Beam loss due to multiple scattering with residual gases is studied for a storage-type electron synchrotron where electrons are injected at 15 MeV and accelerated to 800 MeV in 100 s. The temporal behavior of the beam profile was numerically estimated by taking account of damping and the multiple scattering. It was found that almost no beam loss is expected in the horizontal direction, because the adiabatic damping near the inflector wall exceeds the multiple scattering by CO molecules in the pressure of 10^{-9} Torr right after the injection. In contrast, the scattering in the vertical direction is comparable to the adiabatic damping for about 5 s after the injection. The dependence of the beam loss on the vertical aperture (d) was estimated, showing that the beam loss is less than 1% for d=12 mm and 20% for d=7 mm. The duct half height, thus, can be designed to be 15 mm in this respect.

Introduction

Storage-type electron synchrotrons with low energy single multi-turn injection have been studied both experimentally [1] and conceptually [2,4]. The simplicity of this concept appears to be quite attractive in view of the dependence of the beam loss on the atomic number, we assume that the residual gas is all CO. Eq. (1) is then rewritten as

\[ \langle \sigma^2 \rangle = 5 \times 10^5 \frac{P_{\text{RF}}}{E \pm Z} \]  \hspace{1cm} (3)

where the unit of \( \langle \sigma^2 \rangle \) is rad^2, \( P_{\text{RF}} \) (Torr) is the residual gas pressure, \( \sigma(t) \) is the traveling time of electrons, \( E(\text{MeV}) \) is the electron energy, and the value of \( Z \) was taken to be 10.

To calculate the beam loss by multiple scattering, the distribution function is described with respect to the beam divergence denoted by \( x' \) and \( y' \) for the horizontal and vertical directions, respectively. This is the safer side of approximation, since the kick angle of the electron located near the aperture becomes smaller when it is transposed to the \( x' \) or \( y' \) axis in the phase space. Fig. 1 shows the schematic beam distributions with respect to the beam divergence right after the multi-turn injection.

Computational Method

Electrons travelling around the machine undergo many small-angle scattering with residual gas molecules. Since the events are completely independent, the resulting scattering distribution is Gaussian, the mean square angle of which is given by [7]

\[ \langle \sigma^2 \rangle = 10 \ln \frac{2 \pi e^2}{m v^2} \ln \frac{\sigma_{\text{max}}}{\sigma_{\text{min}}} \]  \hspace{1cm} (1)

with

\[ \ln \frac{\sigma_{\text{max}}}{\sigma_{\text{min}}} = \ln \frac{2 \pi e^2}{m v^2} \]  \hspace{1cm} (2)

where \( \sigma \) is the molecular density, \( Z \) is the atomic number of the molecule, \( m \) and \( v \) are the electron momentum, \( m \) is its velocity, \( \sigma_{\text{max}} \) and \( \sigma_{\text{min}} \) are the maximum scattering and cut-off angles, respectively, and \( d \) is the thickness of gas molecules.

In the actual case, the main components of residual gases are H\(_2\) and CO which are typical in ultra-high vacuum conditions. In view of the dependence of \( \langle \sigma^2 \rangle \) on the atomic number, we assume that the residual gas is all CO. Eq. (1) is then rewritten as

\[ \langle \sigma^2 \rangle = 5 \times 10^5 \frac{P_{\text{RF}}}{E \pm Z} \]  \hspace{1cm} (3)

where the unit of \( \langle \sigma^2 \rangle \) is rad^2, \( P_{\text{RF}} \) (Torr) is the residual gas pressure, \( \sigma(t) \) is the traveling time of electrons, \( E(\text{MeV}) \) is the electron energy, and the value of \( Z \) was taken to be 10.

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Fig. 1 Schematic beam distributions in the (a) horizontal and (b) vertical directions with respect to the beam divergence right after the multi-turn injection.

where \( \sigma_x \) is the horizontal \( \sigma \) function and the subscript inf denotes the inflector. As for the vertical direction, the distribution of the injected beam, which is assumed to be Gaussian, is maintained when the physical acceptance is given by

\[ x_{\text{inf}} = x_{\text{inf}}/\sigma_{x,\text{inf}} \]  \hspace{1cm} (4)

where \( x_{\text{inf}} \) is the horizontal \( x \) function and \( \sigma_{x,\text{inf}} \) is the standard deviation at the inflector. As for the vertical direction, the distribution of the injected beam, which is assumed to be Gaussian, is maintained when the physical acceptance is given by

\[ y_d = y_d/\sigma_{y,\text{max}} \]  \hspace{1cm} (5)

is sufficiently larger than the dispersion of the injected beam. Here, \( y_d \) is the effective value of the duct half height, which is defined as the real size minus COO, and \( \sigma_{y,\text{max}} \) is the maximum vertical \( \sigma \) function. When the value of \( y_d \) is small, which usually happens in large COO, the injected beam is cut by the duct wall, resulting in a step-like distribution at the well position.

Starting from these initial conditions, the time dependence of the beam intensity was estimated by taking account of both scattering and damping. The beam intensity in this case is defined by the rate which is left without suffering initial cut in the vertical direction and succeeding scattering beyond the
The beam distribution function, represented in the $x'$ direction, considering scattering with the residual gas molecules during $\delta t$ is given by

$$P(x'(t + \delta t)) = \int P(x'(t)) \exp \left(-\frac{(x' - x_0)^2}{\sigma_x^2} - \frac{(y' - y_0)^2}{\sigma_y^2}\right) dx' dx'' dy'' dz'', \quad \text{in} \quad E(t) \quad \text{for} \quad E(t + \delta t)$$

where $x_0$ and $y_0$ are the coordinates of the beam at time $t$, and $\sigma_x$ and $\sigma_y$ are the standard deviations in the horizontal and vertical directions, respectively.

