REVIEW OF FEEDBACK SYSTEMS

K. Balewski, DESY, Hamburg, Germany

Abstract

The current strategy to achieve high luminosity for high energy physics experiments as well as high brilliance in synchrotron radiation sources is to store huge currents distributed in many bunches in circular machines. This approach has the disadvantage that the performance of these machines is most likely to be limited by coupled bunch instabilities. To circumvent this problem strong feedback systems are necessary to damp collective instabilities. The principle requirements for these damper systems will be reviewed and the performance of existing systems will be presented. Although single bunch instabilities are not considered a problem for particle factories there is still an interest to combat single bunch instabilities, for example the transverse mode coupling instability. This may be of interest for synchrotron radiation sources operating with only a few intense bunches to allow time resolved measurements. Theoretical predictions to control this instability will be compared with observations.

1 INTRODUCTION

High luminosity in colliders or high brilliance in synchrotron sources requires the storage of intense particle beams. Since single bunch currents are either limited by the beam beam interaction in colliders or by single bunch effects such as the Toushek effect in light sources to modest values it is necessary to distribute the high current over many bunches. The interaction of these bunches with the rf cavities or other high Q resonators in the machine would limit the current to rather low values. To overcome this problem two measures are usually taken: first the impedance of the parasitic cavity modes is reduced substantially and second feedback systems are applied to stabilise the beam.


One reason why feedback systems operate so successfully is certainly that the mechanism of coupled instabilities is well understood. So in the first part the essentials results of coupled bunch instability theory are presented. The impact of this theory on the design and the limitations of multi bunch feedback systems will be given.

An explanation why it may be more difficult to fight single bunch instabilities like the transverse mode coupling instability will be discussed. The next section will cover feedback systems to suppress coupled bunch instabilities. The last section deals with feedback systems to fight the transverse mode coupling instability.

2 INSTABILITIES

2.1 Coupled bunch instabilities

The general theory of bunched beam instabilities was formulated by Sacherer [6,7] more than 20 years ago. In the following only the important facts are presented and for further details the reader should consult the original papers or one of the review papers [8].

The interaction of the beam with the metallic surrounding is described in terms of coherent bunch modes which are denoted by certain mode numbers n, m. For the case of M equally spaced bunches the first mode number n is the coupled bunch mode defining the phase difference \( \Delta \phi \) between adjacent bunches:

\[
\Delta \phi = \frac{2 \cdot \pi \cdot n}{M}, \quad 0 \leq n \leq M - 1
\]

The second mode number m describes the phase relation between particles within a bunch. For example in the longitudinal plane m = 0 denotes dipole and m = 1 quadrupole oscillations etc.

If one considers only dipole oscillations in the transverse plane which are modulated longitudinally then m denotes the so-called head tail modes: m = 0 rigid dipole mode, m = ±1 first head tail mode etc.

For completeness we mention that there exists a third mode number describing so-called radial modes which we will not consider further.

The frequencies of the coherent modes are given by:

(1) Longitudinal modes:

\[
f_{\text{long}} = (n + p \cdot M) \cdot f_0 + m \cdot f_s, \quad -\infty \leq p \leq \infty
\]

(2) Transverse modes:

\[
f_{\text{trans}} = (n + p \cdot M) \cdot f_0 + f_\beta + m \cdot f_s
\]

where \( f_0, f_\beta, f_s \) is the revolution-, betatron- and synchrotron frequency respectively.

Negative frequencies are only introduced for mathematical convenience. If one considers only positive frequencies all coherent modes occur twice in the frequency interval \( [pM f_\sigma, (p+1)M f_\sigma] \) and one finds all modes in the interval

\[
\left[ p \cdot \frac{M}{2} \cdot f_\sigma, (p+1) \cdot \frac{M}{2} \cdot f_0 \right]
\]

which determines the band width of the feedback system.
The interaction of the beam with its environment leads to a complex frequency shift of the coherent modes which is given for example for the transverse plane by:

\[ \Delta \omega_{\text{coh}} = -i \frac{\beta * l_0 * \omega_0 * Z_{\text{eff}}}{4\pi * E / e} \]

\[ Z_{\text{eff}} = \frac{2\pi}{C} \sum_{\mu=-\infty}^{\infty} Z_{\mu}^\perp (\mathcal{O})^* | h(\mathcal{O}) |^2 \]

\[ \mathcal{O} = (\mu * N + k) * \omega_0 + \omega_\beta + m \cdot \omega_s \]

