

# SOLUTION TO THE EINSTEIN AND POINCARÉ PARADOX OF SUPERLUMINAL ADDITION OF VELOCITIES

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## Abstract

A new set of space and time quantized transformations is presented, and for any quantization order it gives invariant transformations for which the light velocity is constant. It gives linear functions with real solutions even above the light velocity value. In the low velocity range it may be approximated to the Galilean transformations and for high order quantization it gives conformity to Lorentz transformations. Its direct application to the composition of high velocities theory showed new results compared to the addition theorem of Einstein and Poincaré (E-P). When the relative velocities are in the same range of values a close fitness to the E-P's theorem is demonstrated up to the  $c$  value. Above  $c$  a strong divergence from the E-P's result will be showed. We concluded that these results give a new insight to the concept of advanced accelerators and to the solution of some astrophysics problems.

## 1 INTRODUCTION TO THE EINSTEIN-POINCARÉ PARADOX

Starting from the proposal of the contraction of the space in the direction of the movement, presented by Lorentz to justify the constancy of the light velocity, Einstein[1] and Poincaré[2] (E-P), independently, deduced the well-known formula of addition of velocities:

$$V = \frac{u + v}{1 + \frac{u v}{c^2}} \quad (1)$$

where  $V$  is the composition velocity relative to a space and time system  $S$ , of a velocity  $u$  measured relative to another system  $S'$ , when the system  $S'$  is moving uniformly relative to  $S$  with velocity  $v$ . The light velocity is denoted by  $c$ .

As a result of this kinematics the combination of velocities less than  $c$  always results in velocities less than  $c$ , or subluminal velocities. Also, this addition equation cannot be applied for the combination of relative velocities larger than  $c$  (superluminals) because according to the proposal of Lorentz the values of the space and time coordinates would be imaginary. Even if this equation has its algebraic results extended for

superluminal velocities, the sum can result in subluminal velocities or in the limit to the zero velocity. This example is illustrated in Fig.1 for the sum of two identical relative velocities. Therefore in the E-P model the velocity of the light is not just constant, it also plays the role of the maximum limit to the velocities.

The objective of this work is, without violating the principle of the constancy of the light velocity, to demonstrate that the solution proposed by E-P is not unique. We introduced a generalized and finite solution which can be applied even for the composition of superluminal velocities. If at least one of the relative velocities is superluminal (or luminal) the result will never be subluminal (see Fig.1). In the subluminal range the numeric results can coincide with the results of the E-P equation.

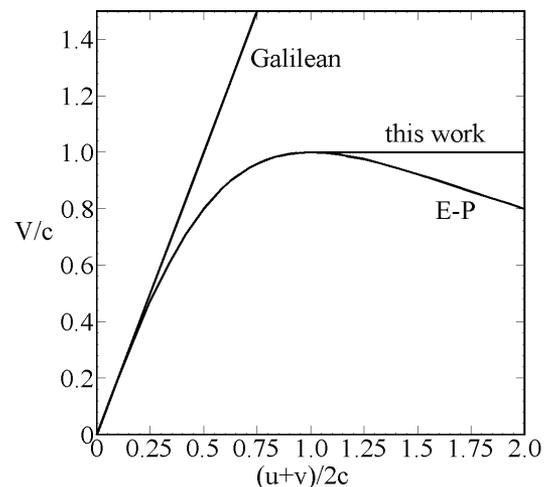


Figure 1: Comparison of the Galilean, E-P, and of this work for the addition of identical velocities ( $u = v$ ).

## 2 BEYOND THE LORENTZ TRANSFORMATIONS

In this section we will demonstrate that the Lorentz transformations are not the only transformations in which the the light velocity can be maintained constant. Starting from there we will show why the Lorentz transformations cannot be applied to superluminal velocities. Finally, we

propose a more appropriate group of transformations, applicable for any velocity.

