EXAMPLE CALCULATIONS OF UNDULATOR RADIATION

The wavelength of the radiation emitted in an undulator is given by Eq. 1.

\[ \lambda = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right) \quad (1) \]

Where \( \lambda \) is the wavelength of the emitted radiation, \( \lambda_u \) is the period length of the undulator, \( \gamma \) is the relative energy and \( K \) is the undulator parameter defined as \( K = 0.934 \cdot \lambda_u / \text{cm} \cdot B / \text{Tesla} \).

Of great interest, for instance for medical applications, is the X-ray region around 20 keV and higher. Typical \( K \)-values of undulators are in between 1 and 2. In order to achieve a certain wavelength the following equation has to be fulfilled:

\[ \gamma \approx \sqrt{\frac{\lambda_u}{\lambda}} \quad (2) \]

For example: if to achieve the photon energy of 20 keV (\( \lambda=0.62 \mbox{ \AA} \)) with an undulator with the period length \( \lambda_u=1 \mbox{ mm} \) a beam energy of 2 GeV would be required. The LBNL Laser-Plasma accelerator cited above does not yet achieve this energy. However ultra-short EUV and soft X-ray pulses would be achievable.

We calculate the expected spectra for a possible advanced superconducting undulator for the Munich LWF accelerator. This undulator is forseen to be based on Nb \(_3\)Sn superconducting wire. The assumed parameters are listed in Table 2.

Very important factors are the energy reproducibility and the energy spread of the electron beam, which is assumed to be \( \sim 10\% \). This might be an upper limit, but for...
Table 2: The parameters of a possible superconducting undulator for the Munich LWF accelerator

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Undulator period</td>
<td>5 mm</td>
</tr>
<tr>
<td>Periods</td>
<td>200</td>
</tr>
<tr>
<td>Gap width</td>
<td>2 mm</td>
</tr>
<tr>
<td>$B_{max}$</td>
<td>0.96 T</td>
</tr>
<tr>
<td>Undulator length</td>
<td>1 m</td>
</tr>
<tr>
<td>K-value</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Figure 1: The calculated first harmonics of the photon spectrum for an electron energy of 0.2 GeV (small peak on the left), 1.0 GeV and 1.4 GeV.

demonstration purposes this conservative value was chosen. Fig. 1 shows the first harmonics of the spectra calculated for 0.2, 1.0 and 1.4 GeV electron energy. The line-broadening effect of the energy spread in the electron beam is clearly visible for the higher energies.

**GENERATION OF MONOCHROMATIC PHOTON BEAMS**

The generation of monochromatic undulator radiation via the SASE effect requires an electron beam with a very low energy spread. The energy spreads achieved in LWF accelerators are one order of magnitude above the energy spread achieved in conventional accelerators. The high Pierce parameter resulting from the high peak current achievable in LWF accelerators helps to compensate the high energy spread. However, to achieve monochromatic light via the SASE effect relying on spontaneous emission, additional methods to compensate the high energy spread are necessary.

One possibility would be the spectral dispersion of the electron beam combined with an undulator with a laterally varying B-field, so that the effective K value changes with the transverse position. The spectral dispersion of the beam can be achieved for example either by deflecting the beam twice by a small angle or by a single deflection by 180°. An additional advantage of the beam deflection in general is, that it allows for an easy separation of the laser from the electron beam and that it facilitates the vacuum design. I.e. getting rid of the remaining plasma before it enters the undulator.

Fig. 2 shows one possibility to achieve an undulator with laterally varying K value by tilting the undulator coils against each other. The electron beam is dispersed in a way that electrons with different energies ‘see’ a B-field and thereby a K-value matched to their energy. The necessary condition for the separation of different energies is, that the dispersion is larger than the beam emittance and the condition for the generation of monochromatic emission is:

$$\frac{1}{\gamma^2} \left( 1 + \frac{K^2}{2} \right) = \text{const.}$$  \hspace{1cm} (3)

From this equation the necessary range of the magnetic field strength to compensate the energy spread can be calculated. Assuming a $\gamma$ of 400 and a K factor of 2, the value of K has to increase to 2.293 if the energy increases by 10%. With $\lambda_u$ fixed, the relative change in the magnetic field is the same as for the K value. In this case the magnetic field variation is 14.7%.

Another possibility to achieve a varying K-value is a lateral variation of the period length. In this case the necessary change in period length can be calculated from Eq. 1. With $\lambda$ and B fixed, $\lambda_u$ has to increase by 7.4% to compensate for a 10% higher energy. Of course both methods can be combined if this proves necessary.

A third possibility would be to use an undulator geometry with an inherently strong field gradient. For example cylindrical undulator coils with the electron beam entering the undulator at a small lateral displacement. Fig. 3 shows a visualisation of the period length variation and the cylindrical design options. A possible drawback of the cylindrical undulator design would be that the K value does not vary linearly with lateral position. That means that it becomes more difficult to match the dispersion to the variation of the K value.

**PHOTON NUMBERS**

If the energy spread can be compensated as described above both the spectrum and the brilliance of the emitted light offer unique experimental opportunities. In this section the spectrum and brilliance for an example undulator are calculated under the assumption that the effect of the

Figure 2: A schematic drawing of the spectral dispersion of the electron beam, combined with a tapered undulator to achieve a laterally varying B-field.
energy spread can be reduced by two orders of magnitude, so that an effective energy spread of 1‰ is achieved.

Fig. 4 shows the calculated spectrum, including the higher harmonics, for the undulator detailed in Table 2 and an electron beam with the normalised emittance of the Munich accelerator and 1 GeV energy. Fig. 5 shows the calculated brilliance for the same undulator. The photon number in the first harmonic is of the order $10^{16}$ in 10 fs and the peak brilliance is $\sim 10^{23}$.

These values are calculated with the natural emittance and source size of the Munich accelerator. If magnet optics are used to fit the $\beta$ factor of the beam to the undulator, the harmonic peaks become much more narrow and the photon number in the first harmonic increases to $2.8 \times 10^{17}$ while the peak brilliance stays essentially the same.

These results show that LWF accelerator driven, superconductive undulators could be a very promising source for ultra-short time physics and other applications requiring a high brilliance.

**SUMMARY**

Laser-plasma / Laser-Wakefield (LWF) accelerators together with specially designed superconductive undulators can be an excellent light source for future femto-second experiments. The comparatively high energy spread of the LWF accelerators can be compensated up to a certain degree by an energy dispersion of the electron beam by a chicanes and specially adapted undulator designs. However, due to the high bunch charge the effect of the emission of synchrotron radiation in the chicanes may have a non-negligible effect, which might pose a limit on this technology. These effects and the actual undulator design are still under investigation.

**REFERENCES**