The imaginary part of the frequency shift determines the damping or growth rate of a particular mode. For both the longitudinal and transverse plane this rate is proportional to the resistive part of the impedance. As long as radiation or Landau damping of the beam exceeds the growth rates the beam remains stable.

There are two requirements that are essential for the successful application of feedback to stabilise a beam.

1. The growth rates of those internal modes that are not affected by feedback must not exceed radiation or Landau damping at the design current.
2. The coherent frequency shifts must be smaller than the synchrotron frequency.

The first requirement is obvious but the second may need a comment.

The mode by mode stability analysis is only valid if the coherent frequency shifts are smaller than the synchrotron frequency. If the second requirement is fulfilled then any motion of bunches with a bunch spacing \( \Delta T \) can be described as a superposition of the above defined coherent modes if:

\[ \Delta T \geq T_0 / M \]

In this case it can be shown theoretically that all coupled bunch modes can be damped by a suitable feedback system [9]. These two requirements define an upper bound for the size of the parasitic cavity impedance. In case of the two B factories damping of the parasitic cavity modes is absolutely necessary to fulfil both requirements.

If the complex frequency shift is bigger than the synchrotron frequency, coupling of internal modes will occur for the coupled bunch [10] as well as for the single bunch case [11], so that a proper functioning of the feedback can no longer be guaranteed.

2.2 Mode coupling instability

Mode coupling may occur in both the longitudinal and transverse plane. We will consider only transverse mode coupling [11,8] since feedback systems have been built with the aim of raising its threshold.

This instability limits the single bunch current in many electron machines but has never been observed in proton machines. Since the vertical impedance is usually bigger than the horizontal impedance the instability occurs in the vertical plane first.

As was already said in the previous section the coherent beam modes are coupled if the strength of the interaction of the beam with its environment is of the order of the synchrotron frequency. It can be shown that in this case the coherent frequencies are the eigenvalues of the coupling matrix. In the simplest case when only two modes are coupled the coherent frequencies are the eigenvalues of a two by two matrix. Typically the two lowest order head tail modes \( (m=0, m=-1) \) couple first and their eigenfrequencies are given by [11]:

\[ \omega_m = \omega_\beta + m \cdot \omega_s + \Delta \omega_m \]

\[ \Delta \omega_m = \frac{1}{4} \cdot (\omega_s + M_{00} + M_{11}) \pm \sqrt{(\omega_s + M_{00} - M_{11})^2 - 4 \cdot M_{12}^2} \]

The exact expression of the coupling elements can be found in the literature. It is only mentioned here that they depend linearly on current and on the coupling impedance.

Fig. 1 shows the current dependence of the coherent frequencies. One can clearly see that an instability occurs when the modes \( m=0 \) and \( m=-1 \) merge. So in contrast to coupled bunch instabilities an exponentially growing mode appears above a certain threshold. This threshold is determined by the synchrotron frequency so the synchrotron frequency forms some kind of instability barrier. This is also illustrated by the fact that the threshold current depends linearly on the synchrotron frequency. The growth rate normally exceeds by far the internal damping given either by radiation or Landau damping.

Since the mechanism of coupled bunch instabilities and mode coupling instability is different this may imply that also a different feedback concept is needed to combat mode coupling. This problem is discussed in section 3.2.

![Figure 1: Transverse mode coupling: current dependence of coherent modes](image-url)
3 FEEDBACK SYSTEMS

3.1 Multi-bunch feedback

A feedback system consists mainly of three parts:
A detection system to measure beam oscillations and to provide the system with an error signal.
A signal processing unit to derive a correction signal
A broad band amplifier and a beam deflector to act with a kick on the beam.

