Abstract

Insertion devices can produce effects reducing the dynamic aperture in a storage ring. To study these effects for the ALBA light source the following insertion devices were introduced in the ALBA lattice: a superconducting wiggler SC-W31 with 31-mm-period and 2.1-T-field amplitude, and two Apple-II type PMM NdFeB undulators with periods of 62 mm (HU62) and 71 mm (HU71). Results of numerical study of the nonlinear beam dynamics by a 6D computer code are presented.

INTRODUCTION

Insertion devices can produce effects reducing the dynamic aperture in a storage ring. To study these effects for the ALBA light source [1] the following insertion devices are considered:

- A superconducting wiggler SC-W31 with 31-mm-period and 2.1-T-field amplitude [2]
- Two Apple-II type PMM NdFeB undulators with periods of 62 mm (HU62) and 71 mm (HU71) [3]

Two Apple-II type undulators are considered in the circular polarization mode only because as it is expected this mode provides strongest effect on the beam dynamics. Other wiggles and undulators with weaker magnetic field should not influence the dynamic aperture much and thus we do not consider them in this document.

A simplectic computer algorithm for particles tracking through the realistic wiggler field is based on the Verlet scheme and is implemented in the Acceleraticum tracking code [4].

SC WIGGLER

The wiggler has 121 dipole magnets with the regular magnetic field (~60 periods) and two side poles with the half field integral of the main poles. The main wiggler parameters are: the nominal peak amplitude of 2.1 T, the period length of 31 mm with the pole gap of 12.4 mm. The transverse field homogeneity is $\left| \frac{\Delta B_z}{B_z} \right| \leq 0.5\%$ at $x = \pm 10$ mm.

The following strategy is applied below to study the wiggler influence to the beam dynamics:

- A simple wiggler model based on a sine and piece-wise representation of the field is elaborated and inserted into the storage ring lattice providing information on the linear wiggler effects and allowing to compensate the linear focusing effect by set of quadrupoles
- The linear optics distortion (even been corrected) in the presence of strong chromatic sextupoles reduces the dynamic aperture and this case is studied numerically by tracking
- Simplectic tracking algorithm with the realistic field map is used to study the wiggler influence generally. Before that a small readjustment of the linear wiggler effect to minimize the tune shift and the beta beating is provided

To construct the linear wiggler model we follow the procedure described in [5]: the sine wiggler field is represented by an array of the piece-wise rectangular dipoles with a drift in between. Two free parameters, the dipole field and length, permit to satisfy the following conditions: (a) conservation of the pole bending angle, (b) conservation of the damping integral, (c) providing the correct focusing effect. These requirements yields the following parameters of the wiggler model

$$B_w = \frac{\pi}{4} B_{w0} \quad \text{and} \quad L_w = \frac{8}{\pi^2} \left( \frac{\lambda_w}{2} \right).$$

One can easily check that the edge focusing of such rectangular magnets produces the same tune shifts as the periodic wiggler field. The model magnets parameters in our case are:

$$B_{w0} = 1.649 \text{ T} \quad \text{and} \quad L_w = 12.564 \text{ mm}.$$ Of course this simple model can not represent all the wiggler effects to the beam motion; however it seems quite adequate to estimate major of them and to recover linear optics. More accurate consideration will be made below by the simplectic integration technique.

Fig.1 The ALBA-25 betatron functions beating (%) due to the SC wiggler insertion before and after correction

The wiggler model has been inserted in one of the M_ID straight sections (the length is 4.193 m) with the low central betatron functions ($\beta_{x0} = 2 \text{ m}$ and $\beta_{y0} = 1.3 \text{ m}$) to reduce the wiggler influence to the beam parameters.
The betatron tunes with and without wiggler are 18.179/8.377 and 18.179/8.372. As was expected from the sine model, the wiggler does not produce the horizontal tune shift while the vertical one $\Delta \nu_z = 5 \times 10^{-3}$ corresponds to the well-known sine-model estimation [6].

The vertical beta distortion can be easily recovered by two quadrupole doublets, adjacent to the wiggler section while the regular lattice quadrupoles corrects the tune shift (Fig.1).

The dynamic aperture with the SC wiggler is shown in Fig.2. This plot (and plots below) refers to the middle of the straight section with the betatron functions $\beta_x = 11.2$ m and $\beta_z = 5.9$ m.

As the linear and the nonlinear wiggler demonstrate approximately same decrease of the aperture, one may conclude that the main source of the reduction is the machine symmetry breaking with respect to the strong chromatic sextupoles pattern. However due to the low betas at the wiggler azimuth the effect is rather small.

