D Gaussian input beam

More on $4^{th}$ order resonance beam distribution evolution

Input: well-matched 3D Gaussian beam

30 mA, $\sigma = 87^\circ$ case
More on 4th order resonance
beam distribution evolution

Input:
well-matched
3D Gaussian beam

Interesting higher order detuning effects

30 mA, $\sigma = 87^\circ$ case
More on 4\textsuperscript{th} order resonance
6\textsuperscript{th} order effects

\[ H_1 = \left( v - \frac{1}{4} \right) I + 5.05 \times 10^{-4} I^2 - 4.95 \times 10^{-4} I^2 \cos 4(\phi) - 2.22 \times 10^{-5} I^3 \]

- 4\textsuperscript{th} order resonance develops a four-fold structure that requires a 6\textsuperscript{th} order detuning term.
- The Hamiltonian describes the system well.
- This 6\textsuperscript{th} order term is caused by the redistribution of the beam by the resonance.
More on 4\textsuperscript{th} order resonance
6\textsuperscript{th} order effects

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More on 4th order resonance theory of 2D Gaussian beam

- Analytical formula exists for 2D Gaussian beam.
- Space charge potential is

\[ V_{SC} = \frac{K_{sc}}{2} \int_0^\infty dt \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+t}\right)\exp\left(-\frac{y^2}{2\sigma_y^2+t}\right)-1}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)}} = \frac{K_{sc}}{2} \int_0^\infty dt \frac{\exp\left(-\frac{2\beta_{x}l_{x}x\cos^2\phi_{x}}{2\sigma_x^2+t}\right)\exp\left(-\frac{2\beta_{y}l_{y}y\cos^2\phi_{y}}{2\sigma_y^2+t}\right)-1}{\sqrt{(2\sigma_x^2+t)(2\sigma_y^2+t)}} \]

- Incoherent tune shift becomes:

\[ \Delta \nu_x \bigg|_{l_y=0} = \frac{2}{2\pi} \frac{d}{dI_x} \frac{\partial H_v}{\partial I_x} = \frac{K_{sc}}{4\pi} \oint ds \left[ -\frac{\beta_x}{\sigma_x(\sigma_x+\sigma_y)} + \frac{2\sigma_x+\sigma_y}{4\sigma_x^3(\sigma_x+\sigma_y)^2} \beta_x^2 I_x - \frac{(8\sigma_x^2+9\sigma_x\sigma_y+3\sigma_y^2)}{48\sigma_x^5(\sigma_x+\sigma_y)^3} \beta_x^4 I_x^3 + \frac{(16\sigma_x^3+29\sigma_x^2\sigma_y+20\sigma_x\sigma_y^2+5\sigma_y^3)}{384\sigma_x^7(\sigma_x+\sigma_y)^4} \beta_x^4 I_x^3 + \cdots \right] \]

(8)
More on 4th order resonance theory of 2D Gaussian beam

- Particle’s phase advance increases monotonically for 2D Gaussian beam, as the oscillation amplitude grows.
- This explains why there is no resonance when $\sigma > 90^\circ$. 
4th order resonance and 2nd order envelope instability
For a constant-$\sigma$ lattice, the 4th order resonance dominates over the envelope instability.

When $\sigma$ is constant, the 4th order resonance structure persists all the way and the envelope instability is not manifested.
For a constant-$\sigma_0$ lattice, the envelope instability follows the 4th order resonance.

The envelope instability is manifested after the 4th order resonance disappears when $\sigma > 90^\circ$.

There is a region where the 4th order resonance is off and the envelope instability is on!
**4th order resonance and envelope instability**

- Question: Is there a case reporting that the envelope instability develops by itself?
- So far, the envelope instability has been reported following the 4th order resonance (non-KV beam) or the 4th order instability (KV beam) for a constant-$\sigma_0$ lattice.

- The envelope instability develops from a mismatch.
- The four-fold structure generated by the 4th order resonance presents itself as a mismatch, which can drive the envelope instability when the 4th order resonance is off.

- All the simulations for lattices with $\sigma > 90^\circ$ show neither the 4th order resonance nor the envelope instability.
- The 4th order resonance should not be mistaken for the 4th order envelope instability.
Around 90° phase advance

• There are three mechanisms around 90° phase advance: 4th order resonance, 2nd order envelope instability, and 4th order envelope instability.

• For non-KV distributions (well-matched),
  - 4th order resonance appears first.
  - For a constant-σ lattice, the 4th order resonance persists.
  - For a constant-σ₀ lattice, the 2nd order envelope instability follows.

• For KV distributions (well-matched),
  - the 4th order envelope instability appears first.
  - the 2nd order envelope instability may follow depending on conditions.
  - It is interesting that the 4th order envelope instability appears first.
Terminology Suggestion

• Two distinct families of space-charge mechanisms exist:
  - Instabilities (or parametric resonances) of the beam envelope,
  - Resonances of the beam particle.

• Instabilities are instabilities of the beam envelope:
  - more specifically envelope instabilities,
  - a.k.a. parametric resonances (of the envelope equation),
  - but would better be called envelope parametric resonances to distinguish them from particle parametric resonances.

• Resonances are resonances of the beam particle, as known in circular accelerators:
  - would better be called particle resonances,
  - a.k.a. single particle resonances.
- Resonances
- Particle resonances

- Instabilities
- Envelope instabilities
- Envelope parametric resonances

D. Jeon, Classification of Space-Charge Resonances and Instabilities in High-Intensity Linear Accelerators, J. Korean Phys. Soc. 72, 1523 (2018)
Thank you for your attention!
감사합니다
Experiment of the 4th order resonance (II) using SNS CCL

• Schematic layout of the SNS CCL showing the wire-scanners used for the experiment.
• Halo of incoming beams were carefully controlled by matching and the MEBT round beam optics.
Experiment of the 4\textsuperscript{th} order resonance (II)

Halo of incoming beam was minimized

- Round beam optics (MEBT) was used to minimize halo formation in the upstream.
- Matching between linac sections was done to avoid the mismatch.
- The beam entering the CCL has little tails.