The energy dependence of $\sigma_y$ and $\sigma_y$ during $\delta t$ was taken into account in the condition that the beam is linearly accelerated to the final value.

Variable time steps, which are twenty every order, were used in calculating the temporal dependence of the beam intensity. The first step was chosen to be $10^{-9}$ s after confirming the results obtained with $10^{-8}$ s and $10^{-9}$ s to be almost the same. Concerning the mesh numbers in the $x'$ and $y'$ directions, they were both chosen to be 50 after confirming that the results with the meshes of 50 and 200 are almost duplicate.

The residual gas pressure is given by

$$P = \rho_{\text{g}}(E) \frac{d\rho}{dE} dE \frac{1}{S}, \quad \text{in} \quad E(t) \quad \text{for} \quad E(t + \delta t)$$

where $\rho_{\text{g}}(E)$ is the radiation loss per unit energy, $d\rho$ is the photon flux, $dE$ the photon energy, and $S$ is the pumping speed. The mechanism of photo-desorption is that electrons are stored current of this machine is $500 \, \text{mA}$.

Results and Discussion

Table 1 shows the parameters of the machine studied in this paper. Electrons are injected in 15 MeV and accelerated to the final energy, 800 MeV, in 100 s. The bending magnetic field is 1.5 T in 800 MeV, the bending angle per the cluster type magnet is 90°, and the magnetic index is 0.4 on the electron orbit. The circumference of the orbit is 21 m which is rather small for 800 MeV machines. Table 2 shows the detailed parameters concerning beam loss calculation. The injector wall is located at $x = 35 \, \text{mm}$ and $y = 2.2 \, \text{m}$. As for the vertical direction, the maximum $\beta = 6.6 \, \text{m}$, and the duct size is $30 \, \text{mm}$. Table 3 shows the beam parameters of an injector linac with a suitable energy compression system. The beam divergence, which is the most critical parameter in this estimation, is 1 mrad. Analysis on multi-turn injection has shown that about 10 turns of the beam can be effectively injected in the conditions shown in tables 1 to 3. Assuming the capture efficiency in the bunching process to be 5%, the designed maximum stored current of this machine is 500 mA.

Table 2 Detailed parameters for beam loss calculation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inflector location</td>
<td>$x = 35 , \text{mm}$</td>
</tr>
<tr>
<td>$y$ at the inflector</td>
<td>$2.2 , \text{m}$</td>
</tr>
<tr>
<td>Maximum $\beta$</td>
<td>$6.6 , \text{m}$</td>
</tr>
<tr>
<td>Vertical duct size</td>
<td>$30 , \text{mm}$</td>
</tr>
</tbody>
</table>

Table 3 Injected beam parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam current</td>
<td>$100 , \text{mA}$</td>
</tr>
<tr>
<td>Beam diameter</td>
<td>$4 , \text{mm}$</td>
</tr>
<tr>
<td>Beam divergence</td>
<td>$1 , \text{mmrad}$</td>
</tr>
<tr>
<td>Energy spread</td>
<td>$0.5%$</td>
</tr>
</tbody>
</table>

Fig. 2 shows the temporal change of the horizontal beam profile during 8.9 s after the injection. It is indicated that the ribbon-like beam shrinks without suffering any kick-off beyond the inflector wall after the injection. This is because the adiabatic damping term of off-axis beam is comparatively large, since the damping term is proportional to $x'$ as was shown in eq. (8). It was confirmed that the behavior of beam shrinkage does not change for a possible COD ($< 5 \, \text{mm}$)
The CO pressure used in this estimation is $10^{-9}$ Torr, since photo-desorption is negligible during this period (the beam energy is still below 95 MeV corresponding to $\lambda_c = 1.16 \times 10^4 \text{ A}$).

Fig. 3 shows the temporal change of the vertical beam profile during the first 8.9 s for $y_d = 15$ mm and 5 mm. When the COD is zero, namely $y_d = 15$ mm, the injected beam is completely included in the duct and starts shrinking right after the injection. This is again because the adiabatic damping exceeds the scattering due to residual gases in large $y'$. When the COD is 10 mm (corresponding to $y_d = 5$ mm), which was assumed to be larger than the horizontal one because of $\lambda_y > 5 \times 10^{-6}$ mrad, the acceptance in the $y'$ direction is only 0.76 mrad. In this case, the beam is cut by the duct wall in the first turn after the injection, and the scattering term exceeds the adiabatic damping term till $t = 5$ s. The radiation damping then becomes comparable to the scattering, and the beam starts shrinking.

Fig. 4 shows the temporal dependence of the beam intensity during the first 8.9 s for various $y_d$ values. The beam cut in the first turn explained in Fig. 3b happens in $y_d < 10$ mm. The beam intensity at $t = 0$ s including this effect is 99, 92, 69 and 25% for $y_d = 10, 7.5, 5$ and 2.5 mm, respectively. Although almost no beam is lost in $y_d \geq 12.5$ mm, general trend is that the beam intensity decays for about 5 s till the radiation damping becomes to exceed the scattering with residual gases. Since the COD is expected to become less than 2 mm by operating correction magnets, almost all the beam appears to survive after the COD correction.

In conclusion, the beam loss due to multiple scattering with residual gases was studied in the storage-type electron synchrotron, the injection energy of which is 15 MeV and the acceleration time to 800 MeV is 100 s. It was found that the beam loss due to the multiple scattering depends on the value of vertical COD and can be reduced to the negligible level when the COD is corrected.

References