LASER PLASMA ACCELERATORS

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Abstract

Particle acceleration by relativistic electron plasma waves generated by intense lasers has been demonstrated in a number of experiments by various mechanisms. Accelerating fields as high as 1 GeV/cm, with electrons accelerated to about 100 MeV in millimetre distances have been achieved. These fields produced by intense lasers in plasmas are the largest ever produced in laboratory experiments. The first experiments are very much “first generation” laser plasma accelerator experiments and are concerned with demonstrating proof-of-principle acceleration in relativistic plasma waves. Attention is now being focussed on other important aspects of plasma accelerators such as beam current and beam quality and not just accelerating gradients. Recent experimental, theoretical and simulation results together with an outline of future experiments will be presented.

1 INTRODUCTION

Particle acceleration by relativistic plasma waves has gained a lot of interest lately due to the rapid advance in laser technology and the development of compact terawatt and petawatt laser systems with ultra-high intensities ($\geq 10^{18} \text{Watts}/\text{cm}^2$), with modest energies ($\leq 100J$) and short sub-picosecond pulses ($\leq 1\text{ps}$). The strength of the electric field at the focus of these high-power short-pulse lasers $E_{\perp}$ is directly related to the laser intensity by $eE_{\perp} = 30\sqrt{2}\text{GeV/cm}$. The electric field $E_{\perp}$ of a laser whose intensity $I$ is $10^{18}\text{W/cm}^2$ is $30\text{GV/cm}$. Direct use of the laser field for particle acceleration is not straightforward. Since the electric field of the laser is perpendicular to the propagation direction the maximum energy gain is limited by the distance the particle moves across the wavefront before the electric field changes sign. However, by using a plasma into which laser energy can be coupled changes the situation. Plasma as a medium for particle acceleration has a number of advantages. It has no electrical breakdown limit like conventional accelerating structures which are limited to a maximum field strength of less than $100\text{MeV/m}$. Plasmas are already ionized. A plasma supports longitudinal plasma waves which oscillate at the plasma frequency $\omega_p \equiv (4\pi n_e e^2/m)^{1/2}$ where $n_e$, $m$ is the electron density and mass respectively and $e$ is the charge. In these waves the plasma electrons oscillate back and forth at $\omega_p$ due to the space charge of the immobile ion background irrespective of wavelength. Therefore, these waves can have arbitrary phase velocities, $v_{ph}$; relativistic plasma waves have $v_{ph} \leq c$. The electric field $E$ of relativistic plasma waves with an oscillatory density $n_1$, can be estimated from Gauss’ law and is given by $E = e\sqrt{n_1} / n$ where $n$ is the plasma density in $\text{cm}^{-3}$ and $e$ is the plasma wave amplitude or fractional density bunching $n_1 / n$. For a plasma density of $10^{18}\text{cm}^{-3}$ accelerating gradients of $1\text{GeV/cm}$ are possible which is more than a 1000 times greater than in conventional accelerators. In the seminal paper on plasma based accelerators Tajima and Dawson showed how intense short pulse lasers with a pulse length $\leq 0.1\text{ps}$ could generate large amplitude relativistic longitudinal plasma waves. This scheme has become known as the Laser Wakefield Accelerator (LWFA). Alternative schemes to excite plasma waves using larger laser pulses are: 1) the Plasma Beat Wave Accelerator (PBWA) where two long intense laser pulses with a frequency separation equal to $\omega_p$ beat together in a plasma to resonantly excite the plasma wave; 2) the Raman forward scattering (RFS) instability where one long intense laser pulse is used this is now called the self-modulated LWFA scheme. Tajima and Dawson showed that the maximum energy gain $\Delta W$ of a particle in a relativistic plasma waves with $v_{ph} \leq c$ is

$$\Delta W = 2\gamma^2 m c^2$$  \hspace{1cm} (1)

Where $\gamma$ is the Lorentz factor, associated with the phase velocity of the plasma wave $\gamma = \left(1 - v^2_{ph}/c^2\right)^{-\frac{1}{2}}$. The phase velocity of the plasma wave is equal to the group velocity $v_g$ of the laser in the plasma $v_{ph} = v_g = \left(1 - \frac{\omega^2}{\omega_p^2}\right)^{\frac{1}{2}}$, where $\omega$ is the laser frequency, therefore $\gamma \approx \omega/\omega_p$ and the maximum energy gain is

$$\Delta W = 2\gamma^2 m c^2$$  \hspace{1cm} (2)

It is clear that for given values of $\omega$ there is a trade-off to be considered in choosing $\omega_p$. From the group velocity $v_g$ we see that a low value of $\omega_p$ is required to minimise the phase slip of extremely relativistic electrons with respect to the wave while a high value of $\omega_p$ is necessary to maximise the accelerating field $E$.

