Ultralow Emittance Light Sources

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Outline

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The Facility
Acknowledgements

What’s Known

• Dedicated third generation light sources: ~20 years of optimizations.
• The horizontal emittance (isomagnetic lattice) is given by

\[ \varepsilon_x \text{ [nm} \cdot \text{rad]} = 7.84 \times 10^3 \cdot \frac{(E \text{ [GeV]})^2 F}{J_x N_b^3} \]

\( N_b \) is the number of dipoles, \( 0 < J_x < 3 \), \( F \geq 1 \). No dipole gradients => \( J_x \approx 1 \).
• Generalized Chasman-Green Lattices: DBA, TBA, QBA, 7-BA [1].
• Effective emittance => chromatic cells.
• Increasing \( N_b \) reduces \( \varepsilon_x \) but also reduces peak dispersion, which makes the chromatic correction less effective => “chromaticity wall” [2].
• Damping wigglers (DWs): damping rings and conversion of HEP accelerators [3,4].
• Mini-Gap Undulators (MGUs), Three-Pole-Wigglers (TPWs) inside DBA [5].
What’s New

• Use of damping wigglers to reduce horizontal emittance and as high flux X-ray sources => achromatic cells and weak dipoles.
• Medium energy ring (3 GeV) with ~30 DBA cells.
• Vertical orbit stability requirements.
• Generalized higher order achromat.
Global Optimization

1. Horizontal emittance (natural): damping ↔ diffusion (fundamental limit is IBS).
2. Optimize (for Insertion Devices (IDs)) [6]:

\[
\varepsilon_x \sim \frac{1}{R^2 \cdot P}
\]

\( R \) bend radius
Challenges

Non-linear dynamics:

• Medium energy: control of Touschek lifetime and momentum aperture.
• 30 DBA cells: control of tune footprint.
• Control of impact of DWs and IDs: include leading order nonlinear effects from DWs in Dynamic Aperture (DA) optimizations.
• Optics requirements for IDs and top-up injection: introduce alternating straights with high- and low horizontal beta function \(\leftrightarrow\) reduced symmetry (30 \(\rightarrow\) 15).
• DBA: momentum dependence of optics functions \(\Rightarrow\) number of chromatic sextupole families.

Technical

• Weak dipoles: introduce TPWs (adjacent to the dipoles) \(\Rightarrow\) control of peak beta functions and horizontal dispersion.
• Vertical orbit stability: sub micron \(\Rightarrow\) pushing the state-of-the-arts [7,8].
# Lattice Parameters

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<td>Circumference</td>
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<td>Beam Current ( I_b )</td>
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<tr>
<td>Bending Radius ( R )</td>
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<tr>
<td>Dipole Energy Loss ( U_0 )</td>
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<tr>
<td>Emittance ( \epsilon_x, \epsilon_y ): bare/w. 8 DWs</td>
<td>(2.1, 0.01)/(0.6, 0.01) nm·rad</td>
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<tr>
<td>Momentum compaction</td>
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<td>RMS Energy Spread: bare/w. 8 DWs</td>
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<td>Working point ( \nu_x, \nu_y )</td>
<td>(32.4, 16.3)</td>
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<td>Chromaticity ( \xi_x, \xi_y )</td>
<td>(-100, -42)</td>
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<td>Peak Dispersion ( \eta_x )</td>
<td>0.45 m</td>
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<tr>
<td>Beta Function ( \beta_x, \beta_y ): long/short straight</td>
<td>(18, 3)/(3, 3) m</td>
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</table>
Intrabeam Scattering (IBS)

