HAMILTONIAN CALCULATIONS ON PARTICLE MOTION IN LINEAR ELECTRON ACCELERATORS

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Abstract

A Hamiltonian theory, in which electromagnetic space waves and longitudinal electric fields are incorporated by means of their vector potentials, is used to calculate particle motion in linear electron accelerators. In particular these calculations have been applied to the Eindhoven 10 MeV travelling-wave linac as well as to the Eindhoven racetrack microtron accelerating cavity. The calculations are in good agreement with simulations performed by particle-tracking codes.

1 INTRODUCTION

The orbit dynamics in an RF linear accelerator has been described using Hamiltonian theory [1]. The various focusing effects that occur in these accelerators: phase focusing in both longitudinal and transverse direction, ponderomotive focusing (i.e. focusing due to higher order terms in the Floquet series describing the electromagnetic field) and magnetic focusing due to solenoidal fields, have been treated in one overall description. The Bessel functions in the vector potential may be expanded, yielding linearized equations; however, the resulting equations of motion show small differences with corresponding simulations using particle-tracking codes.

In this paper, the Hamiltonian theory using the full Bessel functions, i.e. without linearization is used to calculate the equations of motion. The particle motion obtained from this shows excellent agreement with particle-tracking simulations.

In the Hamiltonian also a solenoidal magnetic field has been incorporated. Work on using both the magnetic and electric field is in progress and will not be given in the present paper.

2 THE EQUATIONS OF MOTION

At the EPAC96, a Hamiltonian basis for calculation of particle motion in linear accelerators has been presented [1]. The scaled particle energy h and phase $k\zeta$, as a function of the longitudinal co-ordinate z, can be derived:

$$\frac{dh}{dz} = \sum_{n=-\infty}^{\infty} -\varepsilon_n k \cos\left(\frac{2\pi n}{d} z + k\zeta\right), \frac{d\zeta}{dz} = \frac{-h}{\sqrt{h^2 - e_R^2}} + \frac{k_f}{k}, \quad (1)$$

with

$$h = -H/H_i, \quad e_R = E_R/H_i, \quad \varepsilon_n = \frac{ea_n E_z}{H_i k}, \quad (2)$$

in which H is the particle energy, H_i the initial energy and E_r the particle rest energy, a_n are the components of the Fourier representation of the electric field, with E_z the amplitude of this field, k is the propagation constant in vacuum, furthermore e is the electron charge and d is the cell length.

Trajectory calculations, using the transverse equations of motion, derived directly from [1] are not very accurate, as a result of the linearization of the Bessel function and the expansion of the square root in the Hamiltonian (eq. 6 in [1]). Without linearization and expansion, the following transverse equations of motion result, when no solenoidal magnetic field is applied:

$$\begin{aligned} \frac{dx}{dz} &= \frac{\pi_x}{\sqrt{h^2 - e_R^2 - \pi_x^2 - \pi_y^2}}, \\ \frac{d\pi_x}{dz} &= \sum_{n=-\infty}^{\infty} -\varepsilon_n \left(\frac{k_f k_n - k^2}{\alpha_n}\right) I_1\left(\alpha_n \sqrt{x^2 + y^2}\right) \times \\ &\quad \times \frac{x}{\sqrt{x^2 + y^2}} \sin\left(\frac{2\pi n}{d} z + k\zeta\right), \end{aligned} \quad (3)$$

and similar equations for y,
in which x and y are the transverse co-ordinates,

$$\pi_x = \frac{c}{H_i} p_x, \quad \pi_y = \frac{c}{H_i} p_y, \quad \pi_z = \frac{c}{H_i} p_z, \quad (4)$$

with c the velocity of light and p_x, p_y and p_z the components of the kinetic momentum, I_1 the first order modified Bessel function, α_n is defined by $k^2 = k_n^2 - \alpha_n^2$, where k_n is defined by $k_n = k_f + 2\pi n/d$, with k_f the phase shift per cell.

