Abstract

The purpose of the ATF damping ring is the development of technologies for producing a low emittance beam required in future linear colliders such as JLC. The lifetime of the damping ring is very short (typically a few minutes). It is limited by elastic beam-gas scattering along with a small dynamic aperture, and by single intra-beam scattering (Touschek effect)[1]. The Touschek lifetime strongly depends upon the charge density of the beam, especially, the size of the vertical emittance. In this paper, we report the results of beam lifetime measurements in the ATF damping ring and the estimation of the vertical emittance from these measurements.

1 INTRODUCTION

The beam lifetime in the ATF damping ring is limited primarily by elastic beam-gas scattering and by the Touschek effect; the latter is intra-beam scattering with a momentum transfer into the longitudinal plane that exceeds the longitudinal acceptance. The quantum lifetime is negligible compared with these two effects. For a constant vacuum pressure, the lifetime caused by elastic beam-gas scattering would be constant. However, the ATF vacuum pressure varies with beam current, especially in the arc sections of the ring, as shown in Fig. 1. The reason for the current dependent pressure is photodesorption due to synchrotron radiation. Assuming that the residual-gas composition is constant, and including the effects of both Touschek and gas scattering, the differential equation is [2]

\[
\frac{1}{N(t)} \frac{dN(t)}{dt} = \alpha P(t) = \frac{1}{\tau_{Tou}} \frac{N(t)}{N_0}
\]

\[
\alpha \equiv 7.2 \times 10^{28} \sum_i p_i \sigma_i [Pa^{-1} s^{-1}],
\]

where \(N_0\) is the bunch population at \(t = 0\). \(P(t)\) the average vacuum pressure, \(p_i\) the normalized partial pressure and \(\sigma_i\) the scattering cross section for each gas component. Integration of Eq. (1) yields

\[
n(t) = 1 - \alpha \int_0^t dt' P(t') n(t') - \frac{1}{\tau_{Tou}} \int_0^t dt' n^2(t'), \quad (3)
\]

where \(n(t) = N(t)/N_0\) is the normalized bunch population.

2 TOUSCHEK BEAM LIFETIME

The Touschek lifetime at the initial current \(N_0\) is determined by the momentum acceptance and described by [3]

\[
\frac{1}{\tau_{Tou}} = \frac{\sqrt{n_c} \sigma_{Tou}}{\gamma^2 \langle V_{beam} \sigma_{e'x} \rangle C \left( \frac{\Delta P}{P} \right)^2} \quad (4)
\]

\[
V_{beam} = \frac{8n_c}{\gamma^2 \sigma_e} \quad (5)
\]

\[
F(\epsilon_A) = \int_1^{\infty} du \left( 2u - 2 - \ln u \right) \frac{\exp(-\epsilon_A u)}{2u^2}, \quad (6)
\]

where \(n_c = \gamma^2 \sigma_e \mu \langle V_{beam} \rangle\), \(\sigma_e\) are the speed of light, classical electron radius, momentum acceptance, beam volume and horizontal beam divergence, respectively. The angular brackets denote an average over the circumference, and

\[
\epsilon_A = \left( \frac{\Delta P}{\gamma^2 \sigma_{e'x} \mu c} \right)^2 \quad (7)
\]

is a factor defined by the ratio of the momentum acceptance and the horizontal beam divergence. The momentum acceptance \(\Delta P\) is given either by the height of the rf bucket or by the (physical or dynamic) aperture at a position with nonzero dispersion, whichever value is smaller. Eq. (4) shows that the Touschek lifetime is proportional to the beam volume. Since this volume varies with the beam current due to effects such as potential well distortion and (multiple) intra-beam scattering, in general the factor \(\langle N_0 \tau_{Tou} \rangle\) is not independent of the beam current.