Equilibrium

\[
\varepsilon_x = \varepsilon_{x}^{\text{SR}} + \varepsilon_{x}^{\text{IBS}} = \tau_x (E_{x}^{\text{SR}})(H \cdot (D_\delta^{\text{SR}}(R) + D_\delta^{\text{IBS}}))
\]

where

\[
\sigma_\delta^2 = \tau_\delta (E_{SR}) (D_\delta^{\text{SR}}(R) + D_\delta^{\text{IBS}})
\]

\[
\delta \equiv \frac{E - E_0}{E_0}, \quad H \equiv \tilde{T}^T \tilde{\eta}, \quad \eta \equiv \begin{bmatrix} \eta_x \\ \eta'_x \end{bmatrix}, \quad \tilde{\eta} \equiv A^{-1} \eta, \quad A^{-1} = \begin{bmatrix} 1/\sqrt{\beta_x} & 0 \\ \alpha_x/\sqrt{\beta_x} & \sqrt{\beta_x} \end{bmatrix}
\]
IBS Trade-Off
Touschek Lifetime

Cross section (Møller scattering) × phase space density [9]

\[ \frac{1}{\tau_{1/2}} = \frac{r_e^2 c_0 N_e}{8\pi^3} \frac{F\left(\frac{\delta(s)}{\gamma \sigma_x(s)}\right)^2}{C \sigma_x(s) \sigma_z(s) \sigma_x'(s) \delta(s)} \]

where

\[ F(x) = \frac{1}{2} \int_0^1 \left[ \frac{2}{u} - \ln \left( \frac{1}{u} \right) - 2 \right] e^{-x/u} \, du \]

- \( r_e \) classical electron radius,
- \( N_e \) no of electrons/bunch,
- \( \sigma_s \) rms bunch length,
- \( C \) circumference,
- \( \sigma_x(s), \sigma_z(s) \) horizontal- and vertical rms beam size,
- \( \sigma_x'(s) \) horizontal rms beam divergence,
- \( \delta(s) \) momentum acceptance.
Touschek Life Time Trade-Offs

![Graphs showing lifetime and energy acceptance](image-url)
Damping Wigglers

The natural horizontal emittance scales with the radiated power

\[ \frac{\varepsilon_w}{\varepsilon_0} \approx \frac{U_0}{U_0 + U_w}, \quad \frac{\delta_w}{\delta_0} = \sqrt{1 + \frac{8 B_w U_w}{3 \pi B_0 U_0}} \frac{U_w}{U_0} } \]
Damping Wiggler Trade-Offs

Emittance Control for NSLS-II
Optics Design Guidelines

The traditional approach, i.e., to first design the linear optics and then attempt to control the DA is inadequate for high performance lattices e.g. [10-11]. For a streamlined approach, the nonlinear effects must be considered from the start [12]. In particular, the following guidelines have been provided (the numbers have evolved with time) [13]:

- horizontal chromaticity per cell, $\xi_x \leq 3.5$,
- horizontal peak dispersion $0.3 \, \text{m} \leq \eta_x \leq 0.5$,

and

<table>
<thead>
<tr>
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<th>Hor and Ver Dynamic Acceptance [mm·mrad]</th>
<th>Hor DA [mm]</th>
<th>$\delta$ [%]</th>
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</thead>
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<tr>
<td>Bare Lattice (2.5 D.O.F.)</td>
<td>~25</td>
<td>±20</td>
<td>±2.5</td>
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<tr>
<td>“Real” Lattice (3 D.O.F.)</td>
<td>~20</td>
<td>±15</td>
<td>±2.5</td>
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Optics Constraints

The DBAs have ~6 constraints:

• linear achromat ($\eta_x = \eta'_x = 0$ at the entrance),

• small emittance ($\min(H) \Rightarrow (\alpha_x, \beta_x)$ fixed at the entrance),

• and symmetric ($\alpha_{x,y} = 0$ at the center).

Similarly, the long- and short matching sections have 10 constraints:

• symmetric ($\alpha_{x,y} = 0$ at the center),

• $\beta_{x,y}$ at the center,

• and the cell tune $\nu_{cell}$.

