ADVANCES IN PARAMETRIC-RESONANCE IONIZATION COOLING*

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Abstract

The concept of combining ionization cooling with parametric resonances (PIC) was proposed earlier [1] in order to lead to an additional reduction of muon transverse emittances by an order of magnitude so that high luminosity in a collider can be achieved with fewer muons per bunch. After 6D cooling performed in a *Helical Cooling Channel,* the muon beam is injected in a channel with *alternating dispersion* correlated with particle free oscillations in two planes. A half integer resonance is induced in such a way that the beam becomes focused at *thin wedge absorbers* placed near points of *zero dispersion*. We discuss detuning caused by chromatic and spherical aberrations, non-linear fields, and space charge and the techniques needed to reduce the detuning, including the use of *coupling resonances*.

INTRODUCTION

Experiments at a muon collider require high luminosities, of order 10^{34} $cm^{-2}s^{-1}$ or more, in order to obtain reasonable rates for events having point-like cross sections. High luminosities require small transverse emittances of the muon beams. The reduction of six-dimensional (6D) phase space volume of muons (produced by a proton beam on a target) is possible using ionization cooling (IC). A number of conceptual designs of muon beam capture and transport implementing that include IC are under development [2]. The reduction of initial 6D emittance of muons by a factor of about 50000 has been shown in simulation of a *Helical Cooling Channel* (HCC) with pressurized hydrogen gas as absorber. Innovative designs and test experiments of IC sections with helical magnets and RF cavities filled with pressurized hydrogen gas are under way.

Based on existing technology of superconducting magnets, the HCC concept promises to reach muon normalized emittances about 200 microns in each of three planes [3]. It is possible to attain more transverse cooling, if the HCC is followed by special transport line in which the muon beam is periodically focused at thin absorbers. To avoid the extra extension of the beam and larger magnet apertures, it was proposed to gradually develop the beam envelope along the line by implementing a weak *parametric resonance* to the beam optics. The concept includes an *alternating dispersion* design ("snake channel"). The dispersion is needed for continuation of longitudinal cooling and compensation for chromatic detuning off the parametric resonance, but it should be minimal at the absorbers to minimize the absorber energy straggling impact on transverse emittance.

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Below we report our recent progress in analysing and developing the PIC idea. It includes: 1) conditioning of the linear optics design; 2) use of coupling resonance to organize the PIC process in two planes and simplify compensation for aberrations; 3) calculation of space charge detuning effects; 4) formulation of conditions of compensation for the main aberrations in beam optics; 5) thoughts about minimizing the PIC channel length.

PIC SNAKE CHANNEL

A beam orbit with alternating dispersion as discussed can be created by an alternating dipole field, as shown in figure 1.

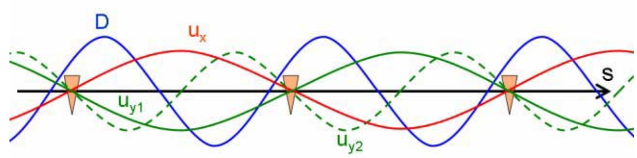

Figure 1: Lattice functions of a beam cooling channel suitable for PIC showing *D*, the dispersion (blue), the horizontal betatron oscillation u_x (red), and two possible solutions for the vertical betatron oscillation, u_{y1} and u_{y1} (green). The triangles represent the wedge absorbers. The dipoles and quadrupoles are not shown.

The absorber plates then are positioned near zero dispersion points. Obviously, the betatron wave length in the horizontal plane must not be the same as the dispersion period, but it can be two times longer. Thus one should design the alternating-bend linear-optics channel such that the dispersion and wedges keep the momentum spread small (~2.5%) and such that the dispersion is small enough at the absorbers to prevent a straggling impact on the horizontal emittance, but large in between absorbers where sextupoles can be introduced to compensate chromatic detuning

It is advantageous to use a symmetric lattice to simplify the design of the linear optics and the addition of aberration correction elements. The symmetry relative to the absorbers can be chosen naturally with the bend curvature *K* and dispersion *D* each antisymmetric and K2 and the field index n each symmetric. Note that the focus point of the parametric resonance should be slightly shifted off the zero dispersion point since some small dispersion is needed at the absorbers in order to create the emittance exchange required for longitudinal cooling.

In the following we choose the betatron wave length in the vertical plane to be half that of the horizontal plane (for combined function magnets with constant field index, the related condition is $5n \approx 4l$). Such a design suggests a few advantages as discussed below.

