Abstract
The SCSS (SPRING-8 Compact SASE Source) XFEL [1] requires an extremely stable RF system in both amplitude and phase. The pulse-to-pulse fluctuation of RF output is mainly caused by modulation of the klystron beam-voltage pulse, which is directly governed by the charging stability of a klystron modulator. During R&D study on beam stability, we found a special operation point, where the beam energy gain is insensitive to the modulator voltage fluctuation. This phase can cancel out the both fluctuations and provide constant accelerating field. The stable phase depends on the klystron parameters such as the length of drift tube, operating voltage, efficiency. It is about +9° in case of the C-band main linac for SCSS XFEL. The bunch length after bunch compressor is so short that additional longitudinal energy spread due to the RF curvature is about 5% of the one caused by the longitudinal wake field. The particle energy is high enough so that longitudinal defocusing is negligible. The reduction of beam energy due to off-crest acceleration is less than 2%. This paper shows the analytical relation of the stable phase. ELEGANT [2] simulation shows no appreciable degradation of the slice parameters.

INTRODUCTION
The stability of the beam acceleration is determined by the fluctuation of both RF power and phase that are mainly caused by the modulation of a klystron voltage pulse, which is directly governed by the PFN (pulse forming network) charging stability of a modulator. Therefore, it is useful to define the sensitivities of the RF parameters such as klystron voltage, RF phase and RF power by its relative stabilities to the one of a charging voltage. This paper analyzes the sensitivities of RF parameters and beam energy, and shows a special operation point, where the beam energy gain is insensitive to the modulator voltage fluctuation.

In general, off-crest acceleration makes the beam energy more fluctuate. However, at certain phase, it is possible for this fluctuation to be same as the one due to power fluctuation with opposite polarity in the falling slope with respect to beam. This phase is preferable to get the stable beam energy even under the voltage fluctuation of the klystron. This paper shows the analytical relation of the stable phase and experimental verification.

The longitudinal profile of a bunch has to be managed to fit the SASE requirements. Energy spread, slice emittance and peak current are analytically evaluated and compared with numerical results obtained by ELEGANT simulation.

RF SENSITIVITY
The klystron voltage is directly determined by the PFN charging voltage in a modulator. The sensitivity of a klystron voltage defined by

\[ s_V = \left( \frac{dV_K}{V_K} \right) = \left( \frac{dV_K}{V_K} \right) = \left( \frac{1 + Z_{PFN}}{Z_K} \right) \left( \frac{1 + 1.5 Z_{PFN}}{Z_K} \right) \] (1)

is obtained by using the Ohm’s law

\[ V_O = V_K + Z_{PFN} \times I_K \] (2)

and the klystron beam current

\[ I_K = k V_K^{1.5} \] (3)

where \( V_o \) is a PFN charging voltage, \( V_K \) is a klystron voltage, \( Z_{PFN} \) is PFN impedance, \( k \) is a klystron perveance, \( Z_0 \) is klystron impedance. At the nominal klystron voltage where the impedance is matched, the typical sensitivity of a klystron voltage becomes 0.8. The RF phase \( \phi_{RF} \) from a klystron [3]

\[ \phi_{RF} = \phi_o - 2\pi f t_{transit} = \phi_o - 2\pi \left( \frac{c}{\lambda_{RF}} \right) \left( \frac{L_{KLY}}{v} \right) \] (4)

is delayed from a driving input RF phase \( \phi_o \) by the transit time \( t_{transit} \) of a drift length \( L_{KLY} \) between the input cavity and the output cavity of the klystron with an electron velocity \( v \) where \( \lambda_{RF} \) is a wavelength in a free-space, \( c \) is the speed of light in vacuum. Therefore, the RF phase fluctuation of a klystron is

\[ \left( \frac{d\phi_{RF}}{2\pi} \right) \left( \frac{dV_K}{V_K} \right) = \left( \frac{L_{KLY}}{\lambda_{RF}} \right) \left( \gamma^2 - 1 \right)^{-1.5} \left( \gamma - 1 \right) \] (5)

where \( \gamma \) is the Lorentz factor of the electron. And the sensitivity of the RF phase is

\[ s_\phi = \left( \frac{d\phi_{RF}}{2\pi} \right) \left( \frac{dV_O}{V_O} \right) = \left( \frac{L_{KLY}}{\lambda_{RF}} \right) \left( \gamma^2 - 1 \right)^{-1.5} \left( \gamma - 1 \right) \times s_V . \] (6)

The typical sensitivity of the RF phase at 350 kV is 1.26 for a C-band klystron.