3 BEAM LIFETIME MEASUREMENT

3.1 Experimental Apparatus

Parameters of the ATF damping ring relevant for this measurement are shown in Table 1. The beam current and vacuum pressure were recorded simultaneously as in Fig. 1. The beam current was measured with a DC Current Transformer (DCCT), and the vacuum pressure was measured with 28 Cold Cathod Gauges (CCG). For the lifetime calculations we used an average of the pressure readings weighted by the distance between individual CCGs.
Table 1: DR Main Parameters in This Measurement

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF Frequency</td>
<td>713.906 MHz</td>
</tr>
<tr>
<td>RF Voltage</td>
<td>280 kV</td>
</tr>
<tr>
<td>Harmonic Number</td>
<td>390</td>
</tr>
<tr>
<td>Natural Emittance</td>
<td>1.122 nm</td>
</tr>
<tr>
<td>Natural Bunch Length</td>
<td>5.211 mm</td>
</tr>
<tr>
<td>Natural Momentum Spread</td>
<td>$5.43 \times 10^{-1}$</td>
</tr>
<tr>
<td>RF Momentum Acceptance</td>
<td>0.01236</td>
</tr>
<tr>
<td>Momentum Compaction</td>
<td>2.141 $\times 10^{-3}$</td>
</tr>
</tbody>
</table>

3.2 Vertical Dispersion

The vertical dispersion in the damping ring was reduced to about 1 cm after dispersion correction [4]. Fig. 2 shows the vertical dispersion before and after the correction. From a simulation reproducing the measured $\eta_y$ by vertical steerings, the vertical emittance was evaluated. The results are shown in Table 2, which includes the contributions to the equilibrium emittance both from the vertical dispersion and from the betatron coupling that is caused by an off-center orbit in the sextupoles. However, it is not included any other errors.

3.3 Estimation of Touschek Lifetime

The lifetime data were taken before and after the dispersion correction described in Sec. 3.2. The fitted values of $\alpha$, defined in Eq. (2), and of the Touschek lifetime are shown in Fig. 3 (a) and (b) as a function of the initial bunch population. The fitting was carried out with Eq. (3). The value of $\alpha$ in Fig. 3(a) should be independent of the bunch current; however, the figure clearly shows some residual beam-current dependence. We conjecture that the main source of this phenomenon was the change in the Touschek lifetime, Eq. (4), due to an intensity-dependent bunch volume, whereas the integration of Eq. (1) assumed a constant value of $(N_0 \tau_{Tou})$. An improved estimate of the Touschek lifetime in the ATF damping ring was obtained employing the following procedure. At first, using the fact that the Touschek lifetime is extremely long at small current, the factor $\alpha$ was determined from a linear extrapolation to zero beam current:

Before Correction $\alpha(0) = 1876$. 

After Correction $\alpha(0) = 1767$. (8)

For the next step, the Touschek lifetime was fitted again, this time keeping $\alpha$ in Eq. 3 constant, equal to $\alpha(0)$ of Eq. (8). The fitted result of $(N_0 \tau_{Tou})$ is shown in Fig. 3(c), as a function of the initial beam current $N_0$. From a linear fit to the data in Fig. 3(c), $(N_0 \tau_{Tou})$ was determined:

Before Correction $N_0 \tau_{Tou} [s] = 4.31 \times 10^{12} + 365N_0$. 

After Correction $N_0 \tau_{Tou} [s] = 1.66 \times 10^{12} + 184N_0$. (9)

So, the product $(N_0 \tau_{Tou})$ increased with the initial beam current. As mentioned, we attribute this increase of the Touschek lifetime to an increase of the beam volume at higher current, possibly caused by intra-beam scattering and potential well distortion.

3.4 Evaluation of Vertical Emittance

Equation (4) can be solved for the vertical emittance,

$$\epsilon_y = G \left( N_0 \tau_{Tou} \right)^2$$  \hspace{1cm} (10)

Assuming that the momentum acceptance is given by the rf bucket height and using the design optics, the factor $G$ is $8.79 \times 10^{-30}$ ms$^{-2}$. From Eqs. (9) and (10), the natural vertical emittance is estimated:

Before Correction $\epsilon_y = 0.1636$ [mm].  

After Correction $\epsilon_y = 0.0243$ [mm]. (12)

These values are 2 times or more larger than the evaluated vertical emittance quoted in Sec. 3.2. Thus, there is some uncertainty in this estimate of the vertical emittance. One explanation might be differences between the actual and the design optics, which were also evident in measurements.

Figure 2: Vertical Dispersion in ATF Damping Ring

Table 2: Evaluated Emittance from Dispersion Data

<table>
<thead>
<tr>
<th>Correction Status</th>
<th>Vertical Emittance [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before Correction</td>
<td>0.062</td>
</tr>
<tr>
<td>After Correction</td>
<td>0.002</td>
</tr>
</tbody>
</table>

Figure 3: Results for Measurement of Touschek Lifetime
of beta functions and dispersion[4]. Systematic errors in the fitted value of $\alpha(0)$, which was determined using a linear extrapolation, represent another possible error source. However, Fig. 3 (d) depicting the vertical emittance for various values of $\alpha(0)$, shows that, for the corrected dispersion case, small changes in $\alpha(0)$ do not much change the result.