### 2.1 Demonstration that an infinite number of transformation functions exist in which the light velocity is constant.

Let the origins of  $S$  and  $S'$  be coincident at  $t = t' = 0$ , and suppose that  $S'$  is moving with velocity  $v$  relative to  $S$  along the  $x$  direction. The coordinate  $x'$  of  $S'$  can be related to coordinate  $x$  of  $S$  system by a general transformation such as

$$x \longrightarrow x' = \beta_1 x - \beta_2 vt \quad (2)$$

where  $\beta_1$  and  $\beta_2$  are functions to be determined. Owing to the symmetry with respect to the direction of movement, the inverse transformation must also exist:

$$x' \longrightarrow x = \beta_1 x' + \beta_2 vt' \quad (3)$$

Then, by simple algebraic manipulation of eqn. (2) and eqn. (3) it can be shown that the time is also transformed according to:

$$t \longrightarrow t' = \beta_1 t + \frac{(1 - \beta_1^2) x}{\beta_2 v} \quad (4)$$

Due to the hypothesis of the constancy of the light velocity the position of a single wave front of light emitted at the origin of both systems when the spaces  $S$  and  $S'$  are coincident, will be described by the corresponding equations,

$$x^2 = (ct)^2 \quad (5)$$

and

$$(x')^2 = (ct')^2 \quad (6)$$

Introducing eqns (2) and (3) into eqns (5) and (6), we have,

$$\beta_1^2 = 1 + \frac{v^2}{c^2} \beta_2^2 \quad (7)$$

Therefore any functions  $\beta_1$  and  $\beta_2$  that obey the above relationship produce space and time transformations in which the velocity of the light is constant, as we wanted to demonstrate. This relationship will be called here as the *hidden equation*.

### 2.2 Why the Lorentz transformation cannot be applied for superluminal velocities.

In the E-P theory there is no distinction among  $\beta_1$  and  $\beta_2$  functions. With this condition, the application of the *hidden equation* gives the Lorentz function ( $\beta_L$ ).

$$\beta_L = \beta_1 = \beta_2 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad (8)$$

In Fig.2 we have an illustration of the hyperbolae given by the *hidden equation*. The Lorentz solutions are determined by the crossing points of these hyperbolae

with the straight line  $\beta_1 = \beta_2$ . The asymptotes of the hyperbolae are given by:

$$|\beta_1| = \frac{v}{c} |\beta_2| \quad (9)$$

As the straight line of Fig.2 is also the asymptote of the hyperbola for the value of  $v = c$ , the Lorentz solution for  $v = c$  is an Euclidian absurd since no hyperbola meets its asymptote. For assuming this, the Lorentz transformations give only space-time continuum solutions. It is also evident that real Lorentz solutions for  $v \geq c$  don't exist.

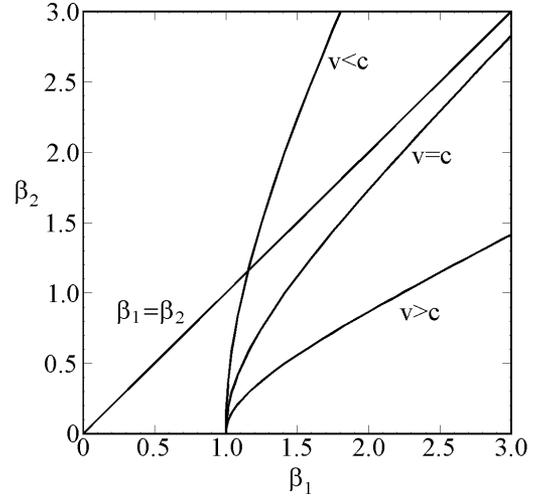


Figure 2: Geometric representation of the *hidden equation* in the Lorentz solution.

### 2.3 Proposal of a new transformation.

Starting from the *hidden equation* we can define pairs of finite functions given by

$$\left(\beta_1^k\right) = \sqrt{1 + \frac{v^2}{c^2} \left(\beta_2^k\right)^2} \quad (10)$$

and

$$\beta_2^{k-1} = \beta_1^k \quad (11)$$

where  $k = 1, 2, \dots$  is the order number of the functions.

The simplest group is obtained being taken  $\beta_2^1 = 1$ .

Applying this result in eqns. (3) and (4) gives

$$x \longrightarrow x' = \beta_B x - \sqrt{\beta_B^2 - 1} ct \quad (12)$$

and

$$t \longrightarrow t' = \beta_B t - \sqrt{\beta_B^2 - 1} \frac{x}{c} \quad (13)$$

with

$$\beta_B = \sqrt{\sum_{n=0}^k \left(\frac{v}{c}\right)^{2n}} = \sqrt{\frac{\left(\frac{v}{c}\right)^{2(k+1)} - 1}{\left(\frac{v}{c}\right)^2 - 1}} \quad (14)$$

These transformations maintain the principle of the constancy of the light velocity for any value of the order  $k$ , and are simple transformations. One can easily observe from eqn. (14) that for subluminal velocities the factor  $\beta_B$  approaches the Lorentz factor  $\beta_L$ . However,  $\beta_B$  can be applied for any velocity  $-\infty < v < +\infty$ .

