ISSUES IN MULTI-BUNCH EMITTANCE PRESERVATION IN THE NLC*

K.L.F. Bane, C. Adolphsen, K. Kubo*, and K.A. Thompson
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309 USA

INTRODUCTION

In the linacs of the SLAC NLC design 1 nC bunches are accelerated in trains 90 bunches long with an interbunch spacing of 42 cm. In this multi-bunch design one important problem that needs to be controlled is the multi-bunch beam break-up instability. One method of controlling this instability is by detuning the transverse modes of the accelerator cavities. This is accomplished by varying the cell dimensions (specifically the cell and iris radii) as one proceeds down the structure in such a way that the transverse modes are detuned, while the fundamental, accelerating mode is left unchanged. In a properly designed, gaussian detuned structure the transverse wakefield excited by the first bunch in the train can be made to cancel sufficiently at the positions of bunches 2 to 25, after which it begins to grow again. To accommodate the long NLC bunch train it was suggested to build the linac out of 4 types of structure types. Another important question for the design of discrete focusing, we find that the final position (designated by subscript f) of the mth bunch is given by

\[ x_{mf} = -e^2 N L_o S_{\text{nm}} \sum_{i=1}^{N_f} \frac{\beta_i}{E_i} \sin \beta_i \sqrt{\frac{L_o}{E_i}} \]  

(2)

where \( \beta \) is the phase advance between positions \( z_i \) and \( z_f \), and \( S_{\text{nm}} = \sum_n W[(m-n')\Delta s] \), a parameter which we will call the sum wake. Note that bunch m's final angle \( x_{mf} \) is given by an expression similar to Eq. (2); in particular, it is also proportional to \( S_{\text{nm}} \), its only m dependence. Therefore, in phase space all bunches lie on a straight line that goes through the origin. (However, also note that for an accelerator with more than one structure type, as is proposed for the NLC, this will no longer be true.)

The growth in projected emittance, if the fractional growth is small, is as follows. Consider linear accelerator and an average beta function variation \( \beta \sim E^{1/2} \), as in the NLC. Then for an ensemble of machines, each of which has normally distributed, uncorrelated, structure offsets with rms \( \langle x_{mf} \rangle_{\text{rms}} \), the final position of the mth bunch will also follow a normal distribution with rms (if \( \beta_f \approx \beta_f^0 \) and \( E_f \gg E_f^0 \))

\[ \langle x_{mf} \rangle_{\text{rms}} \sim e^2 N L_o S_{\text{nm}} \langle x_{\text{a}} \rangle_{\text{rms}} \sqrt{N_o \beta_0} \left[ \frac{1 - (E_f / E_f^0)^{1/2}}{E_0 E_f^0} \right]^{1/2} \]  

(4)

where subscript 0 designates initial conditions. It follows that the emittance growth will follow a \( \chi^2 \) distribution of degree 2, i.e. an exponential distribution \( \exp(-\langle \Delta \text{e}\rangle / \sigma_f) / \sigma_f \), with

\[ \sigma_f = e^2 N \beta_0 N_o L_o \langle x_{\text{a}} \rangle_{\text{rms}}^2 \left( \frac{1 - (E_f / E_f^0)^{1/2}}{E_0 E_f^0} \right) \]  

(5)

where \( \beta_0 \) is the beta function, \( N \) the particles per bunch, \( W \) the transverse wake function, \( \Delta s \) the bunch spacing, \( N_o \) the number of structures, \( x_{\text{a}} \) the offset of structure 1, and \( L_o \) the structure length. We assume that the bunch-to-bunch energy variation is small and can be ignored. Note that \( x_{\text{a}} \), \( \beta \), and \( \sigma_f \) are, in general, functions of \( z \). Of the driving terms in Eq. (1) the first we will call the betatron term, the second the misalignment term. For the model that we present here we limit ourselves to the case where the betatron term is small compared to the misalignment term, so that it can be dropped. Under this assumption, and now generalizing to the case of discrete focusing, we find that the final position (designated by subscript f) of the mth bunch is given by

\[ x_{mf} = -e^2 N L_o S_{\text{nm}} \sum_{i=1}^{N_f} \frac{\beta_i}{E_i} \sin \beta_i \sqrt{\frac{L_o}{E_i}} \]  

(2)

with \( \mu_{mf} \) the phase advance between positions \( z_i \) and \( z_f \), and \( S_{\text{nm}} = \sum_n W[(m-n')\Delta s] \), a parameter which we will call the sum wake. Note that bunch m's final angle \( x_{mf} \) is given by an expression similar to Eq. (2); in particular, it is also proportional to \( S_{\text{nm}} \), its only m dependence. Therefore, in phase space all bunches lie on a straight line that goes through the origin. (However, also note that for an accelerator with more than one structure type, as is proposed for the NLC, this will no longer be true.)