The RF power \( P_{RF} \) of a klystron is given by

\[ P_{RF} = \eta I_K V_K = \eta k V_K^{2.5} \] (7)

where \( \eta \) is the RF conversion efficiency of a klystron. Therefore, the RF power fluctuation of a klystron is

\[ \left( \frac{dP_{RF}}{P_{RF}} \right) \left( \frac{dV_K}{V_K} \right) = \left( \frac{dV_K}{V_K} \right) + \left( \frac{dV_K}{V_K} \right) + 2.5 \] (8)
Figure 1 shows the calculated relative variation of RF power, efficiency and perveance due to the klystron voltage fluctuation of the C-band klystron (Toshiba E3746A). The perveance dependency is relatively so small that it is neglected in the sensitivity of the RF power

\[ s_P = \left( \frac{dP_{RF}}{P_{RF}} \right) \left( \frac{dV_O}{V_O} \right) = s_\eta + 2.5 s_v \]  

where \( s_\eta \) is the sensitivity of efficiency given by

\[ s_\eta = \frac{d\eta}{\eta} \]  

The efficiency variation of a klystron at low voltage has large effect on the sensitivity of RF power. The typical sensitivity of the RF power at 350 kV is 2.4 for the C-band klystron.

**ENERGY SENSITIVITY**

The energy gain of an accelerating unit is

\[ E \propto \sqrt{P_{RF} \cos \phi_{RF}} \]  

Thus, its relative fluctuation by a klystron is

\[ \frac{dE}{E} = 0.5 \times \left( \frac{dP_{RF}}{P_{RF}} \right) - 2 \pi \tan \phi_{RF} \left( \frac{d\phi_{RF}}{2\pi} \right) \]  

Using Eqs. (6) and (9), the sensitivity of energy gain is

\[ s_E = \left( \frac{dE}{E} \right) \left( \frac{dV_O}{V_O} \right) = 0.5 s_\eta + 1.25 s_v - 2 \pi \tan \phi_{RF} s_\eta . \]  

Figure 2 shows the relative energy gain of C-band units as a function of operating RF phase with charging voltage variations at the klystron voltage of 350 kV. On a certain RF phase of the falling slope with respect to beam, which is marked by the circle in the figure, the amplitude is somewhat constant because the both fluctuations are cancelled out, which provides constant accelerating field. The energy gain is insensitive to the modulator voltage fluctuation around the stable phase satisfying following condition

\[ \tan \phi_{RF} = \frac{1}{2\pi s_\eta} \left( 0.5 s_\eta + 1.25 s_v \right) . \]

Figure 2: Relative energy gain vs. RF phase with charging voltage variations at the klystron voltage of 350 with a C-band klystron. The stable phase is +7.2° at 350 kV level.

Figure 3 shows the sensitivity of energy gain for the crest phase and the off-crest phase. The typical sensitivity of the energy gain at 350 kV is 1.2 for a C-band unit. The energy gain is insensitive over wide range of klystron voltage at the off-crest phase of +9°. A little large angle than +7.2° provides better sensitivity for wide operating range below 350 kV. The loss of energy gain by the off-crest acceleration is 1.2%.
150 mm. Figure 4 shows the measured beam energy fluctuation of the C-band main linac. The beam energy stability at crest for two C-band units is 0.34\% (6\(\sigma\)); the stability of energy gain per unit is 0.59\% (6\(\sigma\)). The energy fluctuation is sensitive to the operating phase and also is asymmetric with respect to the crest phase. The energy fluctuation at the off-crest phase of +10° is reduced to 50\% level of the one at crest acceleration.

Figure 4: Beam energy fluctuation vs. RF phase of C-band main linac of SCSS prototype accelerator (0.01 mm/pixel).

Figure 5 shows the stability trend of energy gain per C-band unit normalized by the one at crest phase:

\[
\frac{(dE/E)^2}{(dE/E)_{crest}} = 1 - \frac{2\pi \tan \phi_{RF} s_\phi}{0.5 s_\phi + 1.25 s_v} + (2\pi \tan \phi_{RF})^2 \left( \frac{\sigma_\phi}{2\pi} \right)^2 \left( \frac{dE}{E} \right)^2_{crest} .
\]

(16)

Each curve has different relative phase jitter normalized by the energy stability at crest, \((\sigma_\phi/(2\pi))\)\((dE/E)_{crest}\). For example, with 0.2\% energy stability at crest, 50\% normalized phase jitter corresponds to 0.1\% relative phase jitter that is equivalent to 0.36°. Measured data (C-band 060614) in Figure 5 agree to the case of 50\% normalized relative phase jitter. It means that the phase jitter is about 1.1° that corresponds to the timing jitter of 0.52-\(\phi\)s at C-band frequency.

At the off-crest phase satisfying Eq. (14), the energy stability becomes

\[
\frac{(dE)}{E} = \left( \frac{0.5 s_\phi + 1.25 s_v}{s_\phi} \right) \left( \frac{\sigma_\phi}{2\pi} \right) .
\]

(17)

Then, it becomes \(-\sigma_\phi/(2\pi)\) at 350 kV. Therefore, the total energy stability is directly determined by the phase jitter.