On the other hand, the lattice has only 8 quadrupole families.
Generalized Higher Order Achromat

- Introduce two chromatic sextupole families and choose the cell tune for \( N \) super cells \( \mathcal{M} \) such that

\[
\mathcal{M} = M_1 M_2 \ldots M_N, \quad M_k = \mathcal{A}^{-1} e^{i : h :} \mathcal{R}_k, \quad R^N = \begin{bmatrix} 1 & 0 \\ 0 & \pm i \end{bmatrix}
\]

In particular, so that resonances up to 4th order

\[
n_x \nu_x + n_y \nu_y = n, \quad |n_x| + |n_y| \leq 4
\]

are cancelled (by symmetry):

- Control the residual amplitude dependent tune shift and free up the choice of working point by adding geometric sextupoles [SLS].

- Control the residual nonlinear chromaticity by adding chromatic multipole families as needed.

- Optimize dynamic- and momentum aperture (from tracking) by joint minimization of the driving terms and variation of the cell tune.
A 5-Cell Second Order Achromat (2 chrom families)

$h_{10110}$  
$h_{21000}$  
$h_{30000}$  
$h_{10020}$  
$h_{10200}$
Linear Optics and Tune Scan

• An $11 \times 11$ grid of working points is obtained that meets optics requirements.
• For each working point, the driving terms are minimized and a weighted average of the dynamic- and momentum aperture ($DA/\sqrt{\beta_x\beta_y}$) is computed by tracking.
Frequency Map and Resonance Avoidance

Distance from Resonances

without DWs.
## Optics Tolerances

### Estimated tolerances

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\Delta b_2 / b_2$</th>
<th>$(\Delta \beta_x, \Delta \beta_y)_{\text{rms}}$</th>
<th>$(\Delta v_x, v_y)_{\text{rms}}$</th>
<th>$(\Delta x_{\text{cod}}, \Delta y_{\text{cod}})_{\text{rms}}$</th>
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<tbody>
<tr>
<td>Tolerance</td>
<td>$\sim 5 \times 10^{-4}$</td>
<td>$\sim (2%, 3%)$</td>
<td>$\sim (0.003, 0.002)$</td>
<td>$\sim (50, 50) \mu m$</td>
</tr>
</tbody>
</table>

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**Diagrams:**

1. **Dynamic Aperture vs. $\Delta b_2/b_2$**

2. **Dynamic Aperture vs. Orbit in the Sextupoles**
Impact of Insertion Devices

The averaged Hamiltonian is (planar)

$$\langle H \rangle_{\lambda_w} = \frac{p_x^2 + p_y^2}{2(1 + \delta)} + \left(\frac{B_w}{B_p}\right)^2 \frac{\cosh^2(k_y y)}{4k_z^2(1 + \delta)} - \delta + O(p_{x,y})^4, \quad k_y = k_z = \frac{2\pi}{\lambda_w}$$

To leading order one obtains

$$\Delta \nu_y = \frac{1}{8\pi} \left(\frac{B_w}{B_p}\right)^2 \langle \beta_y \rangle L_w, \quad \frac{\partial \nu_y}{\partial J_y} = \frac{\pi}{4} \left(\frac{B_w}{B_p}\right)^2 \frac{\langle \beta_y^2 \rangle}{\lambda_w^2} L_w$$

Since $\beta(s) = \beta_0 [1 + (s/\beta_0)^2]$, optimum beta for min impact is:

- stay clear $\Rightarrow \beta_0 = L/2$, linear optics $\Rightarrow \beta_0 = L/2^{4\sqrt{3}}$,
- nonlinear dynamics $\Rightarrow \beta_0 = L/2^{4\sqrt{5}}$. 
Conclusions

• The “chromaticity wall” has been avoided by using damping wigglers. Furthermore, these turned out to be useful high-flux X-Ray sources.

• The emittance can be reduced as the facility evolves.

• The nonlinear effects are taken into account for the optics design. In particular, by providing guidelines for chromaticity per cell and peak dispersion.

• The dynamic- and momentum aperture are improved by implementing a generalized higher order achromat. It is optimized by a joint optimization of the driving terms and working point.

