# ESTIMATION OF THE PARTICLE DENSITY IN TRANSVERSE PHASE SPACE USING A MULTI-WIRE PROFILE MONITOR

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*Abstract*

A convenient method for estimating the particle density in transverse phase space using a multi-wire profile monitor or equivalent monitor has been formulated by assuming that the beam particles distribute uniformly along the circumference of an ellipse defined by the twiss parameters. Using this method, the particle density has been estimated. This paper describes the formulation of this method and gives some examples.

## 1 INTRODUCTION

In a low-energy beam-transport line, the particle density in transverse phase space is usually measured by a "destructive" method. In the 40-MeV beam transport line of KEK-PS, the phase-space distribution is measured using the combination of a slit and a Faraday cup[1]. On the other hand, such a "destructive" method cannot be used in a high-energy beam-transport line due to a radiation hazard. If a multi-wire profile monitor or equivalent monitors are installed so as to sufficiently cover the betatron phase of 2π, the phase-space distribution can be estimated using a method such as computer tomography[2]. In almost all transport lines, however, profile monitors are not installed sufficiently to estimate the phase-space distribution.

A beam extracted from a synchrotron is bounded by an ellipse in transverse phase space. Also, the beam particles are expected to distribute uniformly along the circumference of the ellipse. In this case, the phase-space distribution can be simply estimated using a geometrical relation between the phase-space ellipse and the signal of a profile monitor.

In the following sections, the formulation of this method is described and some examples measured in the KEK-PS beam-transport line are demonstrated.

## 2 FORMULATION

As mentioned above, beam particles cover an ellipse in transverse phase space due to betatron oscillation in a synchrotron. Therefore, the beam particles extracted from the synchrotron are bounded by an ellipse defined by the twiss parameters in the transport line. Under this condition, the signal of a multi-wire profile monitor, which is installed at a dispersion-free point, can be simply related to the particle density in phase space. Figure 1 shows the relation between the profile signal and the phase-space ellipse.

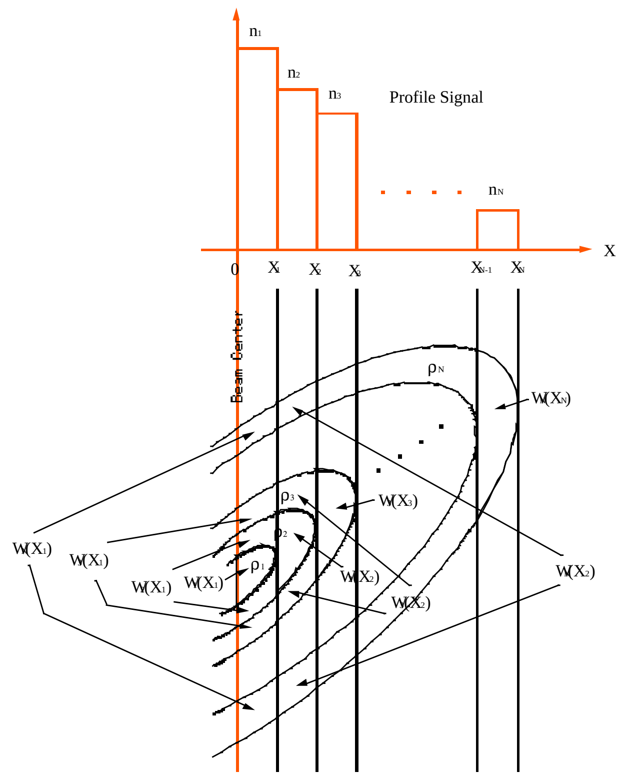


Figure 1: Relation between the profile signal and the phase-space ellipse.