PIC WITH COUPLING RESONANCES

The desired driving parametric resonance can be arranged in two planes by modulation of strength and direction of the gradient field. Here we derive the equations for a perfectly tuned snake channel. The process can be described by equations as follows:

$$y'' + 4k^2 y = 4k^2(\zeta y \sin 4ks + \eta x \sin ks) + f_y$$

$$x'' + 4k^2 x - \frac{K\Delta p}{p} = 4k^2(-\zeta x \sin 4ks + \eta y \sin ks) + f_x$$

Here ζ and η are small numbers characterizing strength of the parametric and coupling resonance, respectively; f_x and f_y are x and y components of the transverse interaction force with the absorbers, including both the systematic and stochastic parts. In the absence of all perturbation forces, the solution of the equations can be written in ordinary form:

$$y = a\cos 2ks + b\sin 2ks$$
$$y' = -2ka\sin 2ks + 2kb\cos ks$$
$$x = D(\Delta p/p) + c\cos ks + d\sin ks$$
$$x' = D'(\Delta p/p) - kc\sin ks + kd\cos ks\,.$$

Here a and $2kb$ represent particle coordinate and angle in the y-plane at points of the beam orbit where we put absorbers. So, c and kd represent the "betatron part" of particle coordinate and angle at these points in the x-plane. We then rewrite equations of motion in these terms:

$$2ka' = -F_y sin2ks$$
$$2kb' = F_y \cos 2ks$$
$$2kc' = -F_x \sin ks - (D\cos ks - D'\sin ks)(p'/p)$$
$$2kd' = F_x \cos ks - (D\sin ks + D'\cos ks)(p'/p)\,.$$

Considering the effects of energy loss in the wedge absorber plates, we find a systematic change of particle energy

$$\Delta\gamma' = \frac{\partial\gamma'}{\partial\gamma}\Delta\gamma + \frac{\partial\gamma'}{\partial x}x = \frac{\Lambda}{2\nu^2}\left(\frac{2}{\gamma^2} - \frac{D_a}{h}\Delta\gamma\right)\cdot$$

Here we introduced the parameter h as an effective height of the absorber wedge:

$$h^{-1} = \frac{1}{\gamma'}\frac{\partial\gamma'}{\partial x},$$

By averaging the equations of motion along the beam path in the PIC channel, we obtain the following:

$$a' = (k/2)(-\zeta a + \eta c); \quad c' = -(k/2)\eta a) + \frac{D_a\Lambda}{h\nu^2}c$$

$$b' = (k/2)(\zeta b + \eta d) - \frac{\Lambda}{2\nu^2}b\,; \quad d' = (k/2)\eta b - \frac{\Lambda}{2\nu^2}d$$

Note that these equations do not take into account angle scattering and energy straggling in the absorber.

We are pursuing an arrangement that makes all three decrements equal to $\Lambda/3$. For this, the coupling resonance must be strong, i.e. $\eta \gg \zeta$. The optimization of cooling leads to the following conditions:

$$\frac{D_a}{h} = 2 - \frac{4}{3}\nu^2; \quad \frac{\zeta k}{\Lambda} = \frac{1}{\nu^2} - \frac{1}{3}$$

According to these equations the beam experiences an exponential shrinkage at points where we put absorbers. The angle spread would grow exponentially if it were not constrained by the ionization cooling due to the absorbers.

Under the conditions $D' << 2\left(\frac{Z+1}{\gamma^2+1}\log\right)^{1/2}$ and $h \ll \frac{w}{(1-2\nu^2/3)}\left[\frac{(Z+1)\log}{12(\gamma^2+1)}\right]^{1/2}$ the equilibrium normalized transverse emittance and beam size σ will be close to minimum values:

$$\varepsilon_\perp \Rightarrow \varepsilon_{\perp 0} = \frac{\sqrt{3}}{4\nu}(Z+1)\frac{m_e}{m_\mu}w, \quad \sigma = \frac{\theta w}{2\sqrt{3}}\cdot$$

Here w is the thickness of the wedge absorber at the reference orbit, and θ is the angle spread at the absorber.

COMPENSATION FOR ABERRATIONS

Compensation for Chromatic Aberrations

The chromatic terms to be compensated are seen in the third power terms of the expansion of the Hamiltonian. The compensation problem is greatly simplified for a lattice which is symmetric about the midpoint between absorbers. Taking this symmetry into account, the compensation conditions are reduced to only two equations:

$$< Dn_s u_y^2 > = < \frac{1}{2}(1-KD)u_y'^2 + \beta_3 u_y^2 + K'Du_y u_y' >$$

$$< Dn_s u_x^2 > = < \frac{1}{2}(KD-1)u_x'^2 - \gamma_3 u_x^2 + KD'u_x u_x') >,$$

where we have introduced the following notation:

$$\beta_3 = D(3\alpha_3 - \frac{1}{2}Kn)$$

$$\gamma_3 = 3D[\frac{1}{2}K(K^2+n) + \alpha_3]$$

$$\alpha_3 = \frac{1}{12}(K'' - 3K^2 - Kn)$$

The parameter n_s is the field index of the introduced sextupole components, and the functions u_x and u_y represent the oscillation modes antisymmetric about the absorber points. To satisfy these two equations, the compensating sextupole field should be designed by reflecting the field behavior along the beam path by two harmonics, $\sin 2ks$ and $\sin 4ks$.