• The ultimate-low emittance limit can be reached by this approach. It is a matter of power consumption and circumference.
References

[3] H. Wiedemann “An Ultra-Low Emittance Mode for PEP Using Damping Wig-
[5] A. Nadji et al “A Modified Lattice for SOLEIL with Large Number of Straight
Sections” SSILS (2001).
NSLS-II Design with Sub-Nanometer Horizontal Emittance”.
[7] C. Steier et al “Operational Experience Integrating Slow and Fast Orbit Feed-
backs at the ALS” p. 2786-2788 EPAC04.
PAC03.
(1989).


Back-up Slides
Damping Wigglers

The natural horizontal emittance scales with the radiated power

\[
\frac{\varepsilon_w}{\varepsilon_0} = \frac{1 + f}{1 + \frac{U_w}{U_0}} \approx \frac{U_0}{U_0 + U_w}, \quad \frac{\delta_w}{\delta_0} = \sqrt{1 + \frac{8 B_w U_w}{3\pi B_0 U_0}} \left(\frac{U_w}{U_0}\right), \quad \frac{U_w}{U_0} = \frac{L_w}{4\pi \rho_0 \left(\frac{B_w}{B_0}\right)^2}
\]

where

\[
f \approx \frac{2 C_q \gamma^2 L_w \rho_0}{3 \pi^2 \varepsilon_0 \rho_w^3} \left[1 + \left(\frac{K_w}{\gamma}\right)^2 \langle \beta_x \rangle + \frac{\eta_{x0}^2}{\beta_{x0}} + \beta_{x0} \eta_{x0}'^2 \right], \quad \frac{K_w}{\gamma} = \frac{\lambda_w}{2\pi \rho_w}, \quad C_q = 3.8 \times 10^{-13},
\]

\[
\beta_x(s) = \beta_{x0} \left[1 + \left(\frac{s}{\beta_{x0}}\right)^2\right], \quad \eta_x(s) = \frac{1}{\rho_w} \left(\frac{\lambda_w}{2\pi}\right)^2 \left[1 - \cos\left(\frac{2\pi s}{\lambda_w}\right)\right] + \eta_{x0} + \eta_{x0}' s
\]
Control of Optics

Control of linear optics perturbation from IDs:

- optics response matrix

\[
\begin{bmatrix}
\frac{\Delta \beta_i}{\beta_i}, \ldots, \Delta \mu_i, \ldots, \Delta \nu_x, \Delta \nu_y
\end{bmatrix}^T = T(\beta, \mu)[\Delta b_{2,1}, \ldots, \Delta b_{2,N_Q}]^T
\]

Control of vertical beam size:

- beam response matrix

\[
\begin{bmatrix}
\frac{\partial x_i}{\partial p_{y,k}}, \ldots, \frac{\partial y_i}{\partial p_{x,k}}, \ldots, \Delta \eta_y, \ldots
\end{bmatrix}^T = U(\beta, \mu)[\Delta a_{2,1}, \ldots, \Delta a_{2,N_{SQ}}]^T
\]

- Include dispersion wave, if needed [14].

Invert $T$, $U$ by SVD, compute, and apply the corrections (i.e., like orbit correction).
The Polymorphic Tracking Code (PTC)

Engineering Tolerances

Initial Conditions

Lie-Lib

(Forest)

Lie generators,
Param. Dep.

Lie-Lib

(Taylor
Series Maps)

Tune
Scans

Frequency
Maps

Optics, ε,

Mod. Symplectic
Integrator

Taylor
Series Maps

NAFF-Lib
(Laskar)

Dynamic
Aperture

Vector Flow

(\(V \cdot \nabla\))

Polymorphic
Number Class

(+, -, *, /)

Phase Space Vector

(+, -, *, /)

Magnetic
Lattice

Euclidian
Group

Polymorphic
Number Class

(+, -, *, /)

TPSA-Lib
(Berz)

(+, -, *, /)

Phase Space Vector

(+, -, *, /)

Engineering Tolerances

Initial Conditions

Lie-Lib

(Forest)

Lie generators,
Param. Dep.

Lie-Lib

(Taylor
Series Maps)

Tune
Scans

Frequency
Maps
A 10-Cell Third Order Achromat

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Tune Footprint

without DWs.
Frequency Map and Resonance Avoidance (w DWs)