The growth in projected emittance, if the fractional growth is small, is as follows. Consider linear accelerator and an average beta function variation \( \beta \sim E^{1/2} \), as in the NLC. Then for an ensemble of machines, each of which has normally distributed, uncorrelated, structure offsets with rms \( \langle x_{mf} \rangle_{\text{rms}} \), the final position of the mth bunch will also follow a normal distribution with rms (if \( \beta_f \approx \beta_f^0 \) and \( E_f \gg E_f^0 \))

\[ \langle x_{mf} \rangle_{\text{rms}} \sim e^2 N L_o S_{\text{nm}} \langle x_{\text{a}} \rangle_{\text{rms}} \sqrt{N_o \beta_0} \left[ \frac{1 - (E_f / E_f^0)^{1/2}}{E_0 E_f^0} \right] \]  

(4)

where subscript 0 designates initial conditions. It follows that the emittance growth will follow a \( \chi^2 \) distribution of degree 2, i.e. an exponential distribution \( \exp(-\langle \Delta \text{e}\rangle / \sigma_f) / \sigma_f \), with

\[ \sigma_f = e^2 N \beta_0 N_o L_o \langle x_{\text{a}} \rangle_{\text{rms}}^2 \left( \frac{1 - (E_f / E_f^0)^{1/2}}{E_0 E_f^0} \right) \]  

(5)

where \( \beta_0 \) is the beta function, \( N \) the particles per bunch, \( W \) the transverse wake function, \( \Delta s \) the bunch spacing, \( N_o \) the number of structures, \( x_{\text{a}} \) the offset of structure 1, and \( L_o \) the structure length. We assume that the bunch-to-bunch energy variation is small and can be ignored. Note that \( x_{\text{a}} \), \( \beta \), and \( \sigma_f \) are, in general, functions of \( z \). Of the driving terms in Eq. (1) the first we will call the betatron term, the second the misalignment term. For the model that we present here we limit ourselves to the case where the betatron term is small compared to the misalignment term, so that it can be dropped. Under this assumption, and now generalizing to the case of discrete focusing, we find that the final position (designated by subscript f) of the mth bunch is given by

\[ x_{mf} = -e^2 N L_o S_{\text{nm}} \sum_{i=1}^{N_f} \frac{\beta_i}{E_i} \sin \beta_i \sqrt{\frac{L_o}{E_i}} \]  

(2)

with \( \mu_{mf} \) the phase advance between positions \( z_i \) and \( z_f \), and \( S_{\text{nm}} = \sum_n W[(m-n')\Delta s] \), a parameter which we will call the sum wake. Note that bunch m's final angle \( x_{mf} \) is given by an expression similar to Eq. (2); in particular, it is also proportional to \( S_{\text{nm}} \), its only m dependence. Therefore, in phase space all bunches lie on a straight line that goes through the origin. (However, also note that for an accelerator with more than one structure type, as is proposed for the NLC, this will no longer be true.)

The growth in projected emittance, if the fractional growth is small, is as follows. Consider linear accelerator and an average beta function variation \( \beta \sim E^{1/2} \), as in the NLC. Then for an ensemble of machines, each of which has normally distributed, uncorrelated, structure offsets with rms \( \langle x_{mf} \rangle_{\text{rms}} \), the final position of the mth bunch will also follow a normal distribution with rms (if \( \beta_f \approx \beta_f^0 \) and \( E_f \gg E_f^0 \))

\[ \langle x_{mf} \rangle_{\text{rms}} \sim e^2 N L_o S_{\text{nm}} \langle x_{\text{a}} \rangle_{\text{rms}} \sqrt{N_o \beta_0} \left[ \frac{1 - (E_f / E_f^0)^{1/2}}{E_0 E_f^0} \right] \]  