In case of an applied solenoidal magnetic field, a transformation (G_2 in [1]) has to be performed, which is only possible after expansion of the square root in the Hamiltonian. As mentioned, this results in rather inaccurate calculations of motion in an electric field. So up to now, only the equations of motion in a solenoidal magnetic field are used, when there is no electric field present:

$$\frac{dx_2}{dz} = \frac{\pi_{x2}}{\sqrt{h^2 - e_R^2}}, \quad \frac{d\pi_{x2}}{dz} = \tfrac{1}{2} x_2 \frac{b^2 k^2}{\sqrt{h^2 - e_R^2}}, \quad (5)$$

and similar equations for y_2, with new co-ordinates

$$x_2 = x\cos\varphi + y\sin\varphi,\ y_2 = y\cos\varphi - x\sin\varphi,$$
$$\pi_{x2} = \pi_x\cos\varphi + \pi_y\sin\varphi,\ \pi_{y2} = \pi_y\cos\varphi - \pi_x\sin\varphi \qquad (6)$$

in which φ arises from the rotation due to the solenoids:

$$\frac{d\varphi}{dz} = \tfrac{1}{2}\frac{bk}{\sqrt{h^2 - e_R^2}} \text{ with } b = \frac{ecB(z)}{H_i k}, \qquad (7)$$

with $B(z)$ is the longitudinal magnetic field.

3 CALCULATIONS COMPARED TO SIMULATIONS

In this section calculations based on the Hamiltonian equations of motion of the previous section are presented and compared to results of the particle-tracking codes Parmela [2] and General Particle Tracer (GPT) [3]. In these codes, the same electric and magnetic field description is specified, however these particle-tracking codes use an entirely different calculation method. Calculations of motion in an electric field have been applied to the Eindhoven racetrack microtron (RTM) accelerating cavity, which is a standing-wave RF structure, calculation of motion in a solenoidal magnetic field have been applied on the Eindhoven 10 MeV linear accelerator.

3.1 Particle energy gain

First the particle energy and phase as a function of position in the RTM cavity have been calculated, using (1). This has been done at different injection phases and at an injection energy of 10 MeV, which is the injection energy in the RTM cavity in the first turn. In figure 1 the energy calculation is compared to the results of Parmela and GPT. The figure shows a very good agreement between the different methods. Differences between calculations and simulations are about 0.01 MeV: 1 ‰ of the injection energy. The particle phase is not provided by Parmela and GPT, so a comparison of phases is not possible. However, because of the agreement in energy calculations, it is obvious that the Hamiltonian phase calculations must be correct.

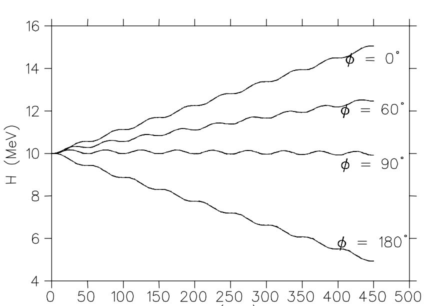

Figure 1: Energy gain of electrons in the RTM cavity. Injection at different phases ϕ, at 10 MeV. Curves presenting Hamiltonian, GPT and Parmela calculations are hardly distinguishable.

3.2 Particle transverse motion

Second the particle transverse motion has been studied. Figure 2 depicts Hamiltonian calculations using (3) and Parmela and GPT simulations of particle trajectories for 10 MeV injection energy and for different injection phases. The left side of the figure shows those particles injected parallelly to the *z*-axis at a displacement of 4 mm from the axis. The right side shows the particles, injected at the axis with an initial divergence of 10 mrad. Again, it is seen that there is excellent agreement between the various computation methods. Maximum differences between the calculations and the simulations are about 0.5% of the displacement from the axis at injection. At an injection phase of 0°, which is about the injection phase to be used, differences are about 0.5 ‰. Differences between calculations and simulations in the divergence ($=\pi_x/\pi_z$) are about 2% of the amplitude of the oscillation in divergence. However, the effects of these differences on the electron displacement from axis are mainly averaged out.

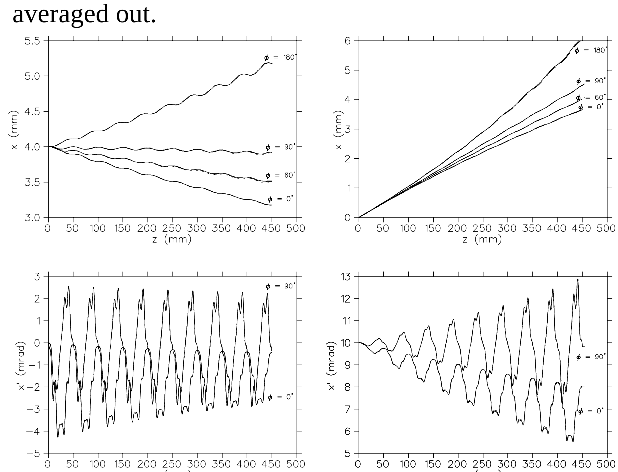

Figure 2: Electron trajectories in the RTM cavity. Injections at different phases (indicated in the figure by ϕ), at 10 MeV. Hamiltonian, GPT and Parmela calculations are hardly distinguishable.