The ellipse is divided into N-rings corresponding to N-bins of the profile monitor. Since the particle density in each ring is expected to be constant, the profile signal is given by

$$\begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ . \\ . \\ n_N \end{pmatrix} = \begin{pmatrix} W_1(X_1) & W_2(X_1) & W_3(X_1) & . & . & W_N(X_1) \\ 0 & W_2(X_2) & W_3(X_2) & . & . & W_N(X_2) \\ 0 & 0 & W_3(X_3) & . & . & W_N(X_3) \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & W_N(X_N) \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \\ . \\ . \\ \rho_N \end{pmatrix} \quad (1)$$

where $n_i$ is the profile signal in the i-th bin and $\rho_i$ is the particle density of the i-th ring of the phase-space ellipse. An element of the matrix, $W_j(X_i)$, is the weight of the j-th ring to the i-th profile bin, and is given by the following formula:

for $i > j$ ($i, j = 1 \sim N$)

$$W_j(X_i) = 0\,; \tag{2}$$

otherwise

$$W_j(X_i) = \frac{1}{\beta} X_j^{\,2}\{\sin^{-1}(\frac{X_i}{X_j}) - \sin^{-1}(\frac{X_{i-1}}{X_j}) + \frac{X_i}{X_j}\sqrt{1-\left(\frac{X_i}{X_j}\right)^2} - \frac{X_{i-1}}{X_j}\sqrt{1-\left(\frac{X_{i-1}}{X_j}\right)^2}\} \quad . \tag{3}$$

Here, $\beta$ is the twiss parameter and $X_0 = 0$.

While the particle density can be obtained by solving equation (1) directly, we solve this problem by taking account of the profile signal error. Using the error ($\sigma_i$) for i-th bin of the profile signal, $\chi^2$ is defined by

$$\chi^2 = \frac{1}{\sigma_1^{\,2}}\{n_1 - (W_1(X_1)\cdot\rho_1 + W_2(X_1)\cdot\rho_2 + \ldots + W_N(X_1)\cdot\rho_N)\}^2 + \frac{1}{\sigma_2^{\,2}}\{n_2 - (W_2(X_2)\cdot\rho_2 + W_3(X_2)\cdot\rho_3 + \ldots + W_N(X_2)\cdot\rho_N)\}^2 + \ldots + \frac{1}{\sigma_N^{\,2}}\{n_N - W_N(X_N)\cdot\rho_N\}^2 \tag{4}$$

Then, in terms of the least-squares method, we obtain the following simultaneous equation:

$$B_j = \sum_{k=1}^{N} A_{jk}\rho_k \quad , \tag{5}$$

where

$$A_{jk} = \sum_{i=1}^{k} \frac{W_j(X_i)\cdot W_k(X_i)}{\sigma_i^{\,2}} \tag{6}$$

and

$$B_j = \sum_{i=1}^{j} \frac{n_i \cdot W_j(X_i)}{\sigma_i^{\,2}} \quad . \tag{7}$$

Ultimately, the particle density can be obtained by solving equation (5).

## 3 EXAMPLE

This example demonstrates three kinds of horizontal phase-space distributions observed in the 500-MeV beam transport line, which transfers the extracted beam from the KEK-PS booster to the 12-GeV PS. The plan of the transport line is shown in Figure 2. The twiss parameters ($\beta$ and $\alpha$) and dispersion ($\eta$) at the observing point are $\beta$=5.89 m, $\alpha$=2.71 and $\eta$=-0.52 m, respectively. The contribution of the dispersion to the beam size was small, since the momentum spread was less than 0.3%.

Figure 3 shows the beam profiles observed in the transport line. Figure 3-1 shows the beam profile obtained in the case of the usual machine operation using the $H^-$ injection scheme. Figure 3-2 and 3-3 show profiles obtained in a feasible study of “beam painting” in the booster at the injection timing. In this study, two kinds of ramping patterns for excitation of the painting magnets were tried, i.e. a fast ramping pattern and a slow pattern, which correspond to Figure 3-2 and 3-3, respectively.

The profile data were analyzed by the following procedure. First, the profile data were fit with a Gaussian and the linear background, as shown in the figures by the broken line. Then, the profile data were corrected by subtracting the background from the raw data. The particle density in the phase space was estimated using these data. Such distributions are shown in Figure 4. Here, open circles, solid circles and crosses denote the particle density in the usual case, the slow-ramping case and the fast-ramping case, respectively. As shown, each distribution can be clearly distinguished and is consistent with the calculated phase-space distribution[3].

