PROPOSAL OF A SORTING EXPERIMENT AT THE CERN-SPS

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Abstract

We propose to study the effectiveness of the sorting strategies on the CERN SPS using eight strong extraction sextupoles independently powered to simulate random errors. In order to find the best sequence of the sextupolar perturbations, some dynamical quantities computed through the perturbative tools of normal forms are used as quality factors. They provide a fast estimate of the beam stability well-correlated with the results of the element-by-element tracking simulations. The most effective quality factor is retained and minimized by an appropriate permutation of the position of the sextupoles along the SPS azimuth. The effectiveness of the ordering procedures is verified for extensive long term six-dimensional tracking. Situations are presented where the beneficial effect of the sorting is expected to be large and well measurable in realistic conditions of operations.

1 INTRODUCTION

In large hadron colliders, such as the LHC [1], the random field-shape errors of the superconducting magnets provoke the excitation of resonances and, as a consequence, a reduction of the dynamic aperture (DA). During the last decade, sorting strategies to optimize the position of the magnets and to partially compensate the harmful effects of the random errors, have been suggested [2, 3, 4]. Several important questions are left unanswered.

- In general, the number of possible configurations is exceedingly large and therefore it is impossible to make an exhaustive search: the optimization procedure is tailored for the limited computing power available, and it is not clear if it brings to the real optimum.

- The sorting strategy is applied to a specific condition of the accelerator model. Quite often both the synchrotron motion and the long-term effects are neglected: moreover, in general the sorting is based on a subset of the expected imperfections. Therefore, it is not clear if the sorting procedure produces a global improvement for realistic operational conditions.

Some experience on these open subjects can be gained by investigating simple scenarios that should be possibly tested on a real accelerator. We propose to carry out these investigations at the CERN-SPS using the eight extraction sextupoles to simulate random sextupolar errors and implementing the sorting strategies to check their effectiveness.

2 MODEL OF THE SPS

In an experimental test the SPS could be operated in coasting mode, without RF, at a beam momentum of 120 GeV/c. A set of 108 sextupoles is used to correct the chromaticity. Residual orbit and linear coupling are assumed to be negligible. The working point is chosen at \( Q_x = 26.6059 \) and \( Q_y = 26.5373 \), i.e. the same as in previous experiments on diffusion [5]. Eight strong sextupoles, placed in symmetric locations around the ring, normally used for the resonant extraction, will produce the main nonlinear perturbation. The eight sextupolar gradients are assigned with a random gaussian distribution cut at one standard deviation \( \sigma = 7.64 \cdot 10^{-7} (m^{-2}) \) is compatible with the maximum gradient allowed at 120 GeV/c in storage mode. In computing the effect of the errors and their sorting procedure we use a 4D description of the particle motion. However in view of a possible experiment with bunched beams and to gain experience for the LHC project, we also check how robust is the beneficial effect of the sorting in presence of synchrotron motion.

3 SORTING STRATEGY

To order the random excitation of the sextupoles along the SPS ring we use the procedure described in Ref. [6]. In the present situation, however, the compensation of the nonlinearities is made in a global manner, whereas in Ref. [6] the local cancellations had an important role. The present situation is rather appealing, because it serves as a test model to the problem of sorting LHC octants using the average random error of each octant. First we try to identify a quality factor (QF), that can be computed fast, well correlated with the short term DA (1024 turns). Good candidates are the norms of the one-turn map, of the tuneshift, of the resonances \( (3, 0) \), called \( Q(3, 0) \) and of the resonance \( (5, 0) \), called \( Q(5, 0) \) (see Ref. [6] for their detailed description). The correlations are investigated using the 4D computer code PLATO [7] for 100 different realizations of the eight random sextupolar gradients. In Fig. 1 we show the results for the norm of the resonances \( (3, 0) \) and \( (5, 0) \) that have the best correlation with the DA and that will be used as quality factors for sorting the eight sextupoles. The sorting procedure consists in generating a small number (typically 100) of random permutations of each realization of the errors, and in selecting the one that gives the least value of the QF. The distribution of the short-term DA for the unsorted and the sorted cases, using the \( Q(5, 0) \) is given in Fig. 2. The av-
4 SPACE OF THE CONFIGURATIONS