3.5 Effect of Dynamic Aperture
As indicated above, the calculated emittance would be different, if the longitudinal acceptance were limited by the transverse aperture at a location with nonzero dispersion and not by the rf bucket height. Since the transverse aperture of the ATF damping ring was found to be extremely small [5], such a transverse limitation of the longitudinal acceptance is not inconceivable. The cross section for particle loss due to elastic scattering on a gas atom of charge $Z_Ae$ is written as

$$\sigma_i \approx \frac{2\pi r_i^2 Z_A^2 \langle \beta \rangle}{\gamma^2 A},$$

where $A$ is the (radial) transverse acceptance, which can be related to $\alpha$ of Eq. (2) via

$$A \approx 7.2 \times 10^{28} \frac{\langle \beta \rangle}{\alpha} \sum_i p_i \frac{2\pi r_i^2 Z_A^2}{\gamma^2}$$

Assuming that the main constituents of the residual gas are $N_2$ or CO molecules, and considering an average horizontal beta function of $<\beta_x >= 3.87$ m (the vertical beta function was slightly larger, about $<\beta_y >= 4.46$ m), the measured value of $\alpha(0)$ corresponds to a full aperture in the horizontal direction of 1.37 m. Since the average horizontal dispersion is 0.0072 m, this aperture, which corresponds to the bucket height of 1.41%, which is comparable to the rf bucket height of 1.24%.

In order to determine whether the transverse aperture had some effect on our emittance estimate, the lifetime measurement was repeated at three different cavity voltages, for the reduced dispersion case. The results are shown in Fig. 4 and Table 3. The Touschek lifetime increased with increasing cavity voltage, while the estimated vertical emittance was roughly constant for cavity voltages up to 300 kV; at 400 kV the Touschek lifetime became independent of the cavity voltage, suggesting that here the momentum acceptance was limited by the transverse aperture. In Fig. 4(b), the solid line shows the vertical emittance calculated for an acceptance determined only by the rf bucket height, while the dashed line shows the vertical emittance calculated assuming as longitudinal acceptance the lower value of either the rf bucket height or the estimated transverse aperture divided by the average dispersion. In the latter case, the estimated emittance is almost independent of the cavity voltage.

4 SUMMARY
The Touschek lifetime in the ATF damping ring was measured, and from the measured Touschek lifetime the vertical emittance was estimated. It seems that this method was effective in the lower emittance region, where the estimated vertical emittance was about 0.020 nm (2% of the horizontal). However, for more precise measurements of the vertical emittance with this method, a better model of the real machine optics will be required. Furthermore, we plan to compare the result with a direct measurement of the emittance, i.e., a wire-scan emittance measurement for the extracted beam, to verify the reliability of this method.

5 ACKNOWLEDGEMENT
We would like to express our thanks to T. Hirose for his continuous encouragement, and to S. Kamada, K. Oide, N. Toge and C. Montag for various useful discussions. We also wish to thank the ATF operation group for their support.

6 REFERENCES

Table 3: Results of Cavity Voltage Dependence : ( ) shows the estimated acceptance due to the transverse aperture.

<table>
<thead>
<tr>
<th>$V_c$ [kV]</th>
<th>$\Delta p/p$ [%]</th>
<th>$\alpha(0)$</th>
<th>$N_0\tau_{Tous}$ [s]</th>
<th>$\epsilon_y$ [nm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.99</td>
<td>1801</td>
<td>0.96 x 10^{12}</td>
<td>0.0185</td>
</tr>
<tr>
<td>300</td>
<td>1.29</td>
<td>1774</td>
<td>1.63 x 10^{12}</td>
<td>0.0197</td>
</tr>
<tr>
<td>400</td>
<td>1.54</td>
<td>1779</td>
<td>1.84 x 10^{12}</td>
<td>0.0130</td>
</tr>
</tbody>
</table>

(c) 0.01
(c) 0.02
(c) 0.03
150 250 350 450
(b) Estimated Vertical Emittance for Various Cavity Voltages

Figure 4: $V_c$ Dependence for Touschek Lifetime