We want to maximise $E$ by increasing $\omega_p$ but this minimises the energy gain $\Delta W$ due to phase slip. As the electron accelerates it slips forward in phase and eventually outruns the useful part of the accelerating field. The maximum energy gain occurs over a distance $L = \Delta W/eE = 2\gamma^2 c/\omega_p$ which is the limit of the de-
phasing length. We now review the three schemes which already have experimental results.

2 PLASMA BEAT WAVE ACCELERATOR

In the plasma beat wave accelerator (PBWA) a relativistic plasma wave is generated by the ponderomotive force of two lasers separated in frequency by the plasma frequency such that the energy and momentum conservation relations are satisfied $\omega_1 - \omega_2 = \omega_p, k_1 - k_2 = k_p$, where $(\omega_{1,2}, k_{1,2})$ are the frequencies and wavenumbers of the two lasers respectively.

The beat pattern can be viewed as a series of short light pulses each $\pi \omega_p$ long moving through the plasma at the group velocity of light which for $\omega_{1,2} \gg \omega_p$ is close to $c$ the speed of light. The plasma electrons feel the periodic ponderomotive force of these pulses.

The growth of the plasma wave is described by the equation $^5$

$$\epsilon = \int_0^\tau \frac{4}{\alpha_1 \omega_p} dt$$

where $\alpha_{1,2} = \frac{E_{1,2} \omega_{1,2}}{m \omega_1 \omega_2}$ is the normalised oscillatory velocity of an electron in the laser fields $E_{1,2}$. As the electron plasma wave grows its electric field amplitude given by eq.1 becomes large enough that the velocity of an electron oscillating in this field becomes relativistic and the plasma frequency $\omega_p$ suffers a small red shift $\Delta \omega_p = -\frac{8}{9} \epsilon^3$ due to the relativistic increase in mass. This red shift in frequency causes the wave to saturate at $^6$

$$\epsilon_{SAT} = \left( \frac{16}{3} \alpha_1 \alpha_2 \right)^{\frac{1}{2}}$$

and the time for saturation is given by $\tau_{SAT} = \frac{8}{\omega_p} \left( \frac{1}{\alpha_1 \alpha_2} \right)^{\frac{1}{2}}$. Other factors which can limit the interaction or acceleration length is diffraction of the laser beams or pump depletion. Diffraction limits the depth of focus to the Rayleigh length which may be overcome by channeling of the laser but this is still not resolved. Pump depletion can be avoided by using more powerful lasers. By using intense short pulse lasers ion instabilities such as the modulational instability can be avoided. A number of experiments have been carried out which demonstrate that the theoretical estimates are in very good agreement with observations.$^5,7,8,9,10$

The experiments carried out at UCLA$^6,10$ focussed a two frequency carbon dioxide laser and injected a $2 MeV$ electron beam to the same point in a hydrogen plasma at a density of about $10^{15} cm^{-3}$. The results showed that approximately 1% or $10^5$ electrons of the randomly phased injected electrons are accelerated from $2 MeV$ to $30 MeV$ in the diffraction length of about 1cm. This corresponds to a gradient of $3 GV/m$. The measured amplitude of the relativistic plasma waves is 30% of its cold wavebreaking limit, agreeing with the theoretical limit given by equation (4).

What is particularly significant about this experiment is it demonstrated that the electrons were “trapped” by the wave potential. Only trapped electrons can gain the theoretical maximum amount of energy limited by dephasing which occurs when the polarity of the electric field of the plasma wave seen by the accelerated electron changes sign.