The standard approach to the problem of searching the global maximum of a given function, defined over a very large set of discrete states, is based on the simulated annealing [8]. It has been applied to the sorting problem in Ref. [9]. Its application to the case of the SPS does not show a significant improvement with respect to the naive choice of the best configuration over 100 random permutations, as described in Section 3. Furthermore, the annealing procedure is very CPU-time consuming because a huge number of permutations must be taken into account. In our case, almost $1.6 \times 10^4$ iterations are necessary for the algorithm to converge, i.e. a figure comparable to the whole number of possible permutations of the eight extraction sextupoles, namely $8!$ The success of the rule based on the choice of the best permutation among a very limited number of random rearrangement is rather surprising. For this reason, we have performed a systematic analysis of this result. A specific case with a rather poor value of the DA has been selected; then, the distribution of the QF and of the DA for all possible permutations of the errors have been computed. The results are shown in Fig. 3. The shape of the distribution of the DA resembles a gaussian and it covers the ranges shown in Tab. 1. The distribution of the QFs has a good correlation with the short-term DA and is well peaked around the good rearrangements, and therefore it is highly improbable not to find a good sequence with a reasonable small number of random permutations. This is a highly positive feature, since it guarantees that the good permutations, i.e. those having a large DA, are not improbable, thus making the sorting rather robust with respect to neglected effects. The dependence of the results on the choice of the initial seed is rather weak. This is another interesting feature, since it ensures that any realization can always be sorted in such a way to obtain a large DA. Furthermore one can argue that a small dynamic aperture is due to a bad disposition of the errors rather than to their values. As a second step, we considered the behaviour of a large sample of machine realizations ($10^4$ cases). The distributions of the DA are shown in Fig. 4 together with the data concerning the QFs. Their pattern is rather similar to that of the previous figure. Hence we can argue that by analyzing only many permutations of one particular realization, it should be possible to reconstruct the average behaviour for a large number of realizations of the machine.

Table 1: Ranges of DA, $Q(3, 0)$ and $Q(5, 0)$ for 100 random permutations and for the complete set of all permutations.

<table>
<thead>
<tr>
<th></th>
<th>100 permutations</th>
<th>all permutations</th>
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<tbody>
<tr>
<td>DA</td>
<td>4.45 - 8.20</td>
<td>3.45 - 9.25</td>
</tr>
<tr>
<td>$Q(3, 0)$</td>
<td>43.0 - 3600</td>
<td>26.0 - 5800</td>
</tr>
<tr>
<td>$Q(5, 0)$</td>
<td>16.0 - 6600</td>
<td>7.0 - 8500</td>
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5 ROBUSTNESS OF SORTING

An important issue is how the improvement depends on neglected effects such as the synchrotron motion or how it persists in the long-term or after a small tune variation. In Tab. 2 we report the values of the dynamic aperture before and after the sorting procedure for the different situations. The beneficial effect of sorting is quite impressive:

Table 2: Dynamic aperture of a bad seed before and after the sorting in different situations.

<table>
<thead>
<tr>
<th></th>
<th>4D motion</th>
<th>4D motion</th>
<th>6D motion</th>
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<tbody>
<tr>
<td></td>
<td>10^3 turns</td>
<td>10^5 turns</td>
<td>10^5 turns</td>
</tr>
<tr>
<td>unsorted</td>
<td>4.45</td>
<td>4.40</td>
<td>4.10</td>
</tr>
<tr>
<td>sorted (5, 0)</td>
<td>8.55</td>
<td>7.95</td>
<td>7.30</td>
</tr>
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</table>

The short-term DA is improved by 90%. The improvement is 80% in the long-term 4D (10^5 turns) tracking simulations. The situation is well depicted in Fig. 5, where the long-term DA is shown in normalized phase-space (left) and in tune-space (right). The improvement still persists when the synchrotron motion is added (δp/p = 10^{-8}) as shown by simulations for 10^5 turns performed with SIXTRACK [10]. In all the cases, the DA is evaluated averaging the radius of the last stable particle for 6 different values of the emittance ratio. The improvement is ≈ 80%.

6 CONCLUSION

A configuration of the SPS with the existing hardware to test the effectiveness of a sorting strategy has been described. The used strategy, based on the quality factors related to the strength of the resonance (5, 0), proves to be very effective. The obtained ordered sequence shows an excellent improvement in the DA also when long-term and 6D motion are included. Due to the limited number of magnets (eight), a complete analysis of the configuration space is possible and it shows that naive methods to search the best configuration over a very limited number of random permutations are already very effective so that more sophisticated strategies based on simulated annealing seem to be unnecessary.

7 REFERENCES