(4)

where subscript 0 designates initial conditions. It follows that the emittance growth will follow a \( \chi^2 \) distribution of degree 2, i.e. an exponential distribution \( \exp(-\langle \Delta \text{e}\rangle / \sigma_f) / \sigma_f \), with

\[ \sigma_f = e^2 N \beta_0 N_o L_o \langle x_{\text{a}} \rangle_{\text{rms}}^2 \left( \frac{1 - (E_f / E_f^0)^{1/2}}{E_0 E_f^0} \right) \]  

(5)

Here (\( S_{\text{nm}} \))_{\text{rms}} \sim \sqrt{\langle S_{\text{nm}} \rangle \langle S_{\text{nm}} \rangle} \). Assuming we can allow a certain emittance growth \( \sigma_f \) we can obtain a misalignment tolerance (\( x_{\text{a}} \))_{\text{rms}} \sim (\sigma_f)_{\text{rms}} / \sigma_f \). Fig. 1 displays the sum wake \( S_{\text{nm}} \) for one of the structure types; here (\( S_{\text{nm}} \))_{\text{rms}} = 7.2 V/PC/mm/m. Four Structure Types

In Ref. 2 it was shown that the use of 4 structure types can greatly reduce the emittance growth due to betatron...
oscillations, particularly for the bunches near the end of the train. Therefore the approximation of dropping the betatron term in Eq. (1) will be more valid than before. As for the emittance growth due to the alignment term it can be shown that its expectation value is still given by Eq. (5), provided that now the sum wake is understood to be averaged over the 4 structure types in quadrature; the emittance distribution, however, will no longer follow a simple exponential distribution.

To benefit from the use of 4 structure types in the misalignment term effects (as we did in the betatron term effects) we need to align the structures particularly well within each group of 4. Let us suppose each group is on its individual girder, to facilitate the alignment. To include the effect of girder misalignments on emittance we need to add a second term to Eq. (5), one that differs only in that the combination of parameters \( N_s L_s / \sigma_m^2 \) is replaced by the one corresponding to the girder scale. If we take subscript \( g \) to represent girder quantities, we have \( N_g L_g = N_s L_s / 4 \), \( L_g = 4 L_s \), and \( S_m \) is found by simply averaging the sum wake over the 4 types. Fig. 2 displays \( S_m \) v.s. bunch number \( m \); here \( (S_m)_{rms} = 0.38 \text{ V/\mu C/mm/m} \).

**Sensitivity to Slight Frequency Changes**

The distribution of dipole modes is approximately gaussian with a central frequency of 15 GHz, an rms of 2.5%, and a total width of 10%, and the bunch spacing is 42 cm. Therefore a kind of resonance can develop between a mode frequency and the 20th, 21st, or 22nd harmonic of the bunch frequency. For one structure type the relative mode spacing at the center of the distribution is \( 4.5 \times 10^{-4} \) [1]; therefore, a small shift in the mode frequencies relative to the bunch frequency can result in a large change in effect. To show this effect, in Fig. 3 we plot \( (S_\delta)_{rms} \) for the 4 structure types as function of small changes in relative bunch spacing (the dashed curves). We see more than a factor of 5 variation when the relative bunch spacing is changed by only \( 2.5 \times 10^{-4} \). (However, in a real accelerator, where each structure has different, random manufacturing errors, the effect of the fluctuations will be reduced.) The solid line in Fig. 3 gives the sum of the contributions of the 4 structure types added in quadrature. This gives the effect on emittance growth when the errors are on the structure scale and 4 structure types are used. We note that the fluctuations are much smaller. Fig. 4 gives \( (S_\delta)_{rms} \) on the girder scale when 4 structure types are used. We note that the average value of \( (S_\delta)_{rms} \) is therefore the expected gain in tolerance on the girder scale (remember the factor \( \sqrt{N_s/N_g L_s/L_g} \) is about 10.