A trapped electron, by definition, moves synchronously with the wave at the point of reflection in the wave potential. At this point the trapped electron has a relativistic Lorentz factor $\gamma = \left( 1 - \frac{v^2_p}{c^2} \right)^{-\frac{1}{2}}$. As the electron continues to gain energy it remains trapped (and eventually executes a closed orbit in the wave potential.) Trapping also bunches the electrons. In the UCLA experiment$^10$ the plasma wave has a Lorentz factor of 33 which is synchronous with $16 MeV$ electrons. Therefore all electrons observed above $16 MeV$ are trapped and move forward in the frame of the wave.

The experiment done at the Ecole Polytechnique$^8$ also accelerated electrons but were limited to very small energy gains from $3 MeV$ to $3.7 MeV$ due to saturation of the relativistic plasma wave by the modulational instability. This instability is important for long pulses of the order of the ion plasma period $\omega_{pi}^{-1}$ and it limits the wave amplitude to very small values. All beat wave experiments confirm earlier simulations$^{11}$ and theoretical work and demonstrate the need to use short pulses to avoid competing instabilities.

The success of the experiments indicate that it should be possible to accelerate electrons to 1 GeV in a single stage laser plasma accelerator. The prospects of such an experiment has been discussed by Chan Joshi and collaborators$^{12}$ and backed up by numerical simulations. In such an experiment an injected $10 MeV$ beam of electrons of 100 A could produce about $10^5$ electrons at 1 GeV energies. The necessary laser power required is $\sim 14TW (14 \times 10^{12}W)$ with a pulse duration of 2ps corresponding to laser energy of 28 Joules and wavelengths of 1.05$\mu m$ and 1.06$\mu m$ in a plasma with density $10^{17} cm^{-3}$ and interaction length $\simeq 3 cm$.

3 THE LASER WAKEFIELD ACCELERATOR (LWFA)

In the LWFA a short laser pulse,$^1$ whose frequency is much greater than the plasma frequency, excites a wake of plasma oscillations (at $\omega_p$) due to the ponderomotive force much like the wake of a motor boat. Since the plasma wave is not resonantly driven as in the beat wave the plasma density does not have to be of a high uniformity to produce large amplitude waves. As an intense pulse propagates through an underdense plasma, $\omega_c \gg \omega_p$, where $\omega_c$ is the laser frequency, the ponderomotive force associated with the laser envelope $F_{laser} \sim \frac{1}{2} m v_{wave} \epsilon_p$ expels electrons from the region of the laser pulse and excites electron plasma waves. These waves are generated as a result of being displaced by the leading edge of the laser pulse. If the laser pulse length, $cT_L$, is long compared to the electron plasma wavelength then the energy in the plasma wave is re-absorbed by the
trailing part of the laser pulse. However, if the pulse length is approximately equal to or shorter than the plasma wavelength $c\tau_p \simeq \lambda_p$, the ponderomotive force excites plasma waves or wakefields with a phase velocity equal to the laser group velocity which are not re-absorbed. Thus any pulse with a sharp rise or a sharp fall on a scale of $c/\omega_p$ will excite a wake. With the development of high brightness lasers the laser wakefield concept first put forward by Tajima and Dawson\(^\text{1}\) in 1979 has now become a reality. The focal intensities of such lasers are $\geq 10^{18} \text{W cm}^{-2}$, with $v_{osc}/c \geq 1$, which is the strong nonlinear relativistic regime. Any analysis must therefore be in the strong nonlinear relativistic regime and a perturbation procedure invalid.

The maximum wake electric field amplitude generated by a plane polarized pulse been given by Sprangle\(^\text{18}\) in the one dimensional limit as $E_{\text{max}} = 0.38 \frac{\sigma_{se}/\epsilon_0^2}{(1 + \sigma_{se}/2d^2)^2} \sqrt{\pi N} \text{V/cm}$ for $v_{osc}/c \sim 4$, and $n_0 = 10^{18} \text{cm}^{-3}$ then $E_{\text{max}} \approx 2 \text{GV/cm}$, and the time to reach this amplitude level is of the order of the laser pulse length. There is no growth phase as in the heat wave situation, which requires many plasma periods to reach its maximum amplitude.

To get larger wave amplitudes it has been suggested by several groups to use multiple pulses with varying time delay between pulses. Conclusive experiments\(^\text{14,15,16}\) have been carried out to demonstrate the excitation of the plasma wakefield. Nakajima\(^\text{14}\) observed injected electrons gaining $13 \text{MeV}$ in the wake of a $10^7 \text{W}$ short pulse laser.