The signal processing unit can be accomplished in either the frequency (mode by mode feedback) or in the time domain (bunch by bunch feedback). The bandwidth required by the feedback system is of course independent of its realisation and is determined by the minimal bunch spacing.

In a mode by mode feedback each mode is identified with the help of a special narrow band filter centred around one of the revolution harmonics. Each mode is then processed individually leading to a feedback that consists of many narrow band systems running in parallel. In the case of HERA and the B factories the number of potentially unstable modes is very high and the amount of electronics for a mode by mode feedback would be extremely large. So a mode by mode feedback is the appropriate choice if only a few coupled bunch modes have to be damped. For example such a system has been in operation in the PS booster for more than 20 years [12].

In a bunch by bunch feedback each bunch is treated individually. It has been shown theoretically that is possible to stabilise all coupled bunch modes by damping each bunch individually [9].

Since very fast digital signal processing electronics like micro-computers have been developed and are commercially available the signal processing unit of many bunch by bunch feedback systems has been constructed using digital electronics [13,14,15,16,17,18]. The digital approach has the advantage that economical and flexible systems can be built and the consideration in the following will be restricted to such systems. In an ideal realisation of a digital bunch by bunch feedback all bunches are treated independently but share the same electronics.

The transverse electron feedback system of HERA (shown in fig. 2) will serve as a prototype to discuss the essential features and limitations of such systems. We will further limit the consideration to feedback systems acting on transverse and longitudinal dipole oscillations only. This is not a principle limitation but is done for the sake of simplicity.

The pick-up in connection with the detector electronics delivers a position signal of each bunch. Any DC component in this signal has to be rejected for two reasons. First of all the dynamic range of the signal processing unit is decreased and secondly any static signal component would result in a constant kick to the beam thereby wasting power. These DC components are the result of orbit offsets at the pick-up or given by the different synchronous phases of the bunches or asymmetries in the detector electronics etc.. In the HERA longitudinal feedback system for example these unwanted components are eliminated by special feedback loops [19]. The pure transverse and longitudinal dipole oscillation amplitudes are then digitised using fast 8 bit analog to digital converters (ADC). Current analog to digital converters are able to convert every 2 ns an analog signal into an 8 bit word. Detector systems with such ADC’s have been tested successfully for the PEP B
factory feedback [20] so there are now detectors available which can deliver every 2 ns a new digitised value for the oscillation amplitude. If the bunch spacing is larger than 2 ns 12 or even 14 bit ADC’s can be used which may be important for feedback systems in proton machines to reduce digitising noise. For that reason a 12 bit ADC is used in the HERA transverse proton feedback.

The digitised signal is then passed to the digital signal processing unit. To achieve damping an overall phase shift of 90° is necessary. Such a phase shift can be realised with a so-called n tap FIR (finite impulse response) filter:

\[ g_{\mu} = \sum_{k=0}^{n} T_k \cdot f_{\mu-k} \]

where \( f_{\mu} \) and \( g_{\mu} \) are the input and output signal respectively and \( T_k \) are the filter coefficients. For example with a three tap filter a phase shifter with arbitrary phase shift can be realised. The correction signal is then properly delayed so that it is applied to the corresponding bunch. If the electronics is shared by all bunches the arithmetic operations must be carried out within the bunch spacing. This requirement is hard to fulfil for a bunch spacing of 2 ns. But there are a few measures to circumvent this problem:

1. To speed the signal processing the FIR filter algorithm can be simplified. For example a 2 tap filter may be used as is done for instance in the KEK B feedback [18]. Their filter design requires only a subtraction which can be carried out rapidly. Usually it is also necessary to run filters in parallel so the error signal of some bunches is grouped with the help of fast multiplexers and the data of a group of bunches is then processed by one signal processor.