Measured data (C-band 060614) indicate that the minimum fluctuation of the beam energy is located at about +8° from the crest as expected. The fluctuation at this phase is limited by the phase jitter. The different set of measured data (C-band 060711) shows that the normalized relative phase jitter is increased to 100\%, which is identified later to be caused by the instability of a maser RF oscillator.

Figure 5: Stability of energy gain per C-band unit normalized by the one at crest phase for relative phase jitter of 0\%, 50\%, and 100\%.

**BEAM QUALITIES**

The geometry of a C-band accelerating structure with iris radius \(a\), cavity radius \(b\), gap length \(g\), and cavity period \(L_{cell}\) is shown in Figure 6. The short-range wakefield of the structure is evaluated by the K. Bane’s formula [5]

\[
W(s) = Z_o c \pi a^2 \exp(-\sqrt{s/s_{oo}})
\]

where

\[
s_{oo} = \frac{g}{8} \left( \frac{a}{\alpha(g_{cell}/L_{cell})L_{cell}} \right)^2 ,
\]

and

\[
\alpha(\gamma) = 1 - 0.4648\sqrt{\gamma} - 0.0704\gamma .
\]

(19)

(20)

and \(Z_o = 120\pi [\Omega]\).

Figure 6: The geometry of a C-band accelerating structure: \(a = 7.6\) mm, \(b = 21.2\) mm, \(g = 15.2\) mm, and \(L_{cell} = 19.682\) mm.

With structure parameters of a C-band unit, \(s_{oo} = 982\) \(\mu\)m, and \(W(0) = 0.623\) kV/\(\mu\)C/m. This paper uses the beam parameters given in the “Optimization of Parameters” of the 6-GeV SCSS CDR [1] where the accelerating gradient is 34.0 MV/m after the second bunch compressor BC2. The compressed bunch length \(s\) after BC2 is 24 \(\mu\)m. It is so short that the wakefield over the bunch is close to \(W(0)\), then the longitudinal energy spread of a bunch due to the short-range wakefield with a bunch charge \(Q\) over a total accelerator length \(L\) is approximately
There are 25 accelerator modules in the C-band main linac of the SCSS 6-GeV machine. Each module has 4 units of 1.8-m long structures. Therefore, total accelerating length is 180 and the longitudinal energy spread becomes -44.6 MeV. Considering the effect of finite bunch length, it is -40.1 MeV. The net change of head-tail energy spread through the C-band linac obtained by ELEGANT code is -41.3 MeV as shown in Figure 7.

\[ \Delta W = -eQW(0)L \]  \hspace{1cm} (21)

There is energy gain difference \( \Delta W_{RF} \) between the head and tail electron of a bunch, which is caused by the phase difference of RF fields along the bunch.

\[ \Delta W_{RF} = |\Delta eE_{L} \cos \phi| \approx -eE_{L} \sin \phi_{o} \Delta \phi \]  \hspace{1cm} (22)

where \( e \) is electronic charge, \( E \) is peak accelerating field. In case of 6-GeV SCSS design, \( eE_{L} = 5200 \text{ MeV} \), and \( \Delta \phi = 0.164^\circ \) for \( s = 24 \mu\text{m} \). Therefore, the energy spread due to the RF fields becomes -2.33 MeV for the off-crest phase of \( \phi_{o} = +9^\circ \). This energy spread is 5.81% of the one caused by wakefield. The ELEGANT simulation shows the difference of -1.53 MeV as shown in Figure 8.

The slice parameters such as slice emittance and peak current are dominant governing parameters of SASE process. We have to keep the slice parameters as good as possible. It is confirmed that there is no appreciable degradation of slice parameters for off-crest acceleration as shown in Figure 9.

**SUMMARY AND DISCUSSION**

The SCSS XFEL is a challenging machine that requires extremely stable RF system. Therefore, it is critical issue to realize stable RF system for both phase and amplitude to provide stable XFEL. The phase-dependent stability characteristics are in detail analyzed and examined. It is confirmed that the off-crest phase of around +9° provides better stability in case of the C-band main linac for SCSS XFEL if phase jitter is small. The low-level RF control has to provide better stability than the one of a klystron modulator for this scheme to be effective. The reduction of beam energy due to off-crest acceleration is about 1%.

The additional longitudinal energy spread due to the RF curvature is about 5% of the one caused by the longitudinal wake field and there is no appreciable degradation of the slice parameters.

The RF unit for a velocity buncher or a bunch compressor is more sensitive to the RF fluctuation because of the off-crest operation to provide a necessary energy chirp. For example, the energy fluctuation is more than 4 times higher than the crest one at the typical off crest phase of -40° for SCSS accelerating units between BC1 and BC2.

**REFERENCES**