### 4 SELF-MODULATED LASER WAKEFIELD ACCELERATOR

Self-Modulated LWFA\(^\text{4}\) is a hybrid scheme combining elements of stimulated Raman forward scattering\(^\text{1}\) (SRS) and the laser wakefield concept. Raman forward scattering\(^\text{1}\) describes the decay of a light wave at frequency $\omega_0$ into light waves at frequency $\omega_0 \pm \omega_p$, and a plasma wave $\omega_p$ with $v_{ph} \simeq c$. Although Raman forward scattering generates relativistic plasma waves and was identified as an instability which generated $\text{MeV}$ electrons in early laser plasma experiments\(^\text{17}\), it was not considered a serious accelerator concept because the growth rate is too small for sufficient plasma wave amplitudes to be reached before ion dynamics disrupt the process. However coupled with the laser wakefield accelerator concept it becomes a viable contender. Short pulse lasers have been demonstrated by Antonsen and Mor\(^\text{a}^\text{18}\) Sprangle et al.\(^\text{19}\) and Decker et al.\(^\text{20}\) to self modulate in a few Rayleigh lengths. This modulation forms a train of pulses with approximately $\pi c/\omega_p$ separation which act as individual short pulses to drive the plasma wave. The process acts in a manner similar to a train of individual laser pulses.

A number of experimental groups\(^\text{21,22,23}\) have recently reported experimental evidence for the acceleration of electrons by relativistic plasma waves generated by a modulated laser pulse. The most impressive results come from a group working with the Vulcan laser at RAL, UK., this group which consisted of research teams from Imperial College, UCLA, Ecole Polytechnique, LLNL and RAL have reported\(^\text{22}\) observations of electrons at energies as high as $44 \text{MeV}$. The observations of the energetic electrons was correlated with the simultaneous observation of $\omega_o + n\omega_p$ radiation generated by Raman forward scattering. The experiments were carried out using a $257 \text{TW}$ laser with intensities $> 10^{18} \text{W cm}^{-2}$ and pulse lengths $< 1 \text{ps}$ in an underdense plasma $n_o \sim 10^{19} \text{cm}^{-3}$. The laser spectrum is strongly modulated by the interaction showing sidebands at the plasma frequency. More recently\(^\text{24}\) electrons with energies up to $100 \text{MeV}$ have been observed in the same experiment, this is in agreement with the theory. Similar results of a modulated laser pulse at $\omega_p$ have been obtained by a separate Livermore experiment\(^\text{41}\) using a $5 \text{TW}$ laser but only observed $2 \text{MeV}$ electrons. The difference in the energy is explained by a difference in the length of plasma over which acceleration takes place. In the RAL experiment the acceleration length is much larger. It is worth pointing out that in these experiments the accelerated electrons are not injected but are accelerated out of the background plasma.

The Raman forward scattering instability is the decay of an electromagnetic wave $(\omega_0, k_0)$ into a forward propagating plasma wave $(\omega_p, k_p)$ and forward propagating electromagnetic waves the Stokes wave at $(\omega_o, -n\omega_p)$ and anti-Stokes wave at $(\omega_o, +n\omega_p)$ where $n$ is an integer, this is a four wave process. One of the earliest papers on forward Raman instabilities in connection with laser plasma accelerators was by Bingham\(^\text{25}\) who discussed a purely temporal theory including a frequency mismatch, more recently a spatial temporal theory was developed by Mori et al.\(^\text{26}\) In this theory the relativistic plasma wave grows from noise, calculating the noise level is non trivial since there are various mechanisms responsible for generating the noise. For example the faster growing Raman backscatter and sidescatter instabilities cause local pump depletion forming a “notch” on the pulse envelope. The plasma wave associated with this notch acts as an effective noise source as seen in the simulations by Tzeng et al.\(^\text{27}\) The simulations carried out by Tzeng et al\(^\text{27}\) are the first to use the exact experimental parameters and show significant growth within a Rayleigh length, there is also remarkable agreement with the experimental results of Modena et al.\(^\text{28}\)