2. For a transverse feedback the signal of two pick-ups approximately 90° in betatron phase apart can be combined to achieve a position signal which is shifted by an arbitrary phase. This method has been used in the transverse feedback of the SPS [21] and LEP [22]. The correction signal is attained on the analog level. So after digitising, the signal has only to be properly delayed which can be done at a rate of 500 MHz. This method is applied in the design of the transverse feedback for PEP II as well as that of the KEK B factory [18]. A prototype of the PEP B factory transverse system has been successfully tested in the ALS [23].

3. In the longitudinal plane one can make use of the fact that the synchrotron frequency is much smaller than the revolution frequency which is especially true for proton machines. Because of this inherent oversampling it is sufficient to sample every bunch only every n-th turn where n clearly depends on the synchrotron frequency. The output signal is only renewed every n-th turn so that the arithmetic burden of the signal processor is reduce by \( 1/n \). This procedure is called down sampling and is used in the design of the longitudinal feedback of the PEP B factory [17]. This method has been successfully tested in a prototype in the ALS [20].

After digital processing the data is converted into an analog signal using fast digital to analog converters. After amplification of the signal the beam is then deflected by a wide band kicker. There are various types of kickers [24] for example ferrite loaded [25] in the transverse and special cavities and so-called longitudinal kickers [26] in the longitudinal plane.

The required power depends quadratically on the gain i.e. on the growth rate that has to be balanced and on the initial amplitude. In section 2.1 two reasons were given why damping of the parasitic cavity modes is necessary. If the parasitic modes of the B factory cavities were not heavily damped an enormous broad band power would be required. So damping of the HOM is absolutely essential.

3.2 Transverse single bunch feedback

Approximately 15 years ago it was proposed to increase the threshold current of transverse mode coupling with the help of a feedback system [27]. This feedback system should either keep the coherent frequency of the dipole mode constant or shift it to higher values. A feedback is termed reactive if it causes only a coherent frequency shift whereas a system causing pure damping is called resistive. The effect of such feedback was investigated using a two particle model and in the framework of the Vlasov equation [27,28]. The first results promised a significant increase in threshold current by a factor between two and four.

This result however was derived making some simplifying assumptions. First the location of the elements causing the instability (cavities, kicker tanks, etc.) and of the feedback elements was not taken into account. In the analysis using the Vlasov equation it was assumed that only the mode m=0 is affected by the feedback which is not true because of the mode coupling.

After removing the above mentioned assumptions the promising results could not be confirmed in a few particle simulation [29] and in an analytical study of the reactive feedback [30]. On the contrary it was found that reactive feedback improves the threshold only under very restrictive conditions.

If the whole impedance of a machine is lumped in an artificial cavity and the betatron phase advance between this cavity and the kicker is a multiple of \( \pi \) then the feedback systems is purely reactive and the promised increase in threshold current is achieved.

Since this requirement is not usually fulfilled the feedback system has a resistive as well as a reactive component. The size of the resistive component depends on the size of the coupling elements and on the strength of the feedback. Because of the mode coupling the
feedback drives some of the higher order head tail modes unstable. As long as these growth rates are compensated by natural damping (Landau or radiation damping) the beam remains stable. The feedback strength is of the order of the synchrotron tune to be comparable with the strength of the instability. So in a machine with a high synchrotron tune the resistive component of the feedback is quite high and the negative effect of the feedback may be so strong that no improvement is possible. That is the reason why the transverse feedback has not helped to increase the mode coupling threshold in LEP.

However in machines with a small synchrotron tune the natural damping may be high enough to compensate the feedback induced instabilities and an increase in threshold current is possible. That may explain why in some machines with a small synchrotron tune a positive effect of a transverse feedback has been observed [31,32,33]. The improvement with resistive feedback was even higher compared to reactive feedback.

Theoretically a positive effect of a reactive feedback could be demonstrated for small synchrotron tunes [28] but the success of a resistive feedback has not yet been convincingly demonstrated. Such an investigation is rather complicated because a detailed knowledge of the coupling impedance and of Landau or radiation damping of higher order head tail modes is needed.

REFERENCES

[23] W. Barry et al., “Commissioning of the ALS Transverse Coupled Bunch feedback system”, HEAC and PAC ’95, Dallas,1995