The undulators were inserted in the straight section with the central betas $\beta_x = 11.2$ m and $\beta_z = 5.9$ m.

The undulators field map required for the simulation was supplied by J.Campmany. With the given end section field we have failed to obtain in the undulators the trajectory with zero angle/drift at the undulator start/end. An example of the trajectory in the EU 62 undulator is shown in Fig.4. However the trajectory drift inside the device is small (tens of microns) and can not influence the non-linear beam motion. The orbit displacement/angle is corrected at the undulator ends artificially by two point-like steering magnets.

**HELICAL UNDULATORS**

Two APPLE-II undulators in the circular polarization mode with parameters given in Table 1 have been studied from the viewpoint of their influence to the dynamic aperture.

<table>
<thead>
<tr>
<th>ID</th>
<th>B (T)</th>
<th>$\lambda_u$ (mm)</th>
<th>$N_{\text{per}}$</th>
<th>$g$ (mm)</th>
<th>L(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HU 62</td>
<td>0.51</td>
<td>62</td>
<td>23</td>
<td>15.5</td>
<td>1.5</td>
</tr>
<tr>
<td>HU 71</td>
<td>0.57</td>
<td>71</td>
<td>22</td>
<td>15.5</td>
<td>1.675</td>
</tr>
</tbody>
</table>

The undulators were inserted in the straight section with the central betas $\beta_x = 11.2$ m and $\beta_z = 5.9$ m.

As the undulators distort the optical functions, a special correction algorithm was applied to the lattice. As the Acceleraticum code has no optical module for the vertical bends, we have constructed two $6 \times 6$ matrices (one for the horizontal and one the vertical planes) from the results of the particle tracking with small initial amplitudes (the motion in this case is linear). These matrices, equivalent to the linear undulator model, were inserted in the ALBA lattice and produced the lattice functions distortion that is shown in Fig.5. The maximum relative deviation for the beta function is $\sim 2\%-4\%$ and for the dispersion function $\sim 6\%-9\%$. To cross-check the results of complicated tracking, we used the expression for the betatron tune shift for the helical undulator found in [7]

$$
\Delta \nu_{z,s} = \left( \frac{k_x^2 + k_y^2}{8 \pi \rho^2 k^2} \right) L \beta_{z,s} \left[ 1 + \frac{1}{12} \left( \frac{L}{\beta_{z,s}} \right)^2 \right],
$$

where $\beta_{z,s}$ is the corresponding betatron function in the middle of the straight section. A comparison of above estimation with the tracking results is given in Table 2.
and shown reasonable (the analytic formula does not take into account the transverse field roll-off) consistence.

Table 2 Linear tune shift induced by the undulator field

<table>
<thead>
<tr>
<th></th>
<th>EU 62</th>
<th>EU 71</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theory</td>
<td>$-1.4 \times 10^{-3}$</td>
<td>$2.5 \times 10^{-3}$</td>
</tr>
<tr>
<td>Simul</td>
<td>$-2.4 \times 10^{-3}$</td>
<td>$3.2 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

The betas distortion was recovered locally by two nearest quadrupole doublets symmetrically placed up- and down stream the undulator. A resulting betatron tune shift was cancelled by all regular quadrupoles in the ring. The results of the lattice functions restoring are also demonstrated in Fig.5; the beta beating was compensated outside the ID at the level ~0.3% and the dispersion function at the level ~1%.

The dynamic aperture reduction for on- and off-momentum particles is shown in Figs.6 and 7. Narrow vertical “peaks” at the DA border indicate stable islands of the high order coupling resonances.

**CONCLUSIONS**

Location of a strong field wiggler in the low-beta straight section is essential to reduce its influence to a storage ring performance. In spite of rather accurate restoring of the lattice functions (<0.5%), distorted by the SC wiggler, their residue distortion is the main source of the DA limitation because of the betas and phase advance disbalance in the strong chromatic sextupoles. Hence, fine adjustment of the linear optics in this case is important.

Despite the field amplitude in the helical undulators much smaller then that in the SC wiggler, the undulators provide comparable (and even larger) reduction of the dynamic aperture. A possible explanation could be in the strong nonlinear coupling producing relevant coupling resonances (one can see in Figs.6 and 7 that both apertures, vertical and horizontal shrink by approximately same ratio). More careful study with the undulators measured field map could be recommended.

**REFERENCES**