### 3 GENERAL SOLUTION TO THE COMPOSITION OF VELOCITIES

Starting from the eqns. (12) and (13) we can derive the corresponding infinitesimal equations for the transformations of the space and of the time:

$$\Delta x' = \beta_B \Delta x - c \Delta t \sqrt{\beta_B^2 - 1} \quad (15)$$

$$\Delta t' = \beta_B \Delta t - \frac{\Delta x}{c} \sqrt{\beta_B^2 - 1} \quad (16)$$

and obtain the composition velocity of  $v$  and  $u$  by

$$v = \frac{\Delta x}{\Delta t} = \left( \frac{\frac{u}{c} \beta_B + \sqrt{\beta_B^2 - 1}}{\beta_B + \frac{u}{c} \sqrt{\beta_B^2 - 1}} \right) c \quad (17)$$

This equation will be denoted here as the *general composition* for velocities. As the proposed space and time transformation doesn't limit the values of  $v$  or  $u$ , the *general equation of composition of velocities* is unrestricted, that is, it can be applied for the sum of subluminals or superluminals velocities, respectively. The three-dimensional plot of  $V$  in function of  $u$  and  $v$  is presented in Fig. 3. We can verify the existence of superluminal solutions and also a break of the symmetry between  $u$  and  $v$ . Thus, it is possible to differentiate the movement of the system  $S'$  relative to  $S$  (velocity  $v$ ) from the movement of the points relative to  $S'$  (velocity  $u$ ).

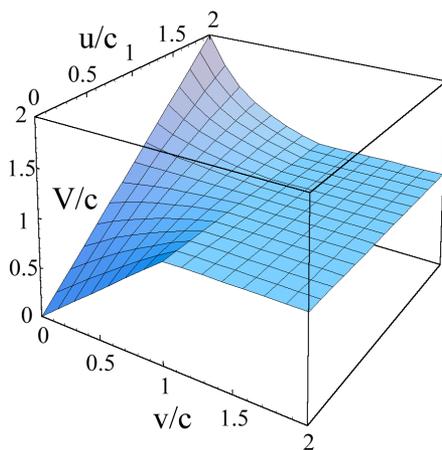


Figure 3: Three-dimensional plot of the composed velocity  $V$  (axis  $z$ ), following the present work.

The resulting composition velocity of an almost stationary  $S'$  system ( $v \approx 0$ ) will be  $V \approx u$ , that can be subluminal or superluminal. If the velocity of the system  $S'$  goes to luminal or superluminal values, the

composition velocity will always be luminal, that is  $V = c$ , independently of the value of  $u$ . In the subluminal range, the present results are close to the E-P theory and the symmetry of velocities exists.

### 4 ASTROPHYSICS OBSERVATIONS

Through the Energetic Gamma Ray Experiment Telescope (EGRET) and of the Very Long Baseline Interferometry (VLBI) 15 Gamma-Ray Sources (GRS) were identified and confirmed with apparent superluminal movements[3]. Its properties include the superluminal movement of its radiation center or of the halo of its expelled jets. They also possess evident asymmetry in its radiation structure. In spite of the orthodox proposals of interpretation, the physical results are still not satisfactory. Sources as the Gamma-Ray blazar NRAO 530 presents jets with velocities above  $26c$ [3]. The theory presented in this work justifies the existence of the superluminal movement of these sources without entering in conflict with the Michelson-Morley results. Furthermore, it justifies the superluminal asymmetry.

### 5 SUMMARY AND CONCLUSIONS

We demonstrated the existence of simple space and time transformations that take into account the constancy of the light velocity and can be applied to superluminal velocities. The proposed transformations, for its own solution, don't need the hypothesis of a continuous space-time and therefore don't contradict the quantum theory. The function  $\beta_B$  shows that high order effects in relation to velocity should be present in the particle accelerators. These effects can be useful for advanced concepts of accelerators. The transformations applied to the E-P paradox of the composition of velocities shown that superluminal velocities are possible, giving an explanation to the cosmological superluminal velocities presented by the space Gama-Ray sources.

### 6 ACKNOWLEDGEMENTS

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