**Comparison with Numerical Results**

For the numerical comparisons we use the NLC parameters: \( eN = 1 \text{ nC}, \gamma_0 = 10 \text{ GeV}, E_f = 280 \text{ GeV}, \beta_0 = 8 \text{ m}, L_s = 1.8 \text{ m}, N_s = 3600, \) and normalized emittance \( \gamma \varepsilon = 3 \times 10^{-8} \text{ rm} \). The lattice is a piecewise 90 degree-per-cell FODO type; the number of structures between quads is given by the integer part of \( \sqrt{4E/E_0} \). Fig. 5 displays the results of numerical tracking when there are 4 independently misaligned structure types, and when \( (S_\delta)_{rms} = 5 \mu \text{m} \). The dashes give an exponential approximation, with \( \sigma_\delta \) given by Eq. (5). The average growth obtained numerically, 0.091, agrees with that obtained analytically, 0.095, but the distributions differ slightly.

In Table 1 we compare the results of three methods of finding the tolerance for 25% emittance growth: (i) using Eq. (5); (ii) numerically performing the sum Eq. (2)
Fig. 5. Tracking results for 4 independently misaligned structure types, with \((x_0)_{rms} = 5 \mu m\).

(and its counterpart for \(x_{si}\), but with \(x_{si}\) replaced by \((x_0)_{rms}\), to find the expected emittance for the real lattice; and (iii) tracking. The error factors give the rms variation due to bunch spacing. We give results for the case of only 1 structure type, and for 4 structure types on both the structure \((a)\) and girder \((g)\) scales. In the first case the analytical tolerances are much larger than the tracking results, indicating that dropping the betatron term of Eq. (1) is not a good approximation. Also, the variation terms are large, showing a great sensitivity to frequency changes. (With random fabrication errors along the linac this sensitivity should decrease.) In case 2 the analytic approximation agrees with the tracking results. Finally, in case 3, 4 structure types on the girder scale, we note that the accurate inclusion of the amplitude and phase of the betatron sum is important for a good estimate.

<table>
<thead>
<tr>
<th>Scale</th>
<th>Ntype</th>
<th>Eq. (5)</th>
<th>Num. sum</th>
<th>Tracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure (a)</td>
<td>1</td>
<td>10.9 ± 6.6</td>
<td>11.5 ± 8.4</td>
<td>4.3 ± 3.2</td>
</tr>
<tr>
<td>Structure (a)</td>
<td>4</td>
<td>8.3 ± 0.6</td>
<td>8.9 ± 0.6</td>
<td>8.7 ± 0.6</td>
</tr>
<tr>
<td>Girder (g)</td>
<td>4</td>
<td>120 ± 60.</td>
<td>32.0 ± 6.4</td>
<td>32.0 ± 4.9</td>
</tr>
</tbody>
</table>

Table 1. Alignment tolerances in microns for 25% emittance growth.

THE HIGHER DIPOLE BANDS

The analysis of the detuned structure has focused almost entirely on the modes of the first dipole band (including the effect of the second band modes on the first) since the kick factors of these modes are at least an order of magnitude larger than those of the other bands. However, when we perform the uncoupled calculation including also the effects of the modes of bands 3-8 we find that the wakefield amplitude is now an unacceptable 10% at \(s = 42\) cm, the position of the second bunch, and it decreases only slowly as we move further back in the bunch train.

Fig. 6a illustrates the cause of the problem. In this figure we plot the dispersion curves representing a cell near the beginning, middle, and end of the detuned structure, for dipole bands 3 to 8, when the iris thickness is kept at 1.46 mm (a), and when it is varied. The bands are alternately given by solid and dashed curves for ease of viewing.

Running URMEL [3] we find that near the light line band 3 is a TM111-like mode, and band 6 a TM121-like mode. This suggests that by varying the iris thicknesses along the structure, with the thinner irises in cells with the larger radii—i.e. near the beginning of the structure—and the thicker ones in cells with smaller radii—i.e. near the end of the structure, we can detune these modes more. Fig. 6b shows the results when the iris thickness is changed to 1.67 mm, 2.06 mm, and 2.45 mm for respectively the representative cell near the beginning, middle, and end of the structure. We see that the 3\(^{rd}\) and the 6\(^{th}\) band curves have separated near the light line. In the actual structure the average iris thicknesses vary as a gaussian with an average of 1.5 mm, an rms of 0.25 mm, and a total variation of 1 mm. We find the contribution of bands 3-8 to the wake at following bunches has been reduced to 1%.

ACKNOWLEDGEMENTS

The authors thank the members of the Structures Group at SLAC for useful comments and suggestions.

REFERENCES