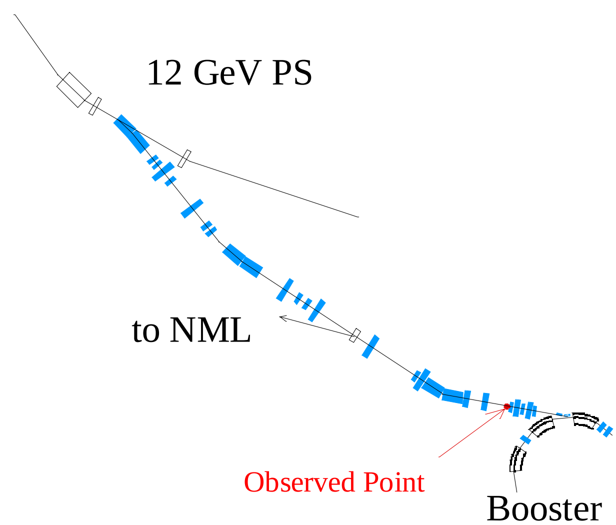


Figure 2: Plan of the KEK-PS 500-MeV beam-transport line.

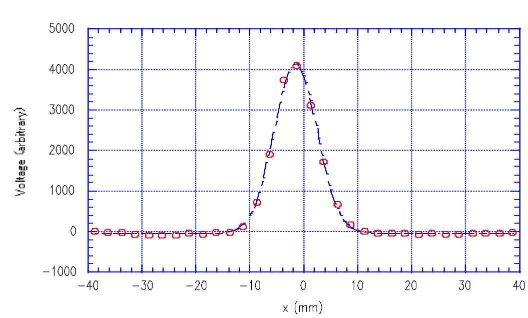


Figure 3-1: Profile signal observed under normal operation.

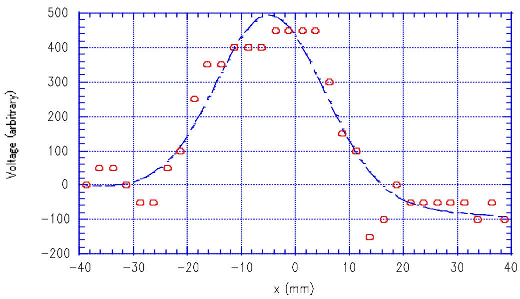


Figure 3-2: Profile signal observed in the fast-ramping case.

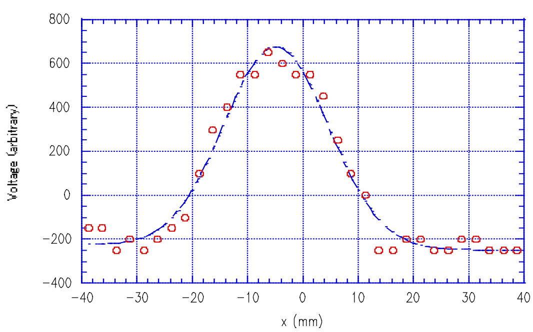


Figure 3-3: Profile signal observed in the slow-ramping case.

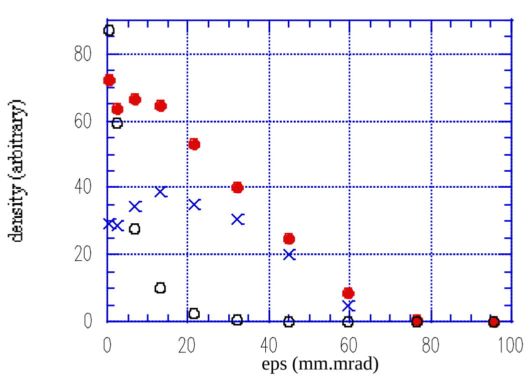


Figure 4: Phase-space density in various injection schemes.

## 4 SUMMARY

A convenient method for estimating the particle density in transverse phase space using a multi-wire profile monitor or equivalent monitor has been formulated, and applied to an estimation of the particle-density distributions in three cases of injection schemes in the KEK-PS booster. The results are consistent with the calculated phase-space distributions. Therefore, it is found that this method can be available for estimating the phase-space distribution of the extracted beam from a synchrotron.

On the other hand, it is interesting to observe a phase space distribution in a synchrotron. Fortunately, a flying wire monitor is installed in the KEK 12-GeV PS. This profile monitor is equivalent to the multi-wire profile monitor. Therefore, we are now preparing to apply this method to the data of the flying wire monitor.

## ACKNOWLEDGEMENT


The authors would like to thank to the KEK-PS group for supporting their beam study and operating machines in good condition.


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