Compensation for Geometrical Aberrations

The spherical and some other geometrical terms to be compensated are seen in the fourth power terms of the expansion of the Hamiltonian. The main compensation conditions are connected to the 4th power of the oscillation modes that are antisymmetric about the absorber points of the snake orbit (the symmetric oscillation components damp!). These conditions are:

$$8 < (K\alpha_3 + \alpha_4 + \frac{1}{3}Kn_{sext} + \frac{1}{4}n_{oct})u_x^4 > = < u_x'^4 >$$

$$8 < (\frac{1}{4}n_{oct} - \alpha_4)u_y^4 > = < u_y'^4 >$$

$$4 < (3K\alpha_3 + \beta_4 - Kn_{sext} - \frac{3}{2}n_{oct})(u_x u_y)^2 > = < (u_x' u_y')^2 >$$

Here we have used the following notations:

$$\alpha_4 = (K^2 n - n'' + \delta)/24; \ \beta_4 = -\frac{3}{2}K\alpha_3 + \frac{1}{4}\delta$$

$$\delta = (9K^4 + 2K^2 n - 4K'^2 - 6KK'')/2,$$

and n_{oct} is the field index of the introduced octupole components. The compensating octupole field design should include a constant component and the two lowest harmonics of the lattice period, $\cos 2ks$ and $\cos 4ks$.

Compensation for Higher Order Aberrations

More terms of the 4th power on transverse variables arrive as a second order effect from the 3rd power terms in the Hamiltonian. Finally, there are several higher power terms in the expansion of the Hamiltonian function which may also require compensation. The compensation for all these terms can be greatly simplified by the use of a coupling resonance, as introduced above. Namely, beam rotation by coupling effectively makes the higher order aberrations axially symmetric to drastically reduce the number of independent aberration parameters to be compensated. This reduction becomes critical while approaching equilibrium at the end of the PIC process.

TUNE SPREAD FROM SPACE CHARGE

Since the tune shift due to space charge is supposed to be small (as for any source of tune spread), the effect can be calculated by perturbation methods. Calculating from first principles the phase advance per betatron period for particles near the beam longitudinal center, we find an estimate of tune shift and tune spread over the beam:

$$\Delta\nu = \frac{\Delta\beta}{\beta} \approx \frac{2Nr_e}{3(Z+1)\gamma^2\sigma_z\sqrt{2\pi}}.$$

It should be noted that the tune spread due to space charge is determined in PIC not by the beam transverse phase space, but by the beam characteristic size between absorbers and the beam optics along the PIC channel. It is determined simply by the beam angle spread at the absorber, which is not influenced by focusing.

For 10^{11} particles/bunch as needed for acceleration and high luminosity collider designs, the tune spread due to space charge in a PIC channel at 100 MeV/c is about 10^{-3}, while for a full PIC effect the tune spread should not exceed 10^{-4}. To overcome the space charge impact, one can implement a beam recombining scheme where several bunches of lower charge are cooled at low energy and then recombined after acceleration [4].

CONCLUSIONS AND OUTLOOK

We have suggested a particular transport scheme using alternating bends to achieve the two requirements of small dispersion at absorbers and large dispersion where aberrations can be compensated. In addition, the use of a conservative coupling resonance is proposed with the purpose of providing equal parametric resonance and cooling rates in the two planes of the beam transport line, and of simplifying the aberration compensation design and control.

We are developing RF schemes to provide regeneration of the energy lost in the absorbers, to use synchrotron motion as an additional correction to chromatic effects, and to include wedge engineering limitations. In order to approach a practical PIC design, the cooling channel parameters can be modified as the beam is cooled.

In order to reduce the required beam path to decrease beam loss due to muon decay, PIC may start with reasonably thick absorbers, then the absorber thickness can be reduced as the parametric resonance beam envelope develops. The initial cooling rate may be limited by the available accelerating RF power. To alleviate this limitation, the beginning cooling sections could be designed effectively isochronous (with no RF cavities), where the strength of magnets scales with decreasing beam momentum. The beam energy could then be restored by RF cavities installed between cooling sections. Such designs also ease the challenge of implementing RF into the PIC magnetic structure. At the beginning and during the middle PIC stages, high precision compensation for aberrations is not required since the beam is not so well focused. Thus the design of the PIC channel for the initial stages can be relatively easy compared to the final stage.

We recently also started to study PIC in an *epicyclic* helical cooling channel [5]. This new PIC concept seems to be a way to combine the advantages of muon beam transport and 6D cooling in a continuous magnetic field (similar to HCC) with alternating dispersion needed to compensate for chromatic aberrations.

Finally, we note that the realization of the PIC concept would allow one to implement another idea to reduce transverse emittance at the expense of the longitudinal emittance, *reverse emittance exchange* (REMEX). PIC and REMEX used together would reduce the final normalized emittance of colliding muon beams to a value of about 1-2 mm·mrad, which is similar to emittances of existing or projected hadron beam colliders.

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