All these experiments rely heavily on extending the acceleration length which is normally limited to the diffraction length or Rayleigh length $L_R = (\omega_0/2c)\sigma_z^2$, where $\omega_o$ is the laser frequency and $\sigma_z$ is the spot size. In present day experiments this is limited to a few $\mu\text{m}$, for example the RAL Vulcan CPA laser has a wavelength of $1 \mu\text{m}$, a spot size of $20 \mu\text{m}$ resulting in a Rayleigh length $L_R \approx 350 \mu\text{m}$. To be a useful accelerator laser pulses must propagate relatively stably through uniform plasmas over distances much larger than the Rayleigh length. Relativistic self-focussing is possible if the laser power exceeds the critical power given by $P_c = 17 \omega_0^2/\omega_p^2GW$ which is easily satisfied for the present high power laser experiments. There are difficulties in relying on the laser pulse to form its own channel since the
beam may break up due to various laser-plasma instabilities such as Raman scattering and filamentation. Relativistic self-guiding over five Rayleigh lengths has recently been reported by Chiron et al.\textsuperscript{28} using a 10TW laser in a plasma of density $5 \times 10^{18} \text{ cm}^{-3}$, they also note that the effect disappears at larger powers and densities. Alternatively plasma channels have been demonstrated\textsuperscript{29} to be very effective in channelling intense laser pulses over distances much greater than the Rayleigh length. In these experiments a two laser pulse technique is used. the first pulse creates a breakdown spark in a gas target, and the expansion of the resulting hot plasma forms a channel which guides a second pulse injected into the channel. Pulses have been channelled up to 70 Rayleigh lengths\textsuperscript{20} corresponding to 2.2 cm in the particular experiment with about 75% of the energy in the injected pulse focal spot coupled into the guide.

In a preformed channel other instabilities may appear. For example it has been shown by Wurtele and Shvets that a laser hose instability exists for parabolic channels.\textsuperscript{80}

Plasma channels are not only important for laser plasma accelerators but has applications in high harmonic generation, for UV and soft X-ray lasers.

5 PROSPECTS FOR HIGH ENERGY ACCELERATORS

The present experiments and future experiments, however, are very far from the parameter range of interest to high energy physicists who require something like $10^{11}$ particles per pulse accelerated to $1 \text{ TeV}$ energies (for electrons) with a luminosity of $10^{-28} \text{ cm}^{-2} \text{sec}^{-1}$ for acceptable event rates to be achieved. The $1 \text{ TeV}$ energy range is $> 1000$ times greater than a single accelerating stage could provide at present, even if the interaction length can be extended by laser channelling there is still going to be the requirement of multiple staging, and more energetic lasers. For a $1 \text{ TeV}$ beam of $10^{11}$ particles per pulse and a transfer efficiency of 50% would require a total of $32 \text{kJ}$ of laser energy per pulse, for a 100 stage accelerator. This would require 100 lasers of about 300 J each with high repetition rates. Compared to the 56 J lasers in the proposed $1 \text{GeV}$ accelerator and the 1 Joule laser used in present day experiments ($10^{10}$ particles per pulse would require $100 \times 30 \text{J}$ lasers).

The parameter range required is out of reach by present day lasers and may never be achievable with laser plasma accelerators.

The work on plasma-based accelerators represents but one area that is being explored by researchers in the advanced accelerator field. Other schemes being investigated at present for high-gradient acceleration are the inverse Cerenkov effect and the inverse free-electron laser effect. Still other researchers, realizing that the next collider will almost certainly be a linear electron-positron collider, are proposing a novel way of building such a device known as a two-beam accelerator, and there are many groups developing an entirely new type of electron lens using focussing by a plasma to increase the luminosity of future linear colliders.\textsuperscript{31}

This plays on the fact that relativistic electron beams can be focussed by a plasma if the collisionless skin depth $c/\omega_{pe}$ is larger than the beam radius. Generally, when a relativistic electron beam enters a plasma, the plasma electrons move to neutralize the charge in the beam on a $1/\omega_{pe}$ timescale. However, if the collisionless skin depth is larger than the beam radius, the axial return current flows in the plasma on the outside of the electron beam and the beam current is not fully neutralized, leading to the generation of an azimuthal magnetic field. Consequently this self-generated magnetic field pinches or focusses the beam in the radial direction. This type of lens exceeds conventional lenses by several orders of magnitude in focussing gradient.

For laser plasma accelerators, the next milestone to be achieved is the $100 \text{MeV} - 1 \text{GeV}$ energy level with good beam quality.

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7 REFERENCES