3.3 Linearity in the calculation

The displacement from axis x_f, and the divergence x_f' at the end of the cavity of a particle injected with initial displacement x_i and divergence x_i' is often calculated by using a transport matrix:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ x' \end{pmatrix}_i. \qquad (8)$$

Using this transport matrix assumes that calculations are linear. For standing wave cavities an analytic approximation of the matrix is given by Rosenzweig and Serafini [4]. The linear character of the Hamiltonian calculations has been examined by comparing particles that are injected close to and far from the axis and by particles that have a small and large divergence at an injection energy of 10 MeV. The same has been done using the simulation codes Parmela and GPT. Figure 3 shows the transport coefficients as a function of ϕ. It appears that the *b* coefficient is linear within the accuracy of the provided data, and that the *a* and *d* coefficient are linear within 1.5% in the Hamiltonian calculations and both the simulations. The *c* coefficient is linear within 20%, so it should be used carefully. For all coefficients a rather good agreement to the Rosenzweig and Serafini theory is shown. From the fact that the particle transverse position is determined by the *a* and *b* component, it can be concluded that linear approximation of particle position is rather accurate.

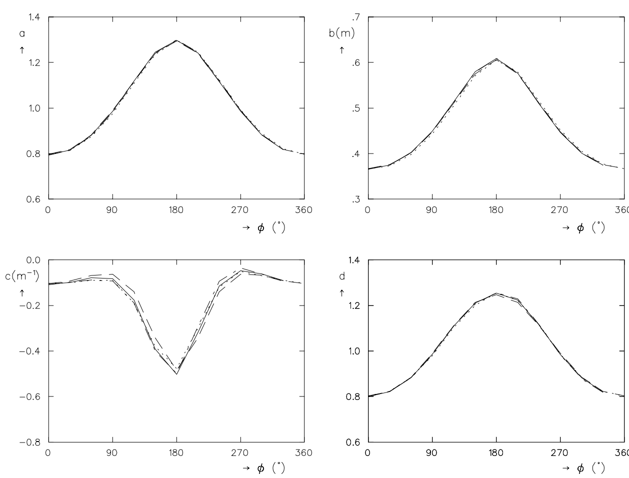

Figure 3: The matrix components of the transfer matrix of the RTME cavity as a function of the injection phase, injection at 10 MeV. Hamiltonian calculations (——), Rosenzweig and Serafini theory (- - -), Parmela simulations (….), GPT simulations (-· - · -).

3.4 Solenoidal magnetic field calculations

To check the validity of the equations of motion in a solenoidal magnetic field, Hamiltonian calculations, using (5) are compared to Parmela calculations. The magnetic field used, equals the solenoidal magnetic field of the Eindhoven 10 MeV linear accelerator [5]. The used particle energy is 1 MeV and no acceleration takes place as it would lead to inaccuracies as mentioned in section 2. In figure 4.a calculations are presented of a parallelly injected particle at (x, y) = (3 mm, 0 mm). Figure 4.b presents a particle injected from the axis with a divergence (x', y') = (10 mrad, 0 mrad). As seen in the figure, agreement between both methods is perfect.

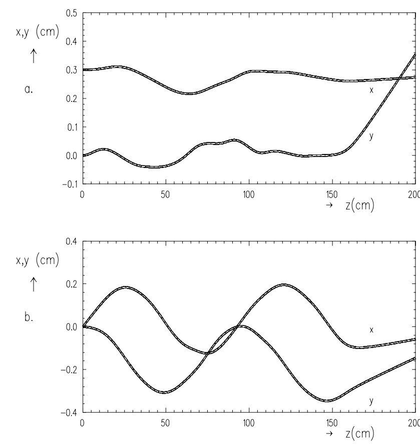

Figure 4: Non accelerated particle (1 MeV) trajectories in the Eindhoven 10 MeV travelling wave linac solenoidal magnetic field, the double line represents Hamiltonian calculations, the dotted line the Parmela simulations.

a. Injection 3 mm off axis without divergence.
b. Injection from the axis with a 10 mrad divergence.

4 CONCLUDING REMARKS

Calculations performed, using Hamiltonian equations, on the Eindhoven RTM cavity have been compared to simulations using the particle-tracking codes Parmela and GPT. Calculations of phase and energy as a function of position in the cavity agree to the simulations within 1‰. Calculations on transverse motion agree with the simulations within 0.5%. The linear character of the calculations has been examined: the particle position given by linear calculations is rather accurate. Furthermore, motion in solenoidal magnetic fields has been calculated. The calculations agree with particle-tracking code simulations. Work on combining solenoidal and